Math-1050

Session #16

Real Zeroes of Polynomial Functions

$$f(x) = x^3 - 4x^2 - 15x + 18$$

This isn't one of the "nice" 3rd degree polynomials:

a) Sum of two cubes: $y = x^3 + 27$

b) Difference of two cubes: $y = 8x^3 - 125$

c) 3rd degree with no "constant" term.

$$y = x^3 - 4x^2 - 12x$$

d) "nice" pattern that can be factored by grouping (or box):

$$y = x^3 + 3x^2 + 3x + 1$$

We are left with "guessing" the first degree polynomial that divides it evenly!

What are the zeroes? $0 = (2x-5)(3x+7)$ $x = \frac{5}{2}, \frac{-7}{3}$

Multiply the two binomials (convert to standard form)

$$0 = 6x^2 - x - 35$$

What do you notice about the first and last terms and the zeroes?

$$0 = 6x^2 - x - 35$$

$$x = \frac{5}{2}, \frac{-7}{3}$$

5(-7) = -35

2(3) = 6

What are the zeroes? $0 = (x-3)(2x+1)(x-4)$

$$x = 3, \quad \frac{-1}{2}, \quad 4$$

Multiply the three binomials (convert to standard form)

$$0 = 2x^3 - 13x^2 + 17x + 12$$

What do you notice about the first and last terms and the zeroes?

$$0 = 2x^3 - 13x^2 + 17x + 12$$

$$x = 3, \quad \frac{-1}{2}, \quad 4$$

$$\pm\frac{1, 2, 3, 4, 6, 12}{1, 2} \qquad \pm\frac{1, 2, 3, 4, 6, 12}{1, 2} \qquad \pm\frac{1, 2, 3, 4, 6, 12}{1, 2}$$

The Rational Zeroes Theorem: the possible rational zeroes of a polynomial are factors of the constant divided by factors of the lead coefficient.

$$0 = 6x^2 - x - 35$$

$$x = \pm\frac{1, 5, 7, 35}{1, 2, 3, 6} \qquad x = \frac{5}{2}, \quad \frac{-7}{3}$$

$$x = \pm 1, 5, 7, 35, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{5}{2}, \frac{7}{2}, \frac{7}{3}, \ldots, \frac{35}{3}, \frac{35}{6}$$

What are the “possible” rational zeroes of the following polynomial?

$$y = x^3 - 4x^2 - 15x + 18$$

$$x = \pm \frac{1, 2, 3, 6, 9, 18}{1} \qquad x = \pm 1 \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$$

If x = 1 is a “zero”, what factor did x = 1 come from?

(x – 1)

If (x – 1) is a factor, then the 3rd degree factors as:

$$y = (x-1)(ax^2 + bx + c)$$

Does (x – 1) divide the polynomial evenly?

$$x^3 - 4x^2 - 15x + 18 \quad = (x-1)(ax^2 + bx + c)$$

Divide left/right sides by (x – 1)

$$\frac{x^3 - 4x^2 - 15x + 18}{(x-1)} = \frac{(x-1)(ax^2 + bx + c)}{(x-1)}$$

$$\frac{x^3 - 4x^2 - 15x + 18}{(x-1)} = ax^2 + bx + c$$

Steps in finding the zeroes of a polynomial (particularly one that is *not easily factored)*.

$$y = x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 16$$

1. Determine the *possible rational zeroes.* $x = \pm\frac{1, 2, 4, 8, 16}{1}$

2. Check each possible zero until the you find the first one that results in a remainder of zero.

1	1	-5	12	-24	32	-16
		1	-4	8	-16	16
	1	-4	8	-16	16	0

$$y = (x - 1)(x^4 - 4x^3 + 8x^2 - 16x + 16)$$

3. Same possible zeroes $x = \pm \frac{1, 2, 4, 8, 16}{1}$

4. Find the next zero.

1)	1	-5	12	-24	32	-16
		1	-4	8	-16	16
	1	-4	8	-16	16	0

1)	1	-4	8	-16	16
		1	-3	5	-11
	1	-3	5	-11	5

'1' is not a repeat zero → x = 1 (multiplicity 1)

$$y = (x - 1)(x^4 - 4x^3 + 8x^2 - 16x + 16)$$

‘1’ is not a repeat zero → x = 1 (multiplicity 1)

5. Same possible zeroes $x = \pm \frac{1, 2, 4, 8, 16}{1}$

6. Find the next zero.

1	1	-5	12	-24	32	-16
		1	-4	8	-16	16
	1	-4	8	-16	16	0

2	1	-4	8	-16	16
		2	-4	8	-16
	1	-2	4	-8	0

$$y = (x-1)(x-2)(x^3 - 2x^2 + 4x - 8)$$

$$y = (x-1)(x-2)(x^3 - 2x^2 + 4x - 8)$$

7. Try "box factoring" of the 3rd – degree polynomial factor or continue trying synthetic division.

	x	-2
x^2	x^3	$-2x^2$
4	$4x$	-8

$$y = (x-1)(x-2)(x^2+4)(x-2)$$

8. List the zeroes.

x = 1, *2i, -2i, 2 (multiplicity 2)*

Descarte's Rule of Signs: The number of *positive zeroes* is the number of sign changes in between the coefficients of *f(x)* or is an even integer less than the number of sign changes.

$$f(x) = x^5 - 5x^4 + 12x^3 + 24x^2 - 32x + 16$$

1 sign change | 1 sign change | 1 sign change | 1 sign change

→ 4 sign changes

→ The *number of positive zeroes* is: 4 or 2

Descarte's Rule of Signs: The number of *negative zeroes* is the number of sign changes of between the coefficients of *f(-x)* or is an even integer less than the number of sign changes.

$$f(x) = x^5 - 5x^4 + 12x^3 + 24x^2 + 32x - 16$$

$$f(-x) = (-x)^5 - 5(-x)^4 + 12(-x)^3 + 24(-x)^2 + 32(-x) - 16$$

$$f(-x) = -x^5 - 5x^4 - 12x^3 + 24x^2 - 32x - 16$$

1 sign change

1 sign change

→ 2 sign changes

→ The *number of negative zeroes* is : 2 or 0