## Math-1050

## Session #16 Real Zeroes of Polynomial Functions

$$f(x) = x^3 - 4x^2 - 15x + 18$$

This isn't one of the "nice" 3<sup>rd</sup> degree polynomials: a) Sum of two cubes:  $y = x^3 + 27$ 

b) Difference of two cubes:  $y = 8x^3 - 125$ 

c) 3<sup>rd</sup> degree with no "constant" term.  $y = x^3 - 4x^2 - 12x$ 

d) "nice" pattern that can be factored by grouping (or box):

$$y = x^3 + 3x^2 + 3x + 1$$

We are left with "guessing" the first degree polynomial that divides it evenly!

What are the zeroes? 
$$0 = (2x-5)(3x+7)$$
  $x = \frac{5}{2}, \frac{-7}{3}$ 

Multiply the two binomials (convert to standard form)

$$0 = 6x^2 - x - 35$$

What do you notice about the first and last terms and the zeroes?

$$0 = 6x^{2} - x - 35$$

$$x = 5 - 7 5(-7) = -35$$

$$2(3) = 6 2^{2} - 3$$

What are the zeroes? 0 = (x - 3)(2x + 1)(x - 4) $x = 3, \quad \frac{-1}{2}, \quad 4$ 

Multiply the three binomials (convert to standard form)

$$0 = 2x^3 - 13x^2 + 17x + 12$$

What do you notice about the first and last terms and the zeroes?

$$0 = 2x^{3} - 13x^{2} + 17x + 12$$

$$x = 3, \quad \frac{-1}{2}, \quad 4$$

$$\bigoplus \frac{1, 2, 3, 4, 6, 12}{1, 2} \quad \bigoplus \frac{1, 2, 3, 4, 6, 12}{1, 2} \quad \bigoplus \frac{1, 2, 3, 4, 6, 12}{1, 2}$$

The Rational Zeroes Theorem: the <u>possible rational zeroes</u> of a polynomial <u>are factors of the constant</u> divided by <u>factors</u> <u>of the lead coefficient</u>.  $0 = 6x^2 - x - 35$ 

$$x = \pm \frac{1, 5, 7, 35}{1, 2, 3, 6} \qquad x = \frac{5}{2}, \ \frac{-7}{3}$$

$$x = \pm 1, 5, 7, 35, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{5}{2}, \frac{7}{2}, \frac{7}{3}, \dots, \frac{35}{3}, \frac{35}{6}$$

What are the "possible" rational zeroes of the following polynomial?

$$y = x^3 - 4x^2 - 15x + 18$$

$$x = \pm \frac{1, 2, 3, 6, 9, 18}{1} \qquad x = \pm 1 \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$$

If x = 1 is a "zero", what factor did x = 1 come from? (x - 1)

If (x - 1) is a factor, then the 3<sup>rd</sup> degree factors as:

$$y = (x-1)(ax^2 + bx + c)$$

Does (x - 1) divide the polynomial evenly?

$$x^{3} - 4x^{2} - 15x + 18 = (x - 1)(ax^{2} + bx + c)$$

Divide left/right sides by (x - 1)

$$\frac{x^3 - 4x^2 - 15x + 18}{(x - 1)} = \frac{(x - 1)(ax^2 + bx + c)}{(x - 1)}$$

$$\frac{x^3 - 4x^2 - 15x + 18}{(x - 1)} = ax^2 + bx + c$$

Steps in finding the zeroes of a polynomial (particularly one that is *not easily factored*).

$$y = x^{5} - 5x^{4} + 12x^{3} - 24x^{2} + 32x - 16$$
  
1. Determine the *possible rational zeroes.*  $x = \pm \frac{1, 2, 4, 8, 16}{1}$ 

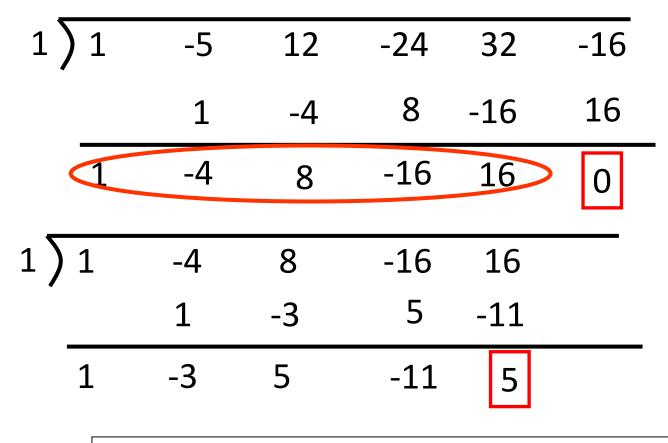
2. Check each possible zero until the you find <u>the first one</u> that results in a remainder of zero.

$1\sum$	1	-5	12	-24	32	-16	
		1	-4	8	-16	16	
-	1	-4	8	-16	16	0	
$y = (x - 1)(x^4 - 4x^3 + 8x^2 - 16x + 16)$							

3. Same possible zeroes x =

$$x = \pm \frac{1, 2, 4, 8, 16}{1}$$

4. Find the next zero.



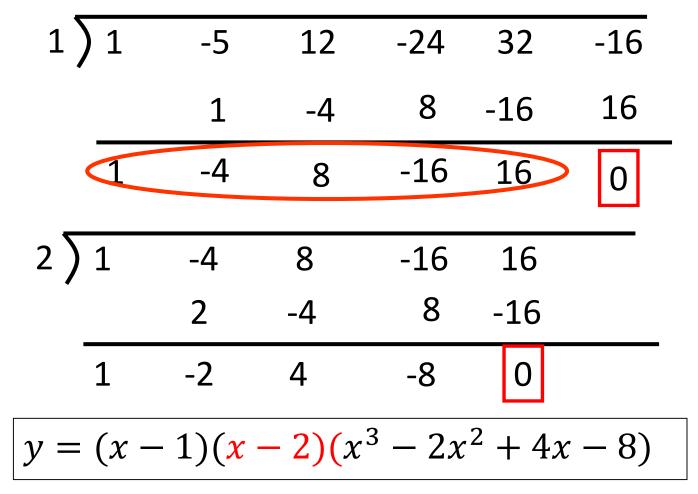
'1' is not a repeat zero  $\rightarrow x = 1$  (multiplicity 1)

$$y = (x - 1)(x^4 - 4x^3 + 8x^2 - 16x + 16)$$

'1' is not a repeat zero  $\rightarrow x = 1$  (multiplicity 1)

5. Same possible zeroes  $x = \pm \frac{1, 2, 4, 8, 16}{1}$ 

6. Find the next zero.



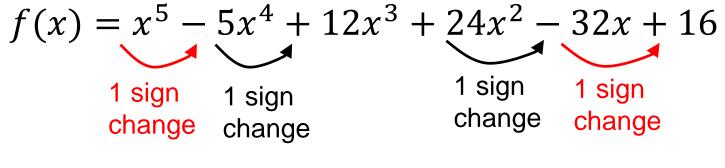
$$y = (x - 1)(x - 2)(x^3 - 2x^2 + 4x - 8)$$

7. Try "box factoring" of the 3<sup>rd</sup> – degree polynomial factor or continue trying synthetic division.

$$y = (x - 1)(x - 2)(x^2 + 4)(x - 2)$$

8. List the zeroes.

<u>Descarte's Rule of Signs</u>: The number of <u>positive zeroes</u> is the number of sign changes in between the coefficients of f(x) or is an even integer less than the number of sign changes.



 $\rightarrow$  4 sign changes

 $\rightarrow$  The <u>number of positive zeroes</u> is: 4 or 2

Descarte's Rule of Signs: The number of *negative zeroes* is the number of sign changes of between the coefficients of f(-x) or is an even integer less than the number of sign changes.  $f(x) = x^5 - 5x^4 + 12x^3 + 24x^2 + 32x - 16$  $f(-x) = (-x)^5 - 5(-x)^4 + 12(-x)^3 + 24(-x)^2 + 32(-x) - 16$  $f(-x) = -x^5 - 5x^4 - 12x^3 + 24x^2 - 32x - 16$ 1 sign 1 sign change change

 $\rightarrow$  2 sign changes

→ The *number of negative zeroes* is : 2 or 0