Math-1050
Session #14
Polynomial and Rational Inequalities

In Textbook Section 4-5 We learned the "Boundary Numbers" Method of solving a quadratic inequality.

$$28 \le x^2 - 12x$$

When solving quadratic equations, we first rearranged the equation to be in standard form.

When solving quadratic inequalities, we first rearrange the inequality into standard form.

$$0 \le x^2 - 12x - 28$$

Find the boundary numbers → solve the equation.

$$0 = x^2 - 12x - 28$$

$$0 = (x - 14)(x + 2)$$

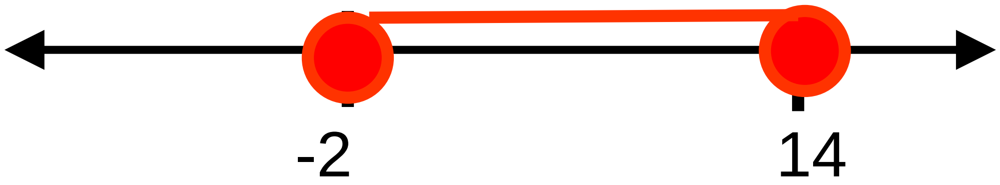
x = 14, -2

$$0 = x^2 - 12x - 28 \qquad 0 = (x-14)(x+2)$$

x = 14, -2

For two boundary numbers, the solution is either:

1) Between the boundary numbers or



2) Outside of the boundary numbers

Test a value to see if it is a solution. Zero is often the best number to test. $0 < (0)^2 - 12(0) - 28 \qquad 0 \leq -28$

Is "0" a solution? (does x = 0 make the inequality true?)

The shaded part of the graph is the solution

→ we must pick the option that "shades" the number "0".

$$(-\infty,\ -2] \cup [14,\ \infty)$$

Solve Single Variable *Polynomial* Inequalities

For 3^{rd} degree and higher degree polynomials, there are often more than two zeroes. So we need to work harder to find the solution.

Method 1: “Sign (+/-) Table” (Required for Math-1050)

Method 2: “Sign (+/-) Chart” (Required for Math-1050

Method 3: Graphically (required for Math-3 Only)

Method 1: "Sign (+/-) Table"

$0 \le x^2 - 12x - 28$ $0 < ($*positive* numbers$)$

1. Find the "real" zeroes of the polynomial equation.

$0 = x^2 - 12x - 28$ $0 = (x-14)(x+2)$ $x = -2, 14$

2. Identify the intervals (between the zeroes)

$$(-\infty, -2] \cup [-2, 14] \cup [14, \infty)$$

3. Build a table to organize the information

intervals	Input	(x – 14)(x + 2)	output	Solution?
$(-\infty, -2]$	-3	$(-)(-)$	$(+)$	yes
$[-2, 14]$	0	$(-)(+)$	$(-)$	no
$[14, \infty)$	15	$(+)(+)$	$(+)$	yes

4. Write the Solution in interval notation $x = (-\infty, -2] \cup [14, \infty)$

Method 2: "Sign (+/-) Chart"

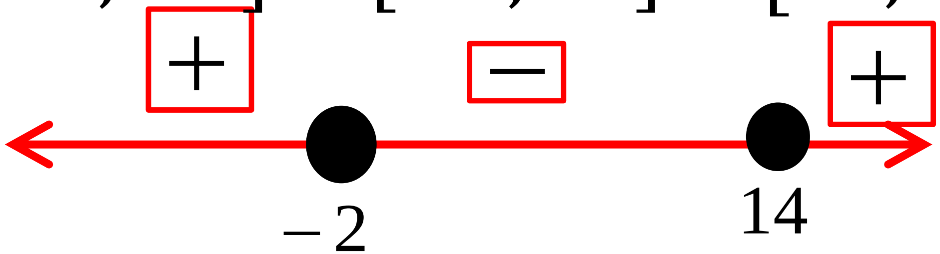
$$0 \le x^2 - 12x - 28 \qquad 0 \le (\textit{positive} \text{ numbers}) \qquad 0 \le (+)$$

$$0 = x^2 - 12x - 28 \qquad 0 = (x-14)(x+2)$$

1. Find the "real" zeroes of the polynomial equation. $x = -2, 14$

2. Label the intervals between the zeroes

$$(-\infty, -2] \cup [-2, 14] \cup [14, \infty)$$



3. Determine the sign (+/-) for each interval

$$(-)(-) = (+) \qquad (-)(+) = (-) \qquad (+)(+) = (+)$$

$$0 \le (x-14)(x+2)$$

4. Write the interval of x-values that make the inequality true (positive intervals).

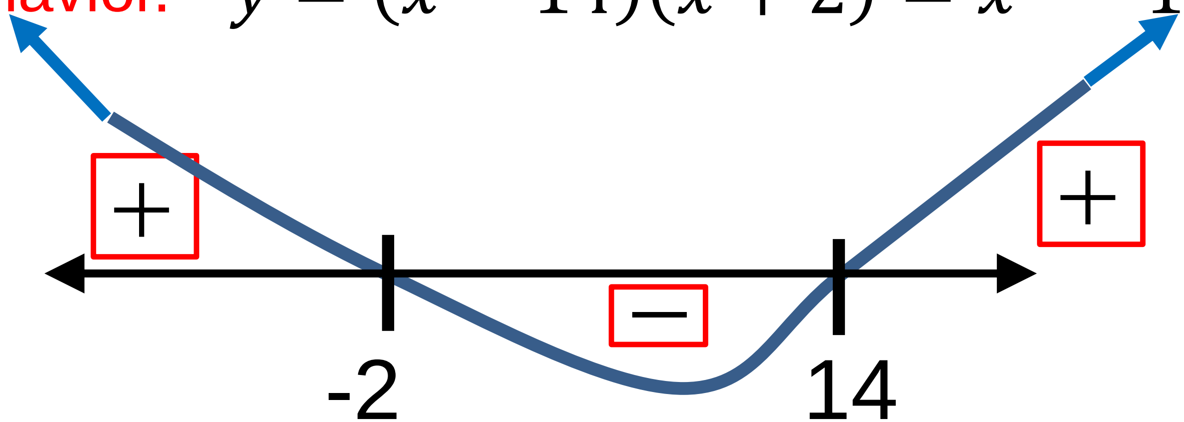
$$(-\infty, -2] \cup [14, \infty)$$

Method 2: "Sign (+/-) Chart"

$0 \le x^2 - 12x - 28$ $\quad$ $0 \le (\textit{positive} \quad \text{numbers})$ $\quad$ $0 \le (+)$

$0 = x^2 - 12x - 28$ $\quad$ $0 = (x-14)(x+2)$ $\quad$ $x = -2, 14$

You can build the *sign chart* by replacing '0' with 'y' then drawing a rough graph using the zeroes (and their multiplicities) and the end-behavior. $y = (x-14)(x+2) = x^2 - 12x - 28$



$(-\infty, -2] \cup [-2, 14] \cup [14, \infty)$

The expression will be positive for regions where the graph is above the x-axis (and negative below)

$x = (-\infty, -2] \cup [14, \infty)$

Method 3: Graphically

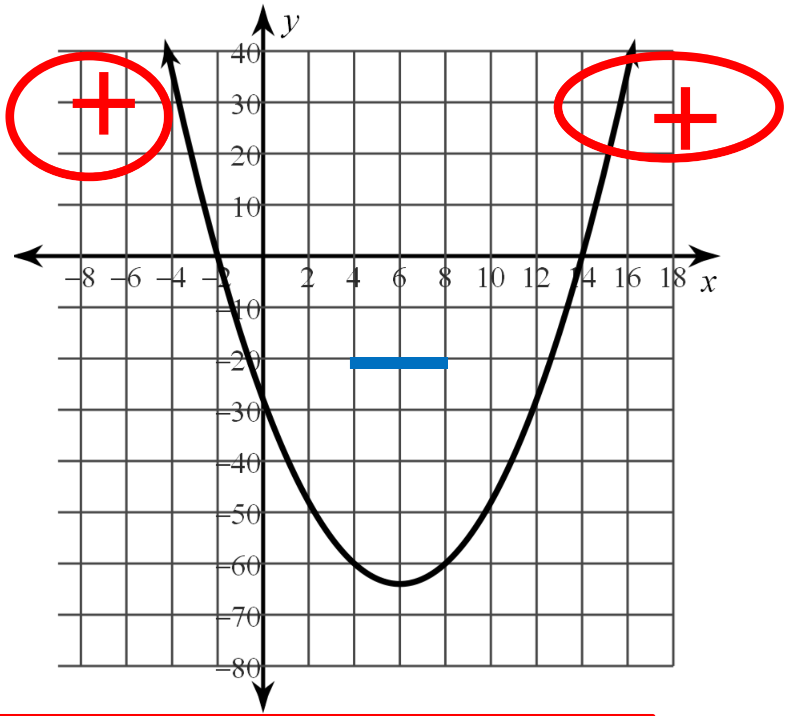
$$28 \le x^2 - 12x$$ Change to "standard form" inequality

$$0 \le x^2 - 12x - 28$$ Change to "standard form" equation

$$0 = x^2 - 12x - 28$$ Replace '0' with 'y'

$$y = x^2 - 12x - 28$$

Graph on calculator



Determine 'x-intercepts'

$x = -2$ $x = 14$

Build function "sign chart"

Solve $$0 \le x^2 - 12x - 28$$

$$0 \le (+\text{ numbers})$$

$$(-\infty, -2] \cup [14, \infty)$$

$0 < (x-1)(x+1)(x-2)$ $0 < (\textit{positive} \text{ numbers})$

1. Find the "real" zeroes of the polynomial equation.

$0 = (x+1)(x-1)(x-2)$ $x = -1, \ 1, 2$

2. Build "Sign (+/-) Table"

	Input	(x – 1)(x + 1)(x – 2)		
$(-\infty, -1)$	-2	$(-)(-)(-)$	$(-)$	no
$(-1, 1)$	0	$(-)(+)(-)$	$(+)$	yes
$(1, 2)$	1.5	$(+)(+)(-)$	$(-)$	no
$(2, \infty)$	3	$(+)(+)(+)$	$(+)$	yes

3. Write the solution.

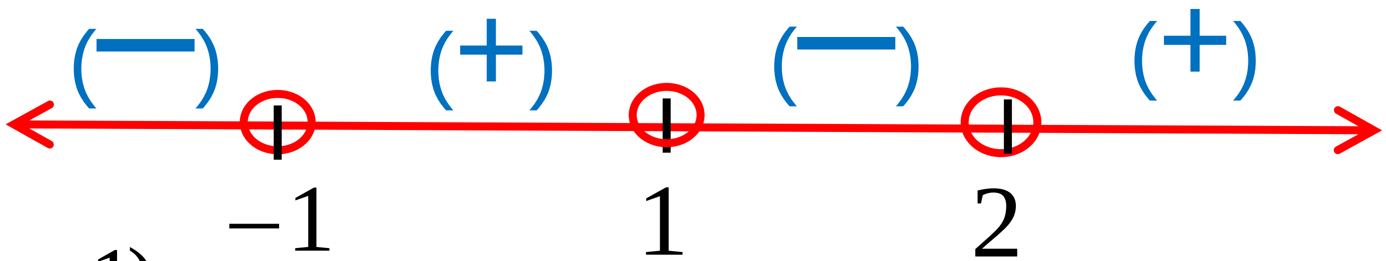
$$x = (-1, 1) \cup [2, \infty)$$

$0 < (x-1)(x+1)(x-2)$ $\quad 0 < (\mathit{positive}\ \text{numbers})$

1. Find the "real" zeroes of the polynomial equation.

$0 = (x+1)(x-1)(x-2)$ $\quad x = -1,\ 1\ ,2$

2. Build "Sign (+/-) Chart"



$x = (-\infty, -1)$

$f(x) = (x+1)(x-1)(x-2)$

$f(-2) = (\ -\ )(\ -\ )(\ -\ )$ $\quad f(-2) = (\ -\ )$

3. All zeroes are odd multiplicities → changes sign at each zero

4. Solve: $0 < (x-1)(x+1)(x-2)$ $\quad x = (-1, 1) \cup (2,\ \infty)$

$0 \geq (\textit{negative} \text{ numbers})$

$0 \geq (-)$

$0 \leq (\textit{positive} \text{ numbers})$

$0 \leq (+)$

Expect the following types of polynomais:

1. "Nice Pattern" → these are the numbers in the box

$$0 \geq x^3 + 2x^2 - 4x - 8$$

2. "Quadratic form" → factor using "m-substitution"

$$0 < x^4 - 10x^2 + 9$$

3. "'x' is a common factor" → factor

$$0 \geq x^3 + 6x^2 - 16x$$

Solving Single Variable Rational Inequalities

We will solve inequalities similar to the following:

$$0 < \frac{3(x+6)(x-2)}{(x+2)(x+1)}$$

$$0 \leq (\,positive\ \text{numbers})$$

$$0 \leq (+)$$

Solution: x-values that make the rational expression positive

$$0 > \frac{x^2+7x+10}{x^2+6x-16}$$

$$0 \geq (negative\ \text{numbers})$$

$$0 \geq (-)$$

Solution: x-values that make the rational expression negative

1) Zeroes of the numerator are x-intercepts

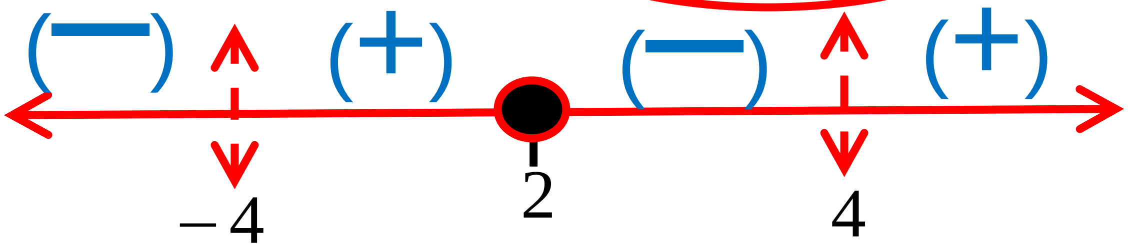
2) Zeroes of the denominator that *do not disappear due to simplification* are vertical asymptotes.

3) Zeroes of the denominator that *disappear due to simplification* are holes.

$0 < \frac{3x-6}{x^2-16}$ $\qquad y = \frac{3(x-2)}{(x+4)(x-4)}$ $\qquad 0 < (\textit{positive} \text{ numbers})$

$0 < (+)$

Solution: x-values that make the rational expression positive



1) Zero of the numerator: x-intercept: (Passes thru at x = 2)

Passes thru means opposite sign on each side of the zero.

2) Zero of the denominator: (Either vertical asymptote OR hole).

No holes and VA at x = -4, 4

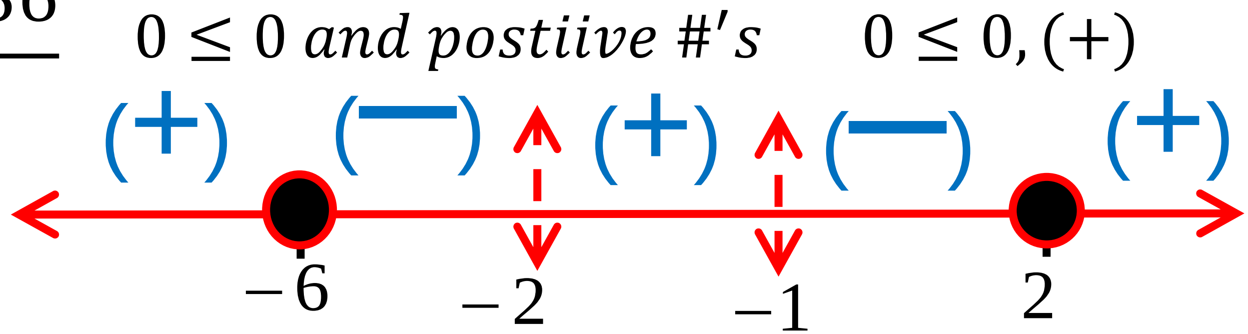
interval	Input	(x – 2)(x + 4)(x – 4)	output	Solution?
$(-\infty, -4)$	-5	$(-)(-)(-)$	$(-)$	no
$(-4, 2)$	0	$(-)(+)(-)$	$(+)$	yes
$(2, 4)$	3	$(+)(+)(-)$	$(-)$	no
$(4, \infty)$	5	$(+)(+)(+)$	$(+)$	yes

$x = (-4, 2) \cup [4, \infty)$

$$0 \le \frac{3x^2 + 12x - 36}{x^2 + 3x + 2}$$

$$y = \frac{3(x^2 + 4x - 12)}{(x+2)(x+1)}$$

$$y = \frac{3(x+6)(x-2)}{(x+2)(x+1)}$$



X-intercepts: $x = 2, -6$

(No holes), VA's are: x = -2, -1

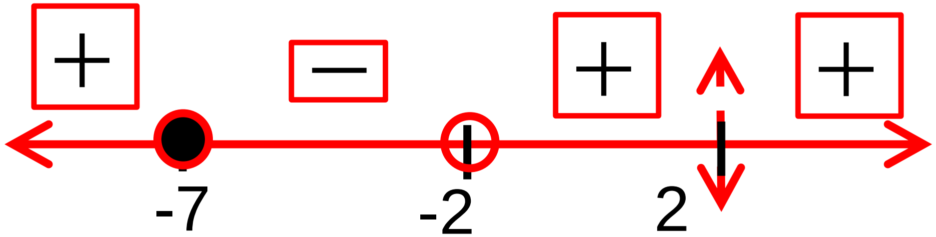
Solution: $x = (-\infty, -6] \cup (-2, -1) \cup [2, \infty)$

interval	Input	$(x+6)(x-2)(x+2)(x+1)$	output	Solution?
$(-\infty, -6]$	-7	$(-)(-)(-)(-)$	$(+)$	yes
$[-6, -2)$	-3	$(+)(-)(-)(-)$	$(-)$	no
$(-2, -1)$	-1.5	$(+)(-)(+)(-)$	$(+)$	yes
$(-1, 2]$	0	$(+)(-)(+)(+)$	$(-)$	no
$[2, \infty)$	3	$(+)(+)(+)(+)$	$(+)$	yes

$$0 > \frac{2x^2 + 10x - 28}{x^2 - 4}$$

$$0 > \frac{2(x+7)(x-2)}{(x+2)(x-2)}$$

$$0 > \frac{2(x+7)}{(x+2)}$$



1. X-intercepts: x = -7

2. Vertical asymptote: x = -2

3. Hole: x = 2

interval	Input	$(x+7)(x+2)$	output	Solution?
$(-\infty, -7]$	-8	$(-)(-)$	$(+)$	no
$[-7, -2)$	-3	$(+)(-)$	$(-)$	yes
$(-2, 2)$	0	$(+)(+)$	$(+)$	no
$(2, \infty)$	3	$(+)(+)$	$(+)$	no

$$0 \geq x^3 + 2x^2 - 4x - 8$$

$$0 = (x^3 + 2x^2) + (-4x - 8)$$

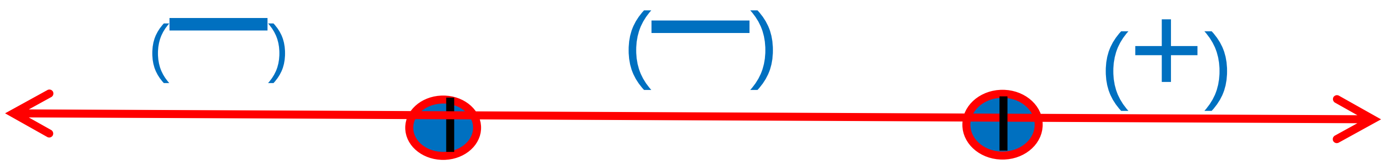
1. Find the “real” zeroes of the polynomial <u>equation</u>.

$$0 = x^2(x+2) - 4(x+2)$$

$$0 = (x^2 - 4)(x+2)$$

$$x = -2,\ 2\ ,-2$$

2. Build “Sign (+/-) Chart”



$$x = (-\infty, -2) \qquad -2 \qquad 2$$

$$f(x) = (x+2)(x-2)(x+2)$$

$$f(-3) = (\ -\ )(\ -\ )(\ -\ )$$

$$f(-3) = (\ -\ )$$

3. Even multiplicity at x = -2 → no sign change at x = -2
4. odd multiplicity at x = +2 → sign change at x = +2

5. Solve: $0 \geq (x+2)(x-2)(x+2)$

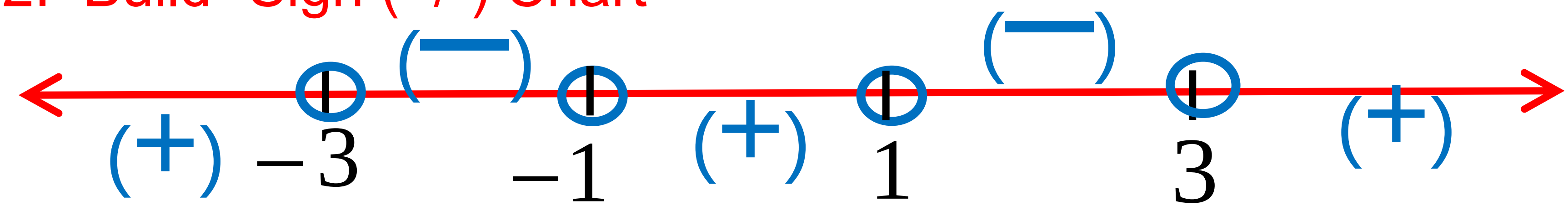
$$\boxed{x = (-\infty, 2]}$$

$0 < x^4 - 10x^2 + 9$ $\qquad$ $0 = (x^2 - 9)(x^2 - 1)$

1. Find the “real” zeroes of the polynomial <u>equation</u>.

$0 = (x+3)(x-3)(x+1)(x-1)$ $\qquad$ $x = -3, -1, 1, 3$

2. Build “Sign (+/-) Chart”



$x = (-\infty, -3)$

$f(x) = (x+3)(x-3)(x+1)(x-1)$

$f(-4) = (\ -\ )(\ -\ )(\ -\ )(\ -\ )$ $\qquad$ $f(-4) = (+)$

3. odd multiplicities → sign change

4. Solve: $0 < (x+3)(x-3)(x+1)(x-1)$

$$x = (-\infty, -3) \cup (-1, 1) \cup (3, \infty)$$