Math-1050

Session #15

R6: Synthetic Division

Find the zeroes of the following 3rd degree Polynomial

$$y = x^3 + 5x^2 + 4x$$ Set y = 0

$$0 = x^3 + 5x^2 + 4x$$ Factor out the common factor.

$$0 = x(x^2 + 5x + 4)$$ Factor the quadratic

$$0 = x(x+1)(x+4)$$ Identify the zeroes

0, -1, -4

An easy method is "box factoring" (if it works).

$$y = 1x^3 + 2x^2 + 2x + 4$$

The 4 terms on the right side are the *terms in the box*.

Find the *common factor* of the 1st row.

Fill in the rest of the box.

	x	2
x^2	x^3	$2x^2$
2	$2x$	4

Rewrite in intercept form.

$$y = 1x^3 + 2x^2 + 2x + 4$$

$$y = (x^2 + 2)(x + 2)$$

Find the "zeroes."

$$0 = (x^2 + 2)(x + 2)$$

$$0 = x^2 + 2 \qquad 0 = x + 2$$

$$-2 = x^2 \qquad x = -2$$

$$x = \pm i\sqrt{2}$$

Sometimes "box factoring" doesn't work.

$$y = 4x^3 - 5x^2 + 10x - 15$$

	$4x$	-2
x^2	$4x^3$	$-5x^2$
2.5	$10x$	-15

$$(-2)(2.5) \neq -15$$

How can we find zeroes of higher-degree polynomials, if
(1) there is no common factor of 'x' (2 slides ago) or
(2) "Box Factoring" doesn't work (1 slide ago)?

Divide Evenly: the remainder when dividing will be zero.

Multiply: $(x+1)(x+4) \rightarrow x^2+5x+4$

Is (x + 4) a factor of: x^2+5x+4 ? Obviously YES

If an expression is a factor of another expression, can we say the factor divides evenly? Try it. Divide x^2+5x+4 by (x + 4)

$$\begin{array}{r|l} & x \quad +1 \\ \hline x+4 & x^2+5x+4 \\ & -(x^2+4x) \\ \hline & \qquad x+4 \\ & \quad -(x+4) \\ \hline & \qquad 0 \end{array}$$

The remainder = 0

I just proved The Factor Theorem

Is (x + 1) a factor of: x^2+5x+4 ?

YES, according to the Factor Theorem.

The Factor Theorem If a polynomial f(x) is divided by (x – k), and the remainder is "0," then (x – k) is a factor of the original polynomial. $f(x) = x^2 + 5x + 4$ divided by (x + 4) = 0

Therefore: (x + 4) is a factor of $f(x) = x^2 + 5x + 4$

What are the zeroes of: $f(x) = x^2 + 5x + 4$

$$0 = (x + 1)(x + 4)$$

$$x = -1, -4$$

→ The zeroes of the factors are the zeroes of the polynomial.

How can we find zeroes of higher-degree polynomials, if
(1) there is no common factor of 'x' (2 slides ago) or
(2) "Box Factoring" doesn't work (1 slide ago)?

We repeatedly test linear factors to see if they divide the polynomial evenly. If they do, we have found a factor, and the zero of that factor is a zero of the polynomial.

Synthetic Division

$$x-1\overline{)\,x^3-4x^2-15x+18}$$

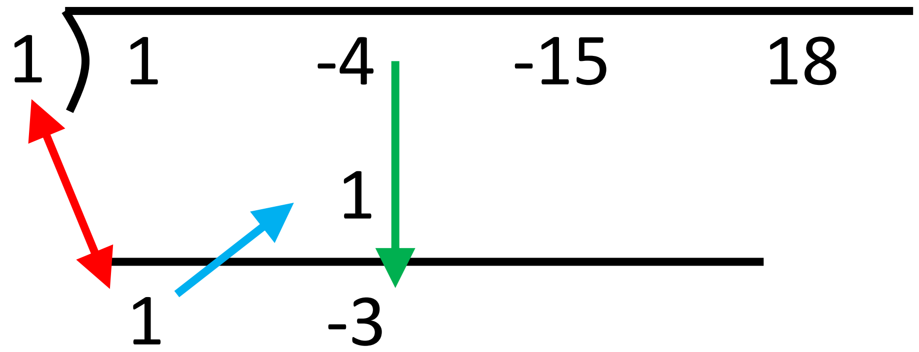
1	1	-4	-15	18
	1			

1st step: Write the polynomial with only its coefficients.

2nd step: Write the “zero” of the linear divisor.

3rd step: Bring down the lead coefficient

$$x-1\overline{)\,x^3-4x^2-15x+18}$$

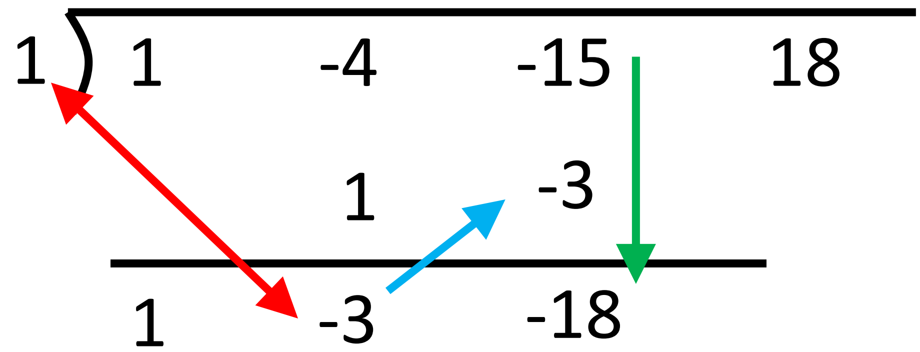


4th step: Multiply the “zero” by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

$$x-1\overline{\big) x^3-4x^2-15x+18}$$

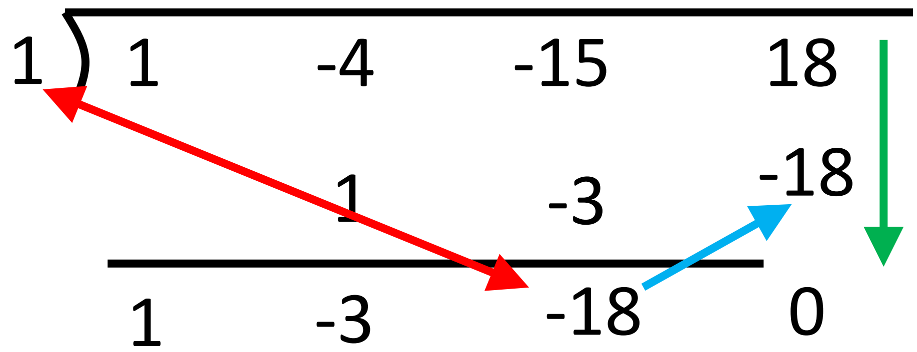


7th step: Multiply the “zero” by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

$$x-1\overline{\smash{\big)}\,x^3-4x^2-15x+18}$$



10th step: Multiply the “zero” by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

$$x-1 \overline{\big) \; x^3 - 4x^2 - 15x + 18}$$

1)	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

remainder = 0

x^2

Quotient

$$(x^3-4x^2 - 15x + 18) \div (x - 1) \;\; = x^2 - 3x - 18$$

Because the remainder = 0, then (x – 1) is a factor AND x = 1 is a zero of the original polynomial.

The “gotcha” of Synthetic Division is that you must account for each term of the polynomial.

$$x^2 - 4 = (x + 2)(x - 2)$$

$x - 2 \overline{)\ x^2 - 4}$

2	1	-4
		2
	1	-2

remainder not = 0

$x - 2 \overline{)\ x^2 + 0x - 4}$

2	1	0	-4
		2	4
	1	2	0

remainder = 0

$$f(x) = x^3 - x^2 + x - 1 \qquad f(2) = ?$$

$$f(2) = (2)^3 - (2)^2 + (2) - 1 \qquad \boxed{f(2) = 5}$$

Now here is something that is really cool.

$$x - 2 \overline{\smash{\big)} x^3 - x^2 + x - 1}$$

2	1	-1	1	-1
		2	2	6
	1	1	3	5

remainder = 5

$$(x^3 - x^2 + x - 1) \div (x - 2) = x^2 + x + 3 + \frac{5}{x - 2}$$

Because the remainder = 0, then (x − 1) is a factor AND x = 1 is a zero of the original polynomial.

$= x^2 - 3x - 18$

$$x - 2 \overline{\big)\, x^3 - x^2 + x - 1}$$

The remainder of division is the output value when the input is the zero of the divisor!

2	1	-1	1	-1
		2	2	6
	1	1	3	5

remainder = 5

$$(x^3 - x^2 + x - 1) \div (x - 2) = x^2 + x + 3 + \frac{5}{x - 2}$$

Now here is something that is really cool.

$$f(x) = x^3 - x^2 + x - 1$$

$$f(2) = ?$$

$$f(2) = (2)^3 - (2)^2 + (2) - 1$$

$$f(2) = 5$$

The Remainder Theorem If a polynomial f(x) is divided by (x – k), then the remainder "r" is: r = f(k)

$$f(x) = 2x^3 + 4x^2 - 3x - 6$$

f(x) ÷ (x + 3) = ?

What is the input into synthetic division? → k = -3

$$\frac{f(x)}{x+3} = 2x^2 - 2x + 3 - \frac{15}{x+3}$$

If the input to *f(x)* is (-3), what is the output value? *f(x)* = *-15*

We say *synthetic division* when we want to find a quotient and a remainder.

We say *synthetic substitution* when we want to find the output value of a function (*the remainder of synthetic division).*

We can find zeroes of polynomials by using synthetic division to see if the remainder is zero (to see if the factor divides “evenly”).

If $f(2) = 0$, then $(x - 2)$ is a factor.