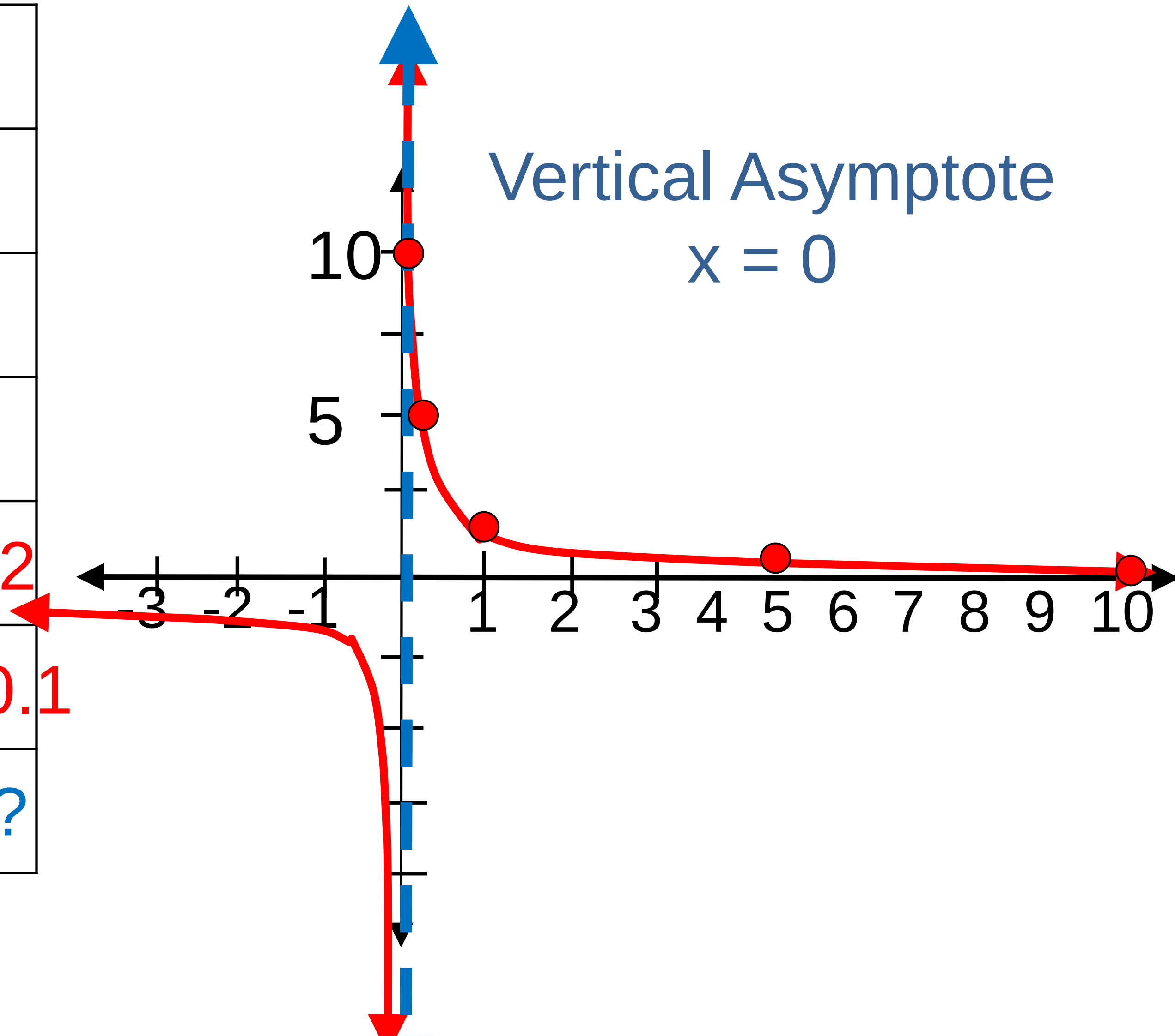
Math-1050
Session #12
(Textbook 5.2: Properties of Rational Functions)

Reciprocal Function

$$f(x) = \frac{1}{x}$$

x	f(x)
1/10 = 0.1	10
1/5 = 0.2	5
1	1
5	1/5 = 0.2
10	1/10 = 0.1
0	1/0 = ??



$$f(x) = \frac{1}{x}$$

Why is there a vertical asymptote?

What is the output when we "approach" x = 0 from the "+" side?

'f' is *unbounded* in the positive direction.

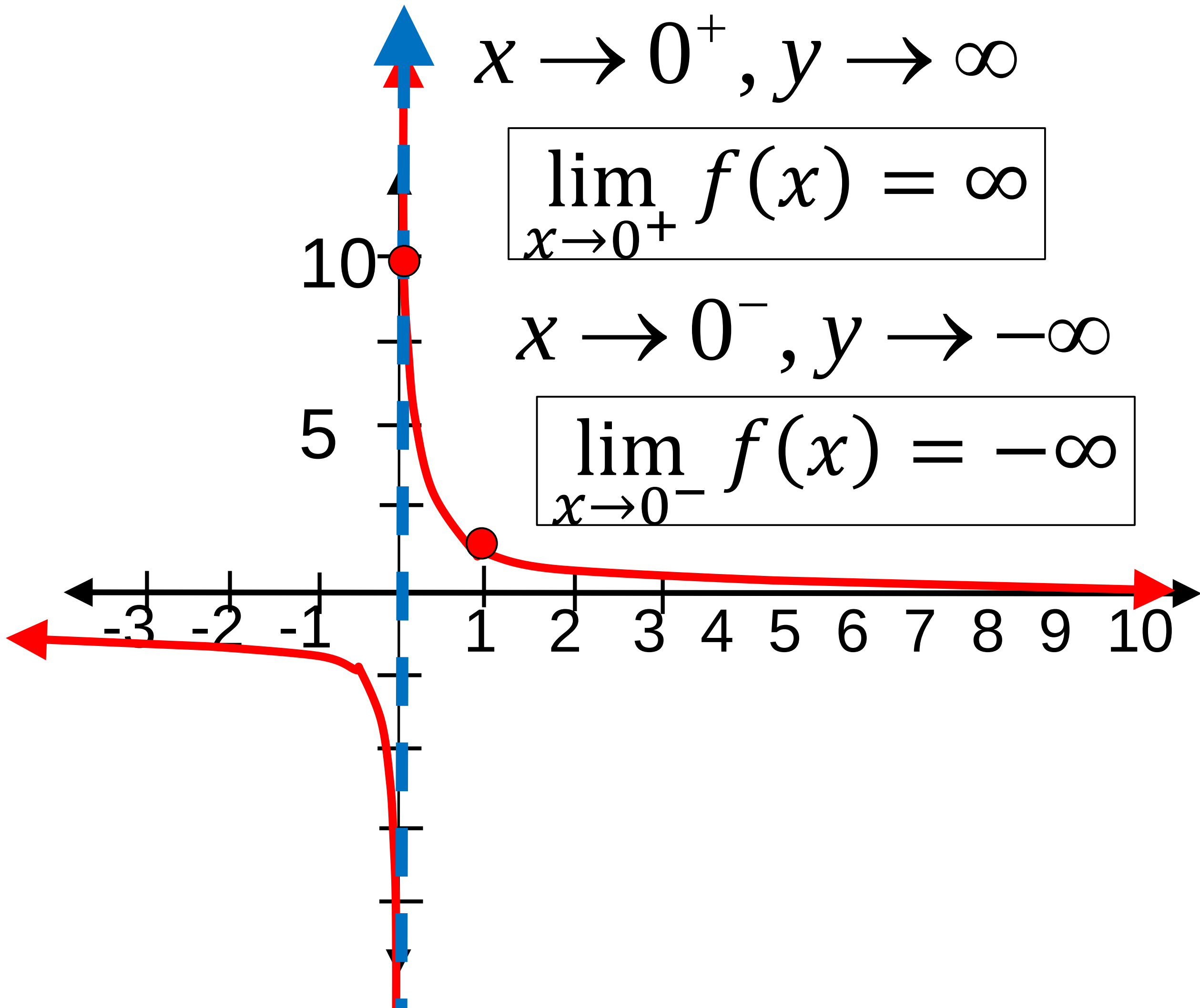
x	f(x)
1	1
0.1	10
0.01	100
0.001	1000
10^{-4}	$10^{4} = 10{,}000$
10^{-12}	10^{12}

$$x \to 0^{+}, y \to \infty$$

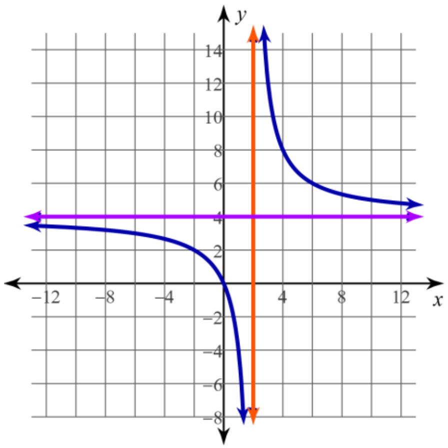
$$\boxed{\lim_{x \to 0^{+}} f(x) = \infty}$$

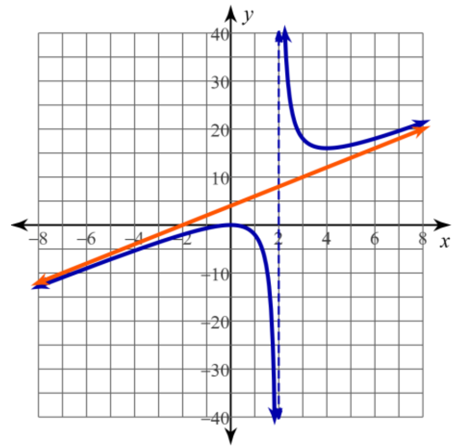
$$x \to 0^{-}, y \to -\infty$$

$$\boxed{\lim_{x \to 0^{-}} f(x) = -\infty}$$



Asymptote: A vertical, horizontal, or oblique line that the graph approaches but NEVER reaches.





Asymptotes are not part of the graph but you can see them easily. We show them as dotted lines.

Vertical Asymptote: is caused by a zero of the denominator that does NOT disappear due to simplification.

Fractions and the number zero.

$$y = \frac{1}{x}$$

Can the denominator a fraction equal to zero?

$$0 = \frac{1}{x} \rightarrow 0 * x = \frac{1}{x} * x \rightarrow 0 = 1$$

There is no solution to this equation → the denominator can never make a fraction equal to zero.

What part of the fraction makes it equal to zero? $y = \frac{m}{x}$

$$0 = \frac{m}{x} \rightarrow 0 * x = \frac{m}{x} * x \rightarrow 0 = m$$

Only the a zero of the numerator can make a fraction equal zero. $\frac{0}{3}$

Fractions and the number zero. $y = \frac{1}{x}$

Division by zero is not a number.

→ Vertical asymptote: x = 0.

Only the a zero of the numerator can make a fraction equal zero.

Is there any input value for ‘x’ that will make the numerator = 0?

The output value “y” of this function will never equal zero.

→ Horizontal asymptote: y = 0.

$$f(x) = \frac{1}{x}$$

Domain?

Domain:

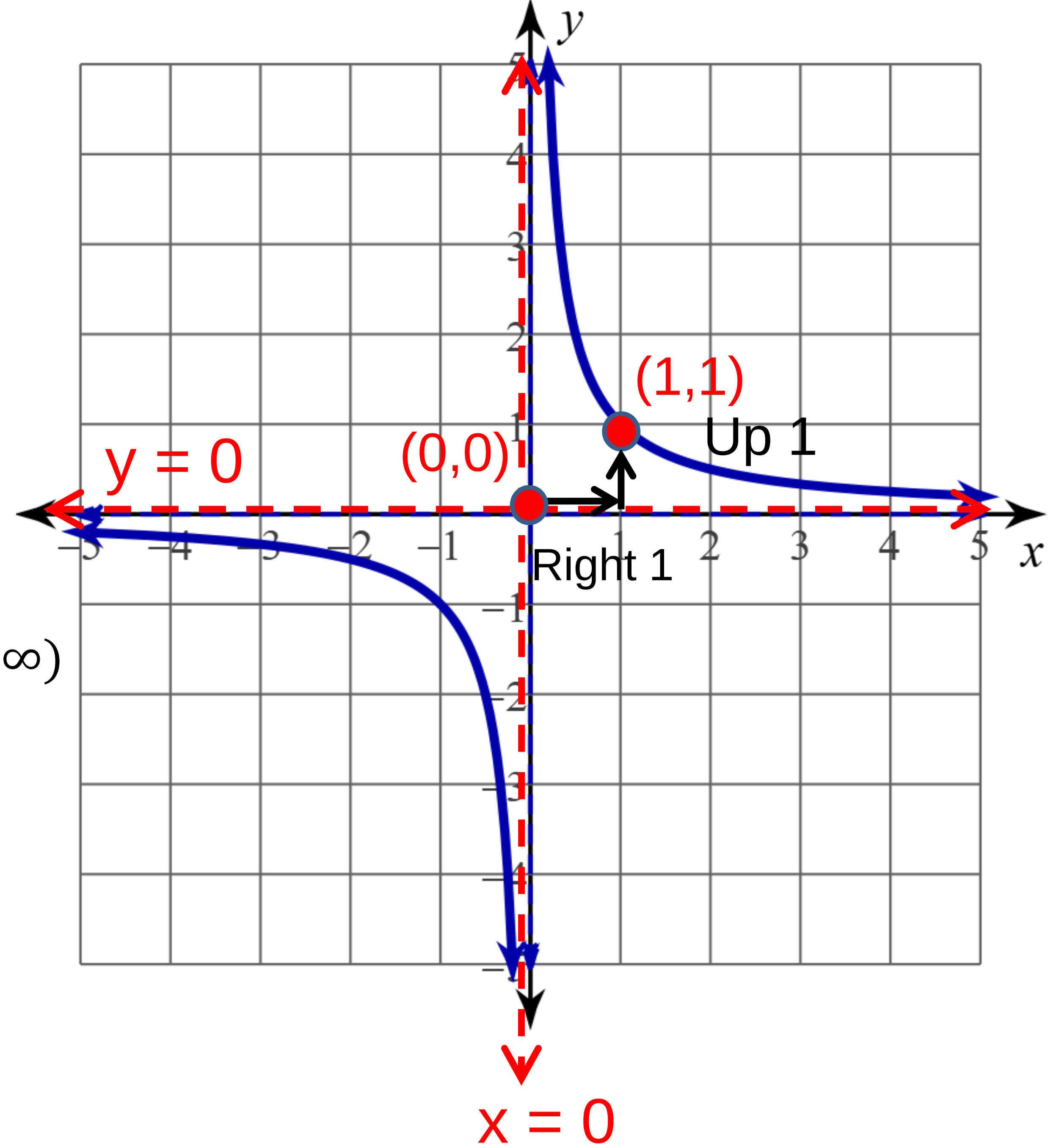
$$x = (-\infty, 0) \cup (0, \infty)$$

$$Domain: x \neq 0$$

Range?

$$Range: y = (-\infty, 0) \cup (0, \infty)$$

$$Range: y \neq 0$$



Generalized Transformations of the Square Function:

$$f(x) = x^2 \qquad y = (-1)a(x-h)^2 + k$$

Reflection across x-axis

vertical stretch factor

Translates left/right

translating <u>up</u> or <u>down</u>

$$y = -2(x-3)^2 + 4$$

Reflected (x-axis), VSF=2, right 3, up 4

Vertex: $(3,4)$

General Transformation Equation

Reflection across x-axis

Vertical stretch factor.

$$f(x) = \frac{(-1)a}{x-h} + k$$

Vertical shift (Horizontal Asymptote)

Horizontal shift (Vertical Asymptote)

(h, k) The point of intersection of the vertical and horizontal asymptotes.

$Domain: x \neq h$ $Range: y \neq k$

a) Describe the transformations of the reciprocal function.
b) What is the intersection of the asymptotes?
c) What is the horizontal asymptote?
d) What is the vertical asymptote?
e) What is the domain?
f) What is the range?

$$g(x) = \frac{1}{x} + 7$$

(a) Up 7
(b) (0, 7)
(c) x = 0
(d) y = 7
(e) x ≠ 0
(f) y ≠ 7

$$h(x) = \frac{5}{(x-2)}$$

(a) VSF=5, right 2
(b) (2, 0)
(c) x = 2
(d) y = 0
(e) x ≠ 2
(f) y ≠ 0

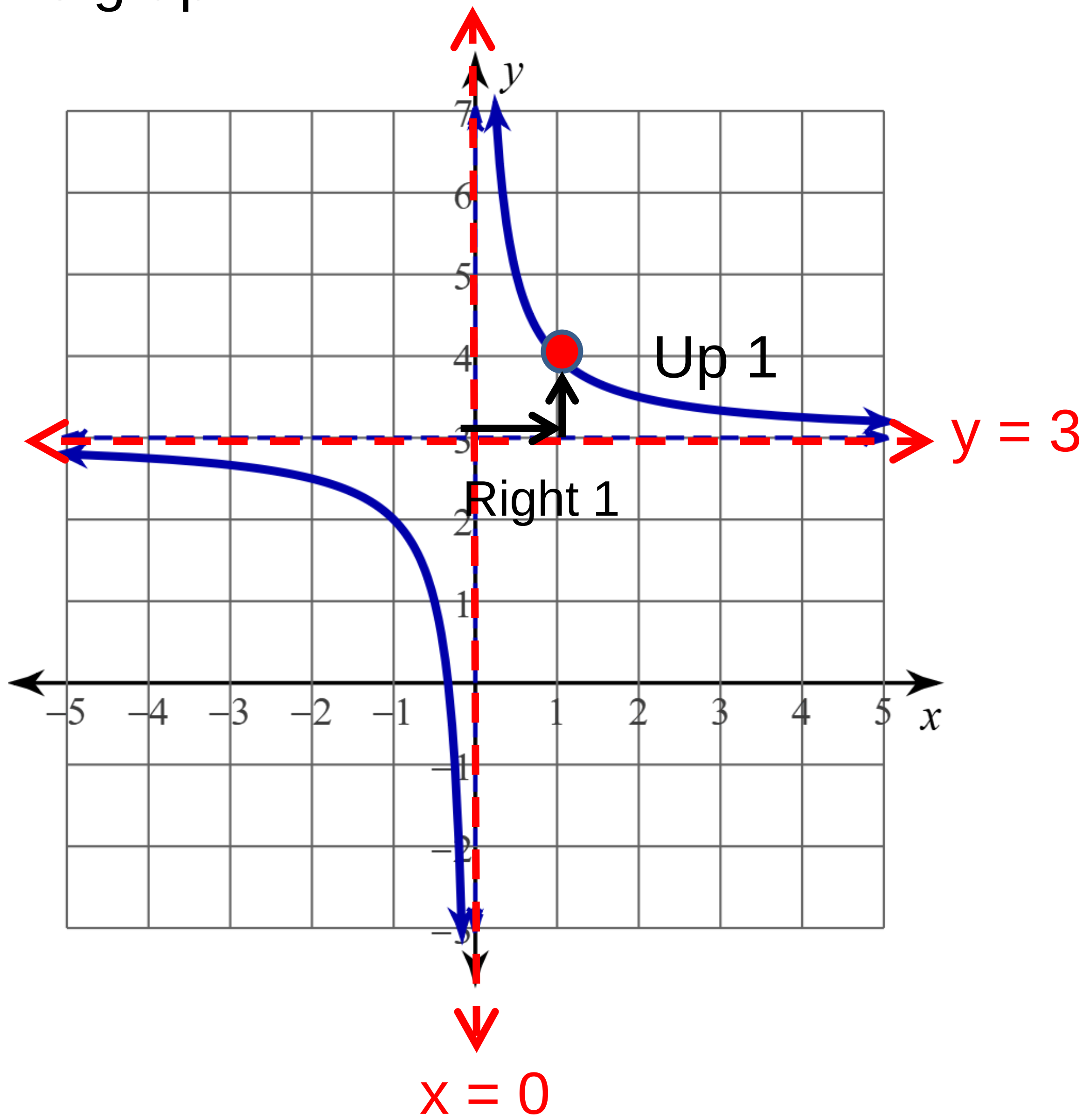
$$f(x) = \frac{-3}{(x+3)} - 5$$

(a) Reflect (x-axis), left 3, down 5
(a) (-3, -5)
(b) x = -3
(c) y = -5
(d) x ≠ -3
(e) y ≠ -5

What is the equation of the graph?

$$f(x) = \frac{1}{x}$$

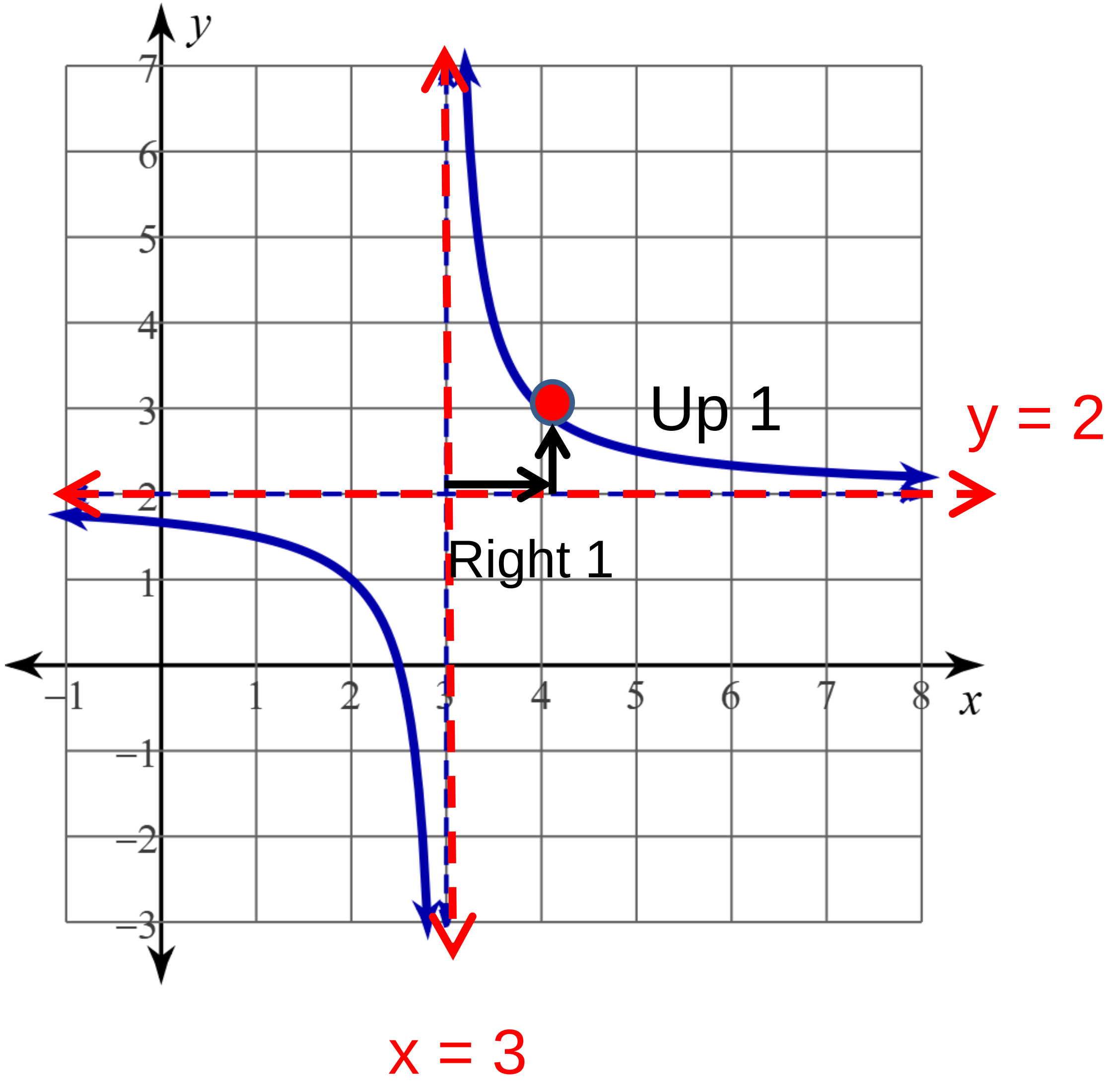
$$g(x) = \frac{1}{x} + 3$$



What is the equation of the graph?

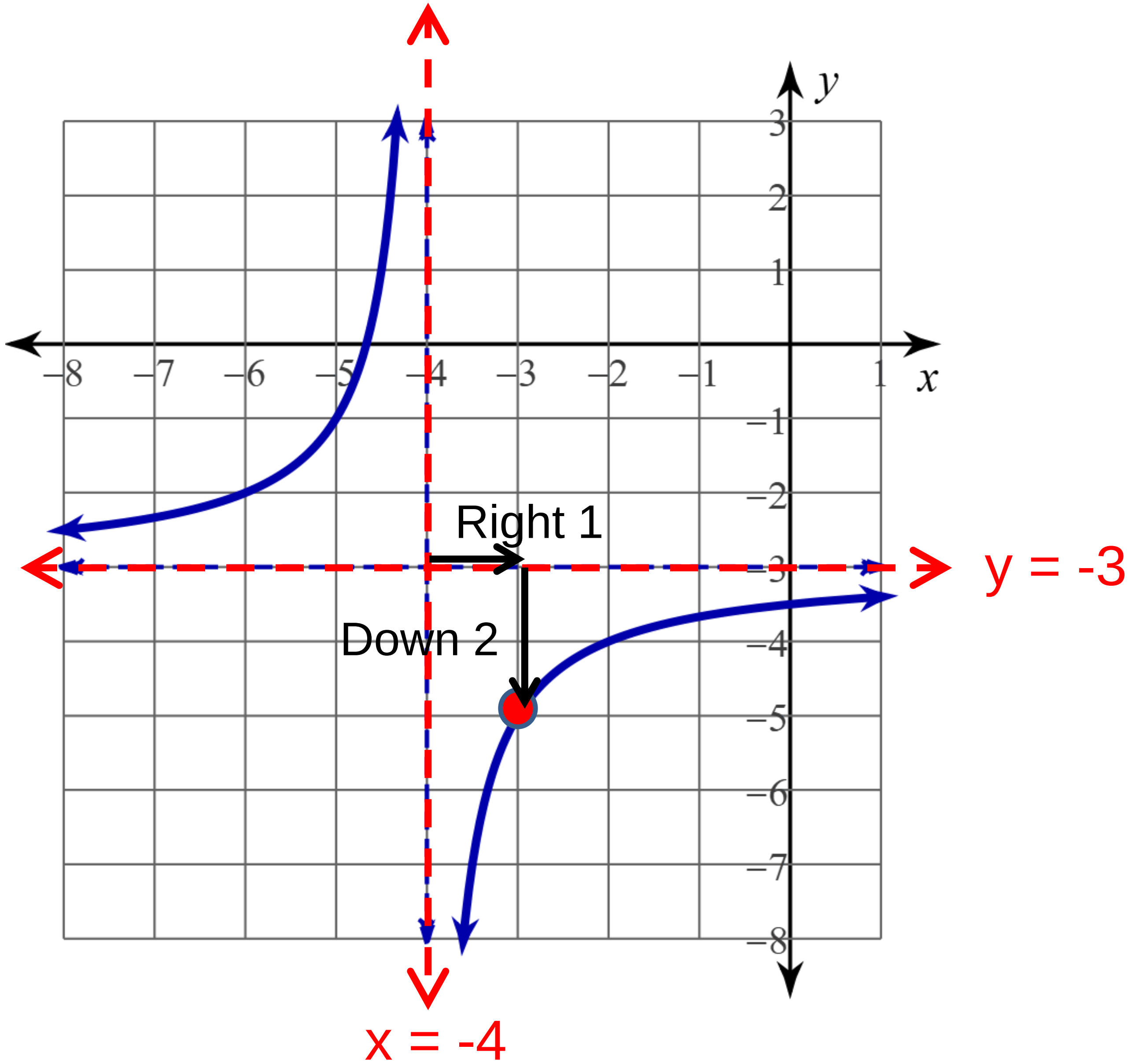
$$f(x) = \frac{1}{x}$$

$$g(x) = \frac{1}{(x-3)} + 2$$



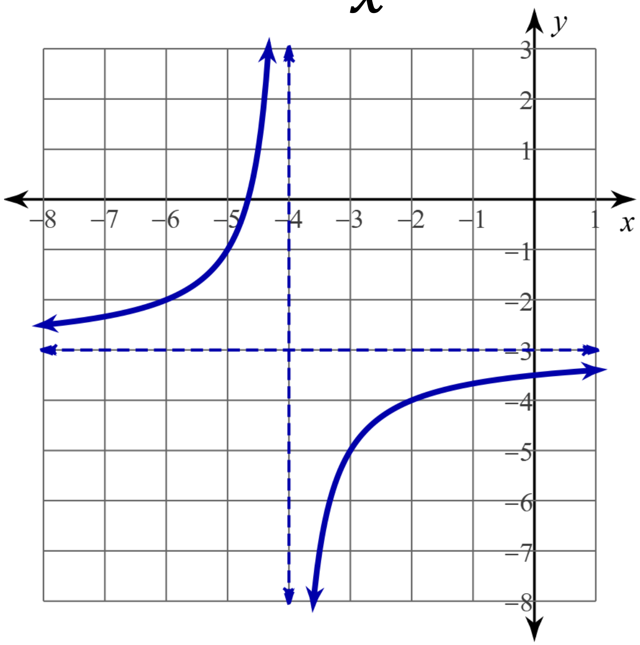
$$f(x) = \frac{1}{x}$$

$$g(x) = \frac{-2}{(x+4)} - 3$$



Another way to understand the horizontal asymptote:

$$g(x) = \frac{1}{x} + 3$$



End behavior!

On right end of the graph, y-value approaches the horizontal asymptote.

If the x-value is ***very*** large, what does y-value approach?

$$g(x) = \frac{1}{x} + 3$$

$$\frac{1}{10} + 3 = 3.1 \qquad \frac{1}{100} + 3 = 3.01 \qquad \frac{1}{1000} + 3 = 3.001$$

(Remember from Math-2), right end behavior is given by:

$$g(x) = \frac{1}{x} + 3 \qquad x \to \infty, \ \ g(x) \to ? \qquad g(x) = \cancel{\frac{1}{x}}^{\,0} + 3$$

$$\boxed{x \to \infty, \ \ g(x) \to 3}$$

Horizontal/Oblique Asymptote: the quotient when you divide.

$$g(x) = \frac{2x}{x-3}$$

$$x-3\overline{)\,2x}$$

$$\begin{array}{r} 2 \\ x-3\overline{)\,2x} \\ -(2x-6) \\ \hline 6 \end{array}$$

$$g(x) = 2 + \frac{6}{x-3}$$

$$x \to \infty,\ g(x) \to ?$$

$$g(x) = 2 + \frac{6}{x-3}$$

0

$$x \to \infty, \quad g(x) \to 2$$

Horizontal/Oblique Asymptote: y = 2

Horizontal/Oblique Asymptote: the quotient when you divide.

$$g(x) = \frac{2x}{x-3} \qquad g(0) = \frac{2(0)}{(0)-3} \qquad g(0) = \frac{0}{-3} = 0$$

x-intercept (zero of the numerator) (x, y) = (0, 0)

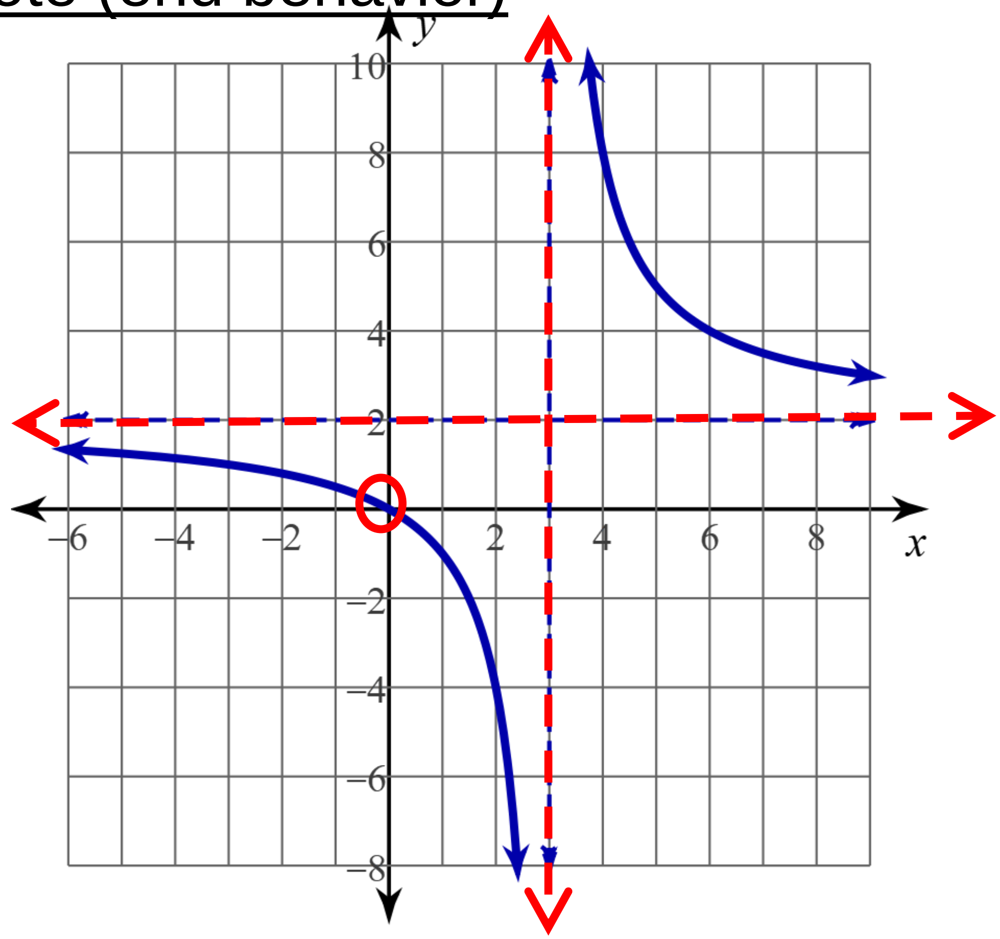
Horizontal/Oblique Asymptote (end behavior)

$$x \to \infty,\ g(x) \to 2$$ y = 2

Vertical Asymptote (excluded value):

x = 3

$$g(x) = \frac{6}{x-3} + 2$$



Asymptotes cross (3, 2)

VSF = 6

$$g(x) = \frac{3x-6}{x-4}$$

x-intercept = ?

$$0 = \frac{3x-6}{x-4}$$

$$0 = 3x - 6$$

$$\boxed{2 = x}$$

Horizontal Asymptote?

	3	remainder
x	$3x$	6
-4	-12	

$3x - 6$

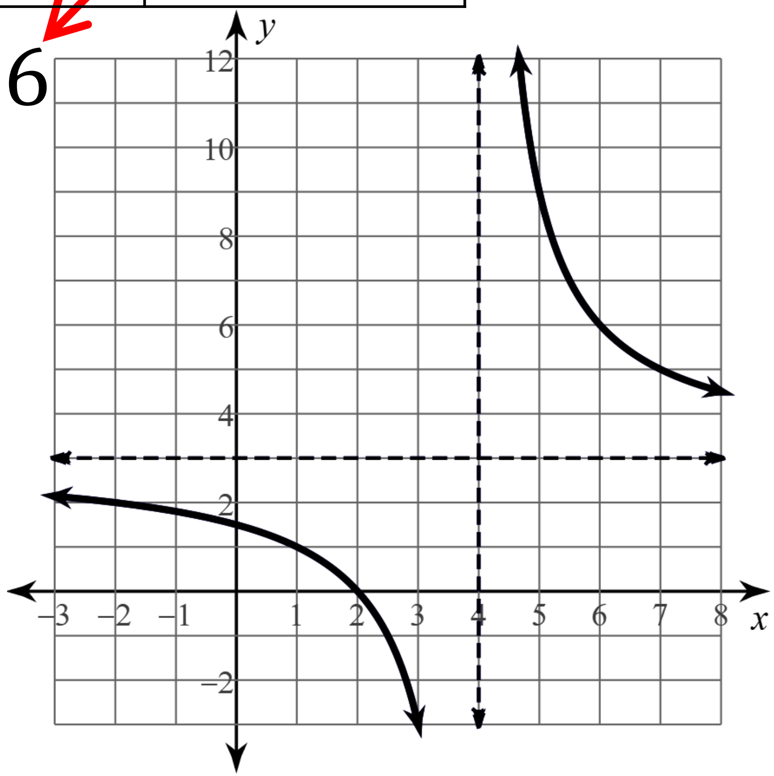
Vertical Asymptote ?

$0 \neq x - 4 \qquad 4 \neq x$

$$g(x) = 3 + \frac{6}{x-4}$$

$$g(x) = \frac{6}{x-4} + 3$$

$$x \to \infty,\ g(x) \to 3$$



$$\boxed{y = 3}$$

Asymptotes cross (4, 3)

VSF = 6

The ‘quotient’ when you divide a rational equation determines the ‘end behavior.”

$$y = \frac{x^3}{x^2 - 1}$$

$$\begin{array}{r|l} & x \\ \hline x^2 - 4 & x^3 \\ & -(x^3 - 4x) \\ \hline & \quad 4x \end{array}$$

$$y = x + \frac{4x}{x^2 - 1}$$

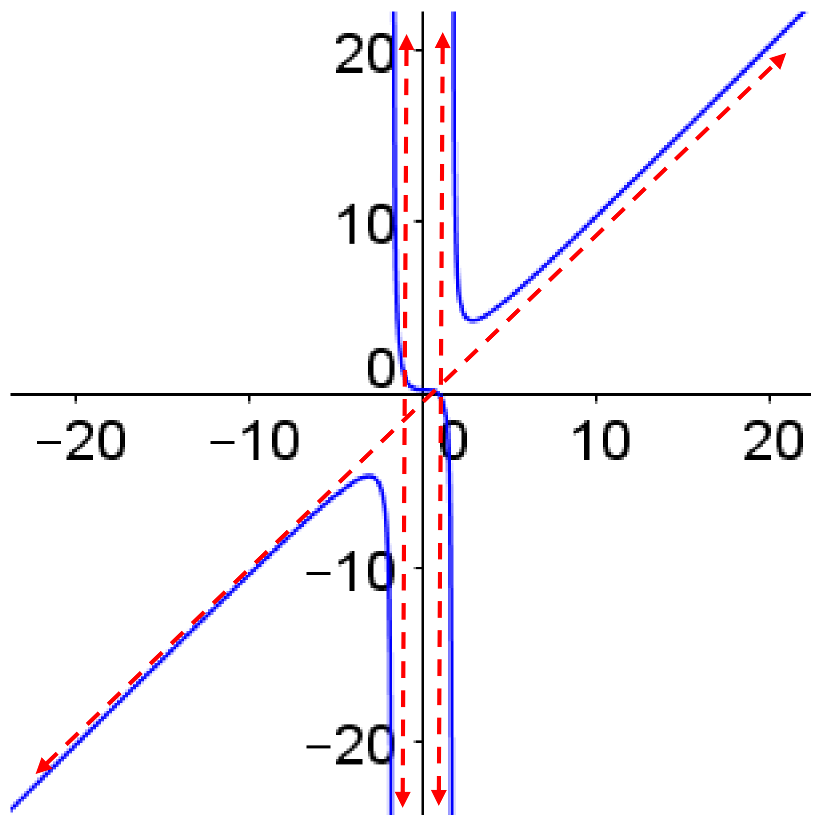
Oblique asymptote:

y = x

VA:

y = -1

y = 1



Will there be a horizontal asymptote?

$$g(x) = \frac{x^2 + 2x + 4}{x - 1}$$

$$\begin{array}{r}
x + 3 \\
x - 1 \overline{\big) x^2 + 2x + 4} \\
-(x^2 - x) \\
\hline
3x + 4 \\
-(3x - 3) \\
\hline
7
\end{array}$$

$$g(x) = x + 3 + \frac{7}{x - 1}$$

X→∞   y→?

Use long division.

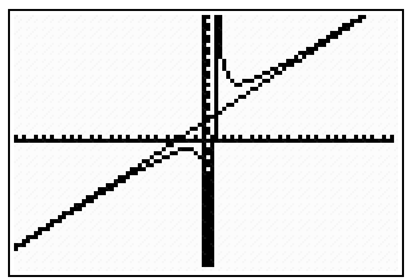
0

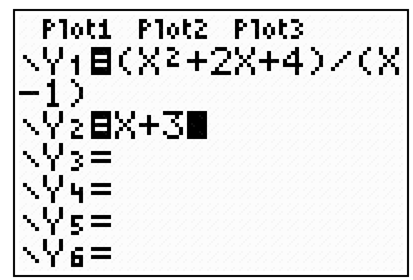
$$g(x) = x + 3 + \frac{7}{x - 1}$$

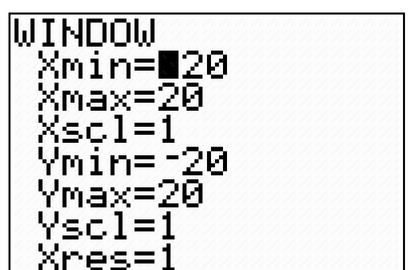
The fraction approaches <u>zero.</u>

X→∞   g(x) → x + 3

<u>Oblique</u> asymptote: y = x + 3







find the oblique asymptote

$$g(x)=\frac{6x^2+5x-2}{3x-2}$$

$$\begin{array}{r}
2x+3 \\
3x-2\,\overline{)\,6x^2+5x-2} \\
-(6x^2-4x) \\
\hline
9x-2 \\
-(9x-6) \\
\hline
4
\end{array}$$

$$g(x)=2x+3+\frac{4}{3x-2}$$

X→∞ y→?

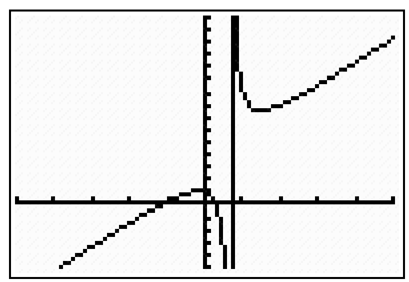
Use long division.

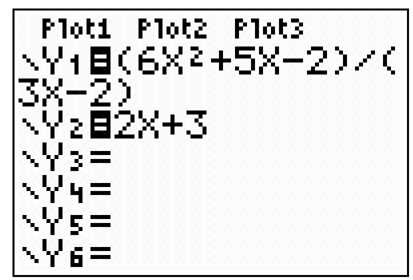
$$g(x)=2x+3+\cancelto{0}{\frac{4}{3x-2}}$$

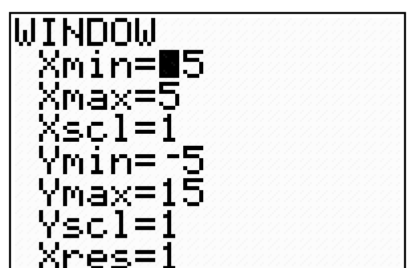
The fraction approaches <u>zero.</u>

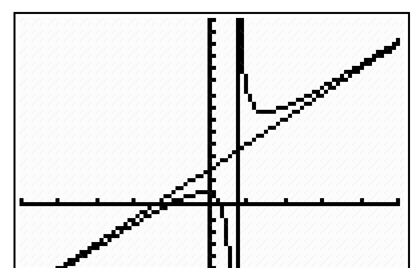
X→∞ g(x) → 2x + 3

<u>Oblique</u> asymptote: y = 2x + 3









$$y = \frac{6(x+4)\cancel{(x-1)}(x+2)}{2\cancel{(x-1)}(x+3)} \qquad y = \frac{6(x+4)(x+2)}{2(x+3)}$$

x-intercept? (-4, 0), (-2, 0)

y-intercept? (0, 8)

Vertical asymptote? x = -3

holes? f(1) = 45/4

Oblique asymptote? $y = 3x + 9$

$$\begin{array}{r} 3x \quad +9 \\ 2x+6 \overline{\big) 6x^2 + 36x + 48} \\ -(6x^2 + 18x\ ) \\ \hline 18x + 48 \\ -(\ 18x + 54\ ) \\ \hline -6 \end{array}$$

$$y = 3x + 9 + \frac{-6}{2x+6}$$

$$y = 3x + 9 + \frac{-3}{x+3}$$

4

$$g(x) = 2x + 3 + \frac{4}{3x-2}$$

x→∞ y→?

Your turn: find the asymptote

$$g(x)=\frac{x^3+2x^2+3x-4}{x+2}$$

$$\begin{array}{r l}
 & x^2+3 \\
x+2 \,\big) & x^3+2x^2+3x-4 \\
 & -(x^3+2x^2) \\
\hline
 & 3x-4 \\
 & -(3x+6) \\
\hline
 & -10
\end{array}$$

Use long division.

$$g(x)=x^2+3-\frac{10}{x+2} \quad 0$$

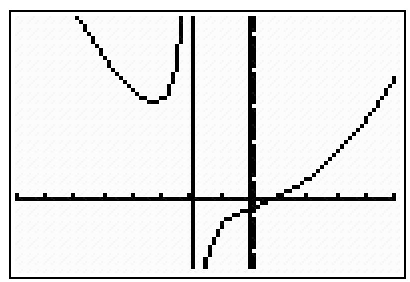
The fraction approaches zero.

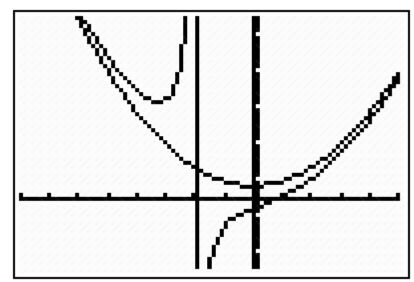
X→∞ g(x) → x^2+3

Oblique asymptote: $y=x^2+3$

$$g(x)=x^2+3-\frac{10}{x+2}$$

X→∞ y→?





```
WINDOW
  Xmin=-8
  Xmax=5
  Xscl=1
  Ymin=-15
  Ymax=40
  Yscl=1
  Xres=1
```

```
Plot1  Plot2  Plot3
\Y1=(X^3+2X²+3X-
4)/(X+2)
\Y2=X²+3
\Y3=
\Y4=
\Y5=
\Y6=
```