Math-1050
Session #13
(Textbook 5.3: Graphs of Rational Functions)

Find the zeroes

$$f(x) = \frac{(x-4)(x+5)}{x}$$

x* $0 = \frac{(x-4)(x+5)}{x}$ *x

$$0 = (x-4)(x+5)$$

(4, 0), (-5, 0) $x = 4, -5$

$$g(x) = \frac{(x-5)(x+4)}{x-1}$$

x* $0 = \frac{(x-5)(x+4)}{x}$ *x

$$0 = (x-5)(x+4)$$

(5, 0), (-4, 0) $x = 5, -4$

$$k(x) = \frac{(x-6)(x+3)}{x-2}$$

x* $0 = \frac{(x-6)(x+3)}{x}$ *x

$$0 = (x-6)(x+3)$$

(6, 0), (-3, 0) $x = 6, -3$

List everything you can about the previous 3 problems.

(1) all 3 were ratios of polynomials (rational functions)

(2) zeroes of the functions are x-intercepts

(3) the zeroes moved one unit right each time

(4, 0), (-5, 0)

(5, 0), (-4, 0)

(6, 0), (-3, 0)

$$\mathrm{f}(x) = \frac{(x-4)(x+5)}{x}$$

$$g(x) = \mathrm{f}(x-1) = \frac{((x-1)-4)((x-1)+5)}{(x-1)}$$

$$g(x) = \frac{(x-5)(x+4)}{x-1}$$

(4) the zeroes of the function come from the numerator only.

$$\mathrm{f}(x) = \frac{(x-4)(x+5)}{x} \qquad 0 = (x-4)(x+5)$$

$$g(x) = \frac{(x-5)(x+4)}{x-1} \qquad 0 = (x-5)(x+4)$$

$$k(x) = \frac{(x-6)(x+3)}{x-2} \qquad 0 = (x-6)(x+3)$$

(5) the domain of each excludes zeroes of the denominator

$f(x)\ Domain: x \neq 0$ $\qquad$ $g(x)\ Domain: x \neq 1$

$k(x)\ Domain: x \neq 2$

(6) the oblique asymptote is the quotient of the function

$$g(x) = \frac{(x-5)(x+4)}{x-1}$$

$$f(x) = \frac{x^2 - x - 20}{x-1}$$

$$f(x) = x + 2 - \frac{18}{x}$$

$$
\begin{array}{r}
x \quad + 2 \\
x - 1 \,\big)\overline{x^2 - x - 20} \\
-(\,x^2 - x\,) \\
\hline
2x - 20 \\
-(\,2x - 2\,) \\
\hline
-18
\end{array}
$$

(7) the degree of the oblique asymptote equals the numerator degree subtract the denominator degree.

(8) the y-intercept can be found by: f(0), g(0), and k(0)

$$f(x) = \frac{(x-4)(x+5)}{x}$$

$f(0) =$ Does not exist

$$g(x) = \frac{(x-5)(x+4)}{x-1}$$

$g(0) =$ 20

$$k(x) = \frac{(x-6)(x+3)}{x-2}$$

$k(0) =$ 9

(9) None of the ratios could be simplified

Graph every item we discovered so far.

$$g(x) = \frac{(x-5)(x+4)}{x-1}$$

X-int: (5, 0), (-4, 0)

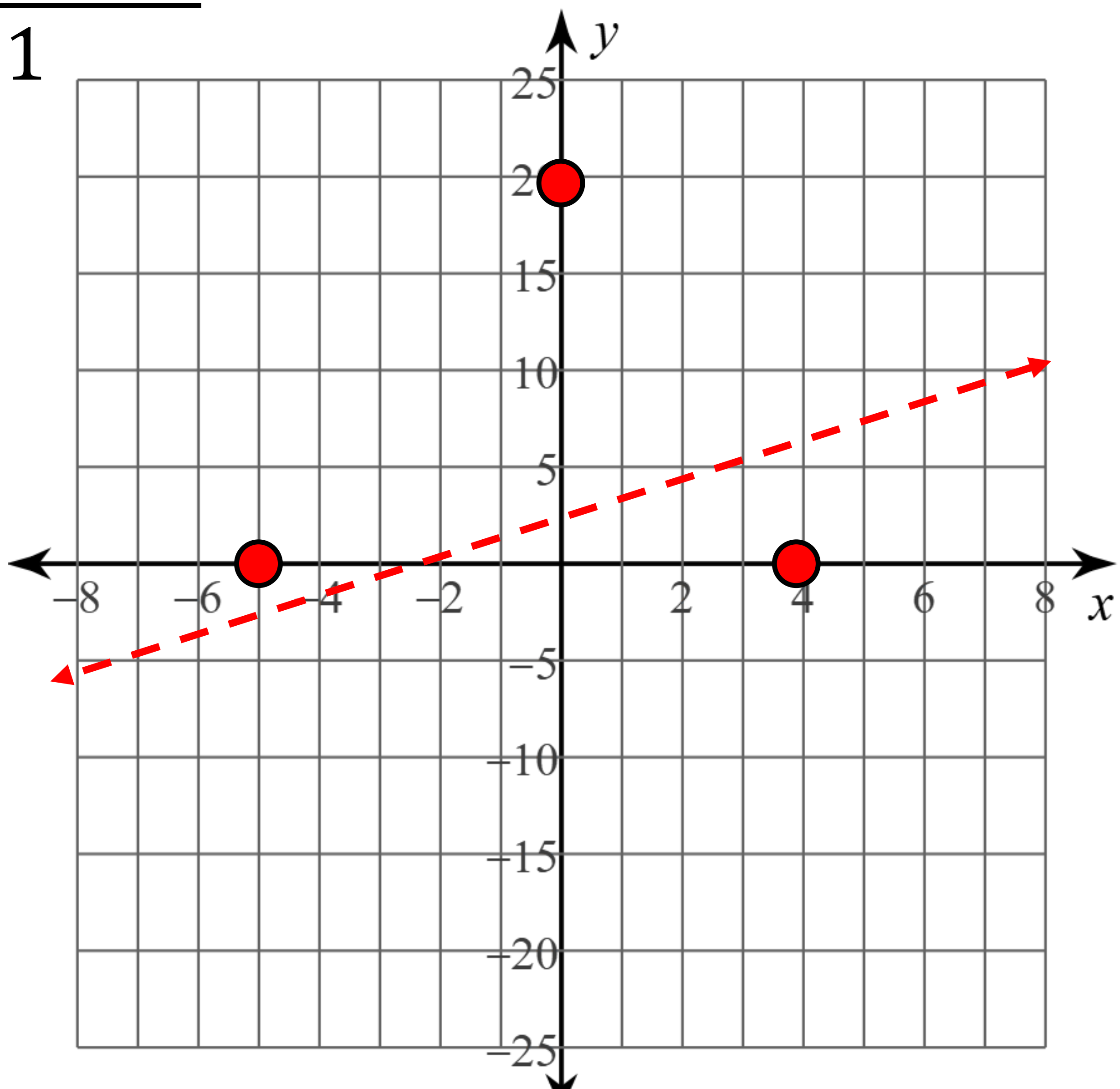
Y-int: (0, 20)

Oblique Asymptote:

y = x + 2

(8, 10), (-7, -5)

Domain: $x \neq 1$



$$f(x) = (x+1)^2(x+3)(x-4)$$

How does the graph ***"behave"*** near the zero: x = -1 ?

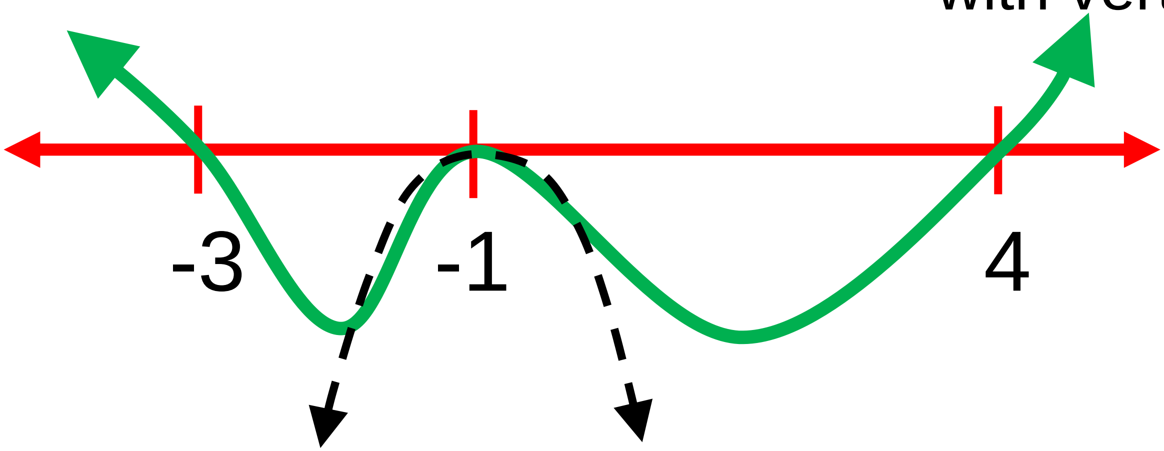
Substitute x = -1 into every linear factor *except the one causing the zero* of the function.

$$f(x) = (x+1)^2(-1+3)(-1-4)$$

$$f(x) = (x+1)^2(2)(-5)$$

$$f(x) = -10(x+1)^2$$

Downward opening parabola with vertex at (0, -1)



What is the behavior of the function near the zero of the denominator?

$$g(x) = \frac{(x-5)(x+4)}{x-1}$$

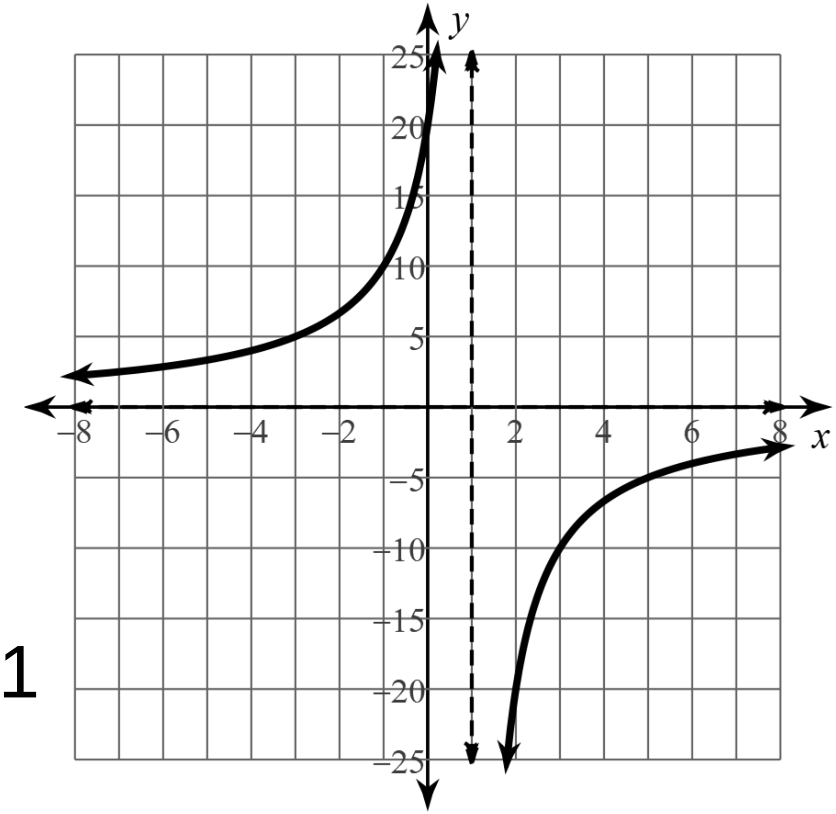
Substitute x = 0 into every linear factor *except the one causing the zero* of the function.

$$\mathrm{f}(x) = \frac{(0-5)(0+4)}{x-1}$$

$$\rightarrow \mathrm{f}(x) = \frac{-20}{x-1}$$

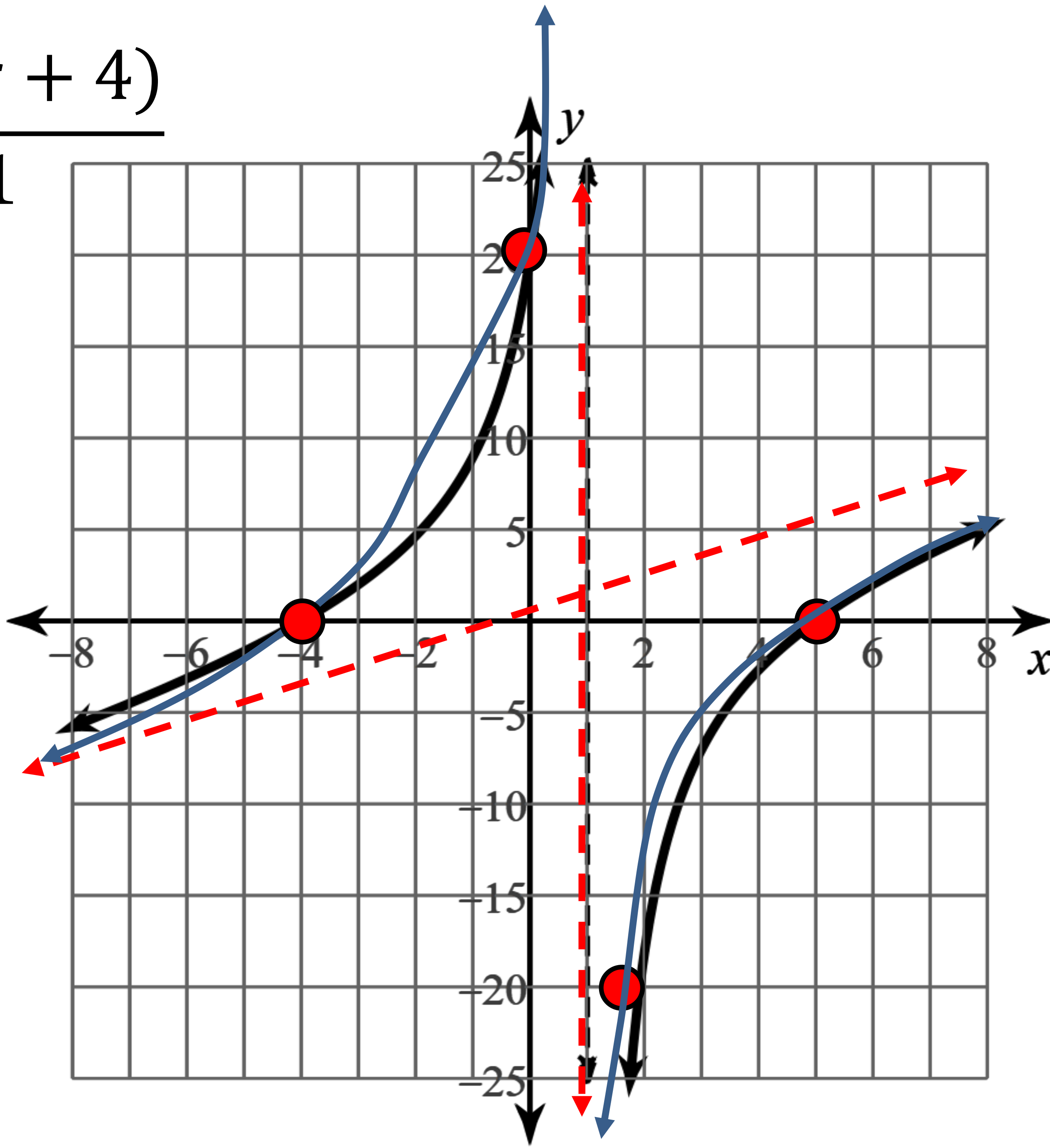
→ The Reciprocal Function

→ Vertical Asymptote at x = 1



Graph every item we discovered so far.

$$g(x) = \frac{(x-5)(x+4)}{x-1}$$



X-int: (5, 0), (-4, 0)

Y-int: (0, 20)

Oblique Asymptote:

y = x + 2

(8, 10), (-7, -5)

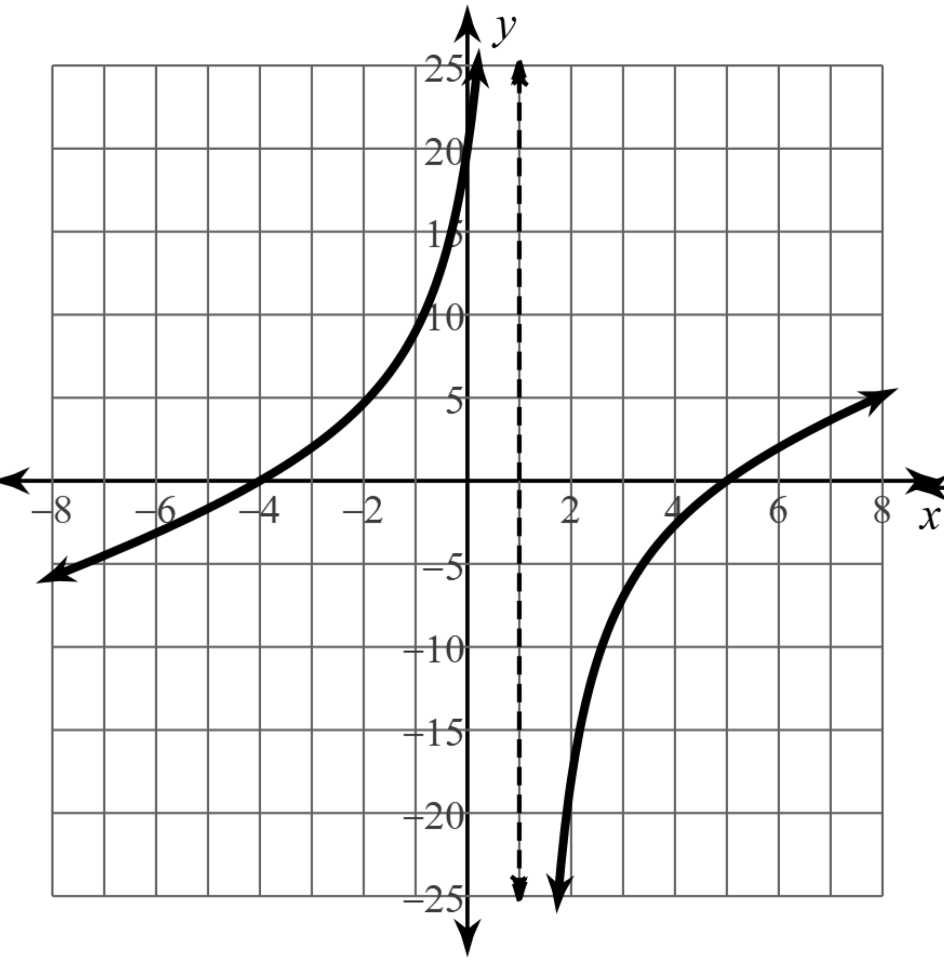
Domain: $x \neq 1$

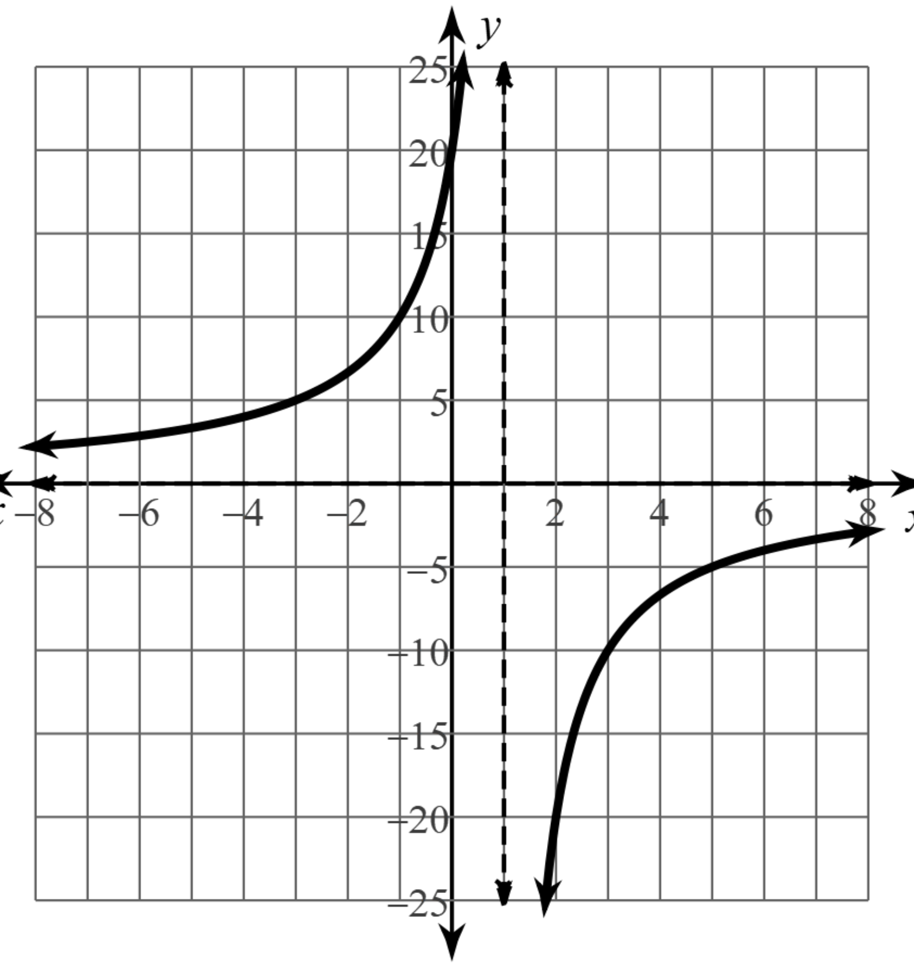
Vertical Asymptote:

$x \neq 1$

I'll show you g(x) and the "behavior near x = 0" function

$$g(x) = \frac{(x-5)(x+4)}{x-1} \qquad \rightarrow \mathrm{f}(x) = \frac{-20}{x-1}$$





What about this?

$$g(x) = \frac{(x-1)(x+4)}{(x-1)}$$

Domain = ? x ≠ 5

Simplified equation = ? $g(x) = x + 4$

X-intercepts? (-4, 0)

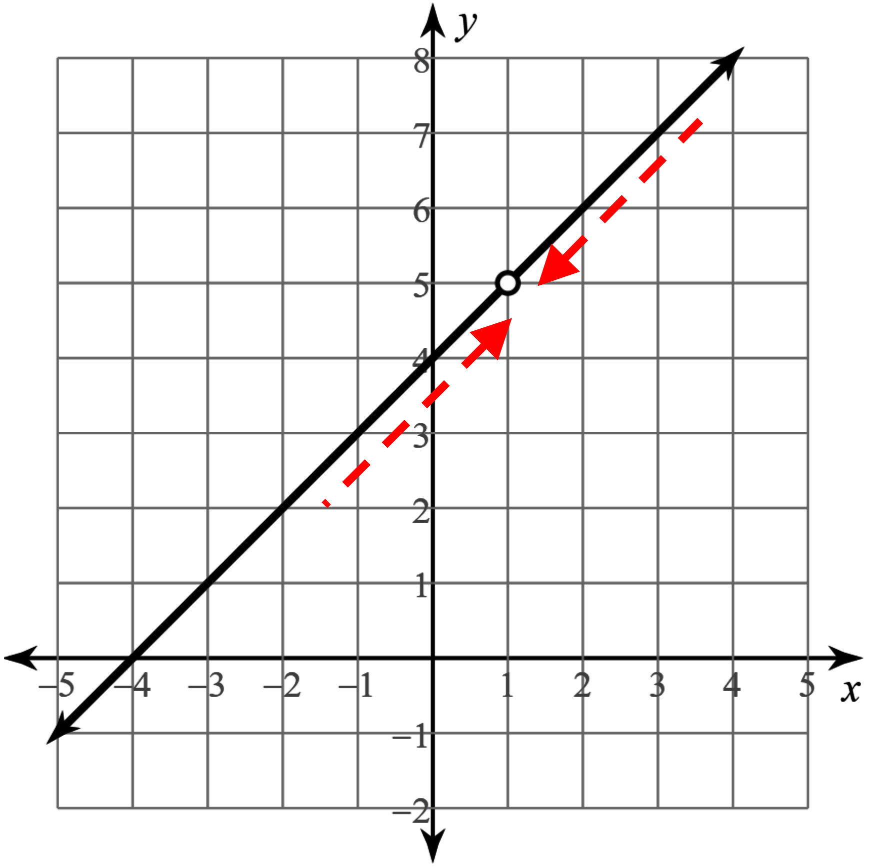
Asymptotes? none

$x \to 1^+, y \to$ 5^+

$x \to 1^-, y \to$ 5^-

$x = 1, y \to ?$

***Hole in the graph*: (1, 5)**



X-intercepts: Zeroes of Numerator (unless it is a hole)

y-intercept f(0)

Oblique Asymptote

Numerator Degree (ND)
Denominator Degree (DD)

ND > DD Quotient of Long Division

horizontal Asymptote

ND = DD Quotient of Long Division

ND < DD y = 0

Vertical Asymptote(s)

Zeroes of the denominator that *do not disappear* due to simplification

Holes Zeroes of the denominator that *disappear* due to simplification

Behavior near VA’s

Input the VA x-value into the other linear factors

Requirements of the graph:

$$k(x) = \frac{(x-2)(x+3)(x-4)}{(x-2)(x-1)}$$

X-intercepts

y-intercept

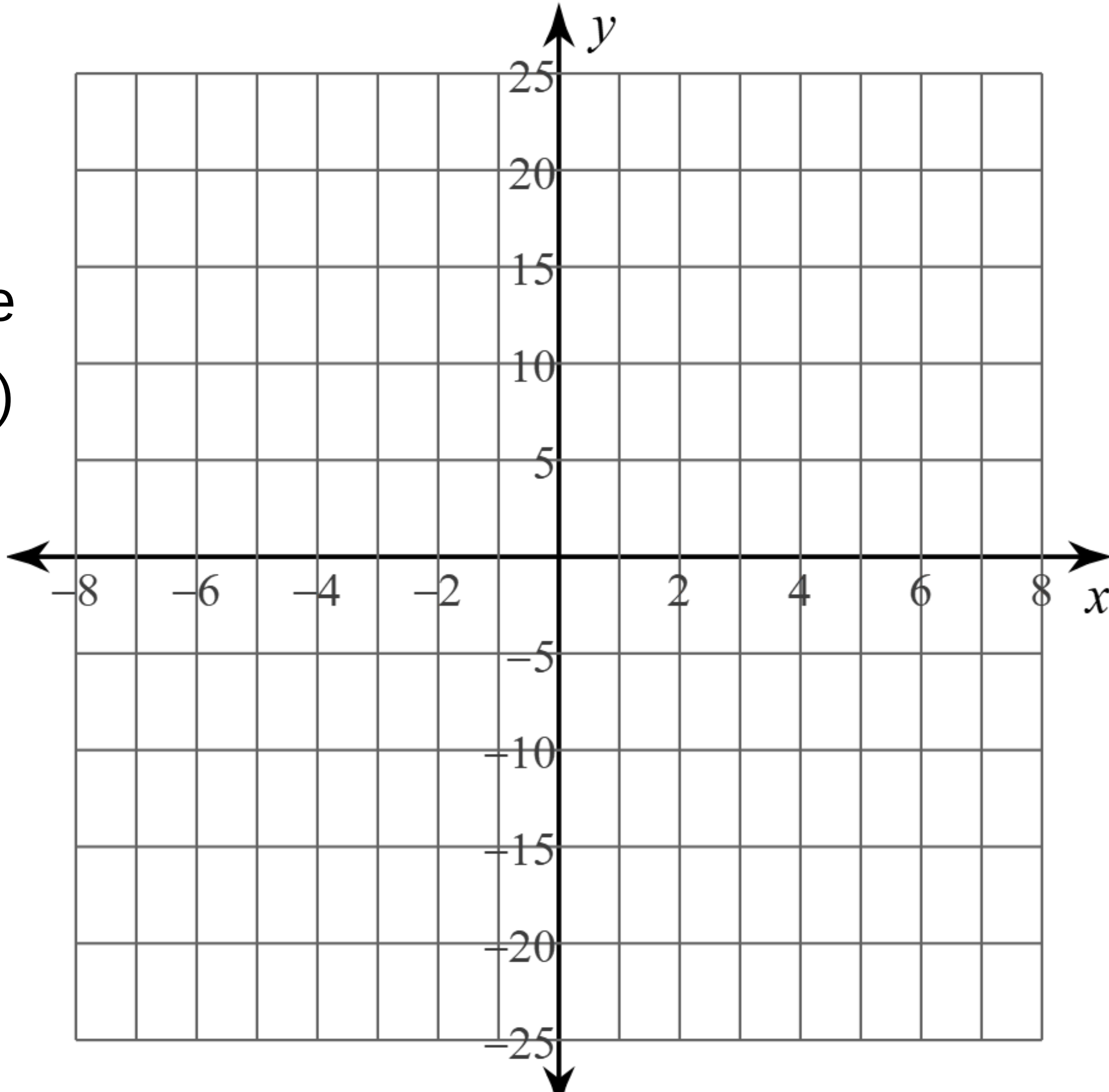
Oblique Asymptote

horizontal Asymptote

Vertical Asymptote(s)

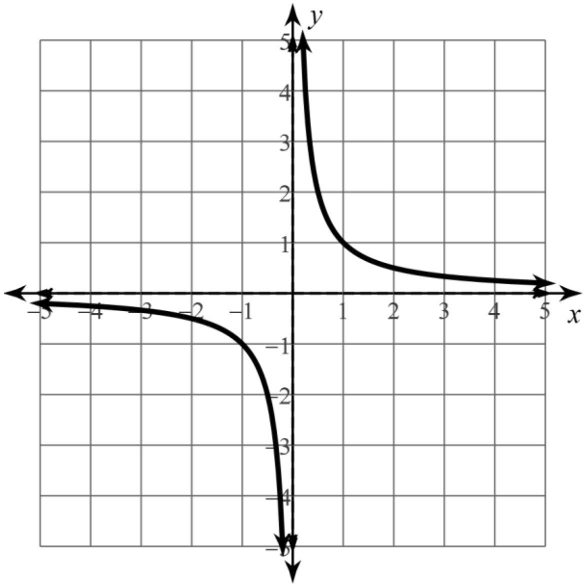
Holes

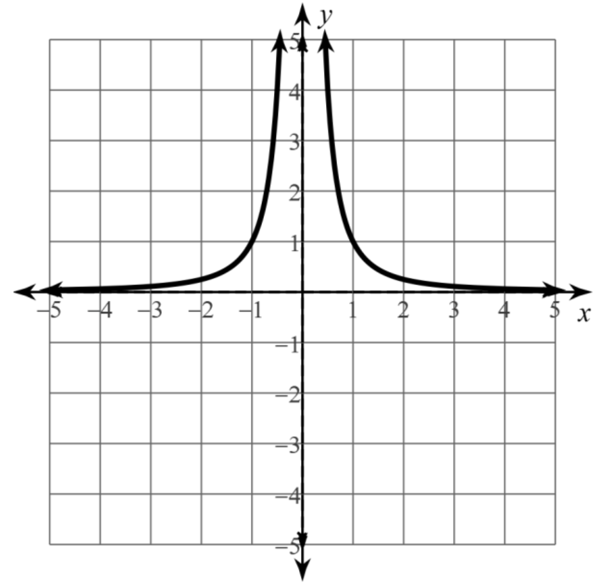
Behavior near VA's



VA Asymptotes and their "multiplicities"

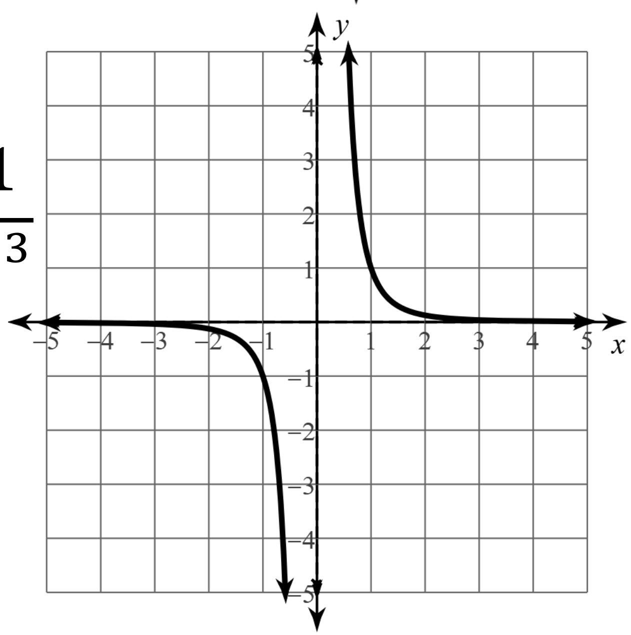
$$f(x) = \frac{1}{x}$$

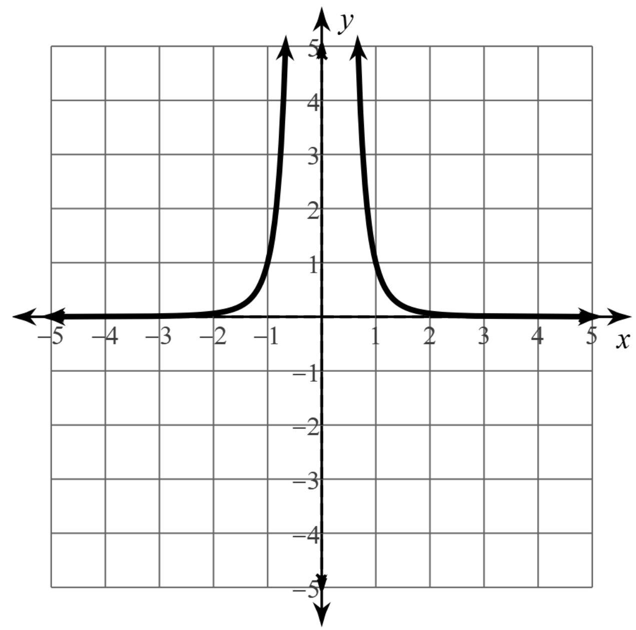




$$g(x) = \frac{1}{x^2}$$

$$k(x) = \frac{1}{x^3}$$

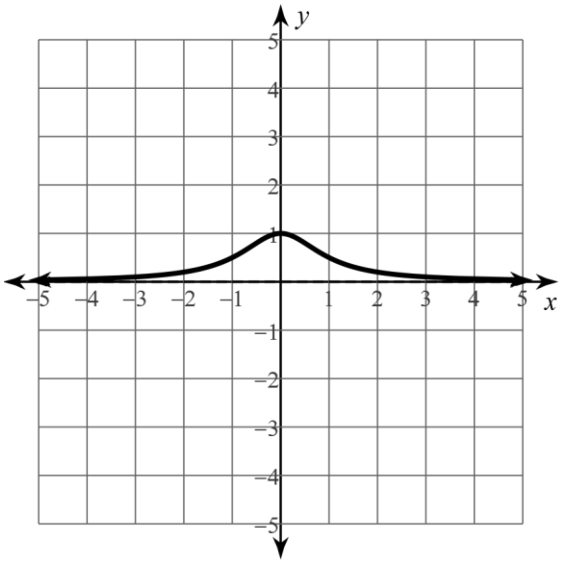




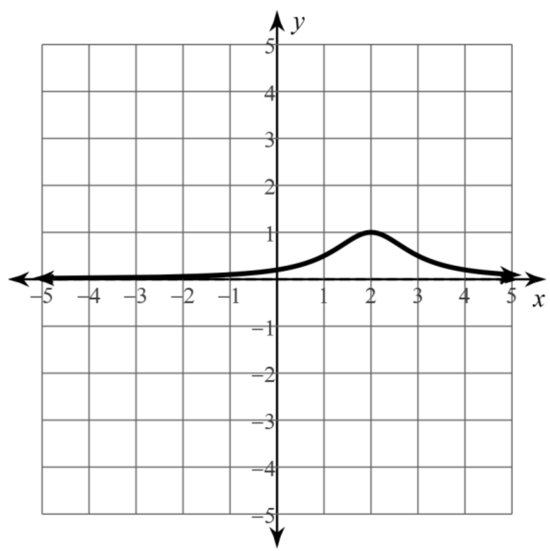
$$g(x) = \frac{1}{x^4}$$

Rational Functions without Vertical Asymptotes

$$g(x) = \frac{1}{x^2 + 1}$$



$$k(x) = \frac{1}{(x-2)^2+1}$$



$$h(x) = \frac{1}{x^2 + 1} + 2$$