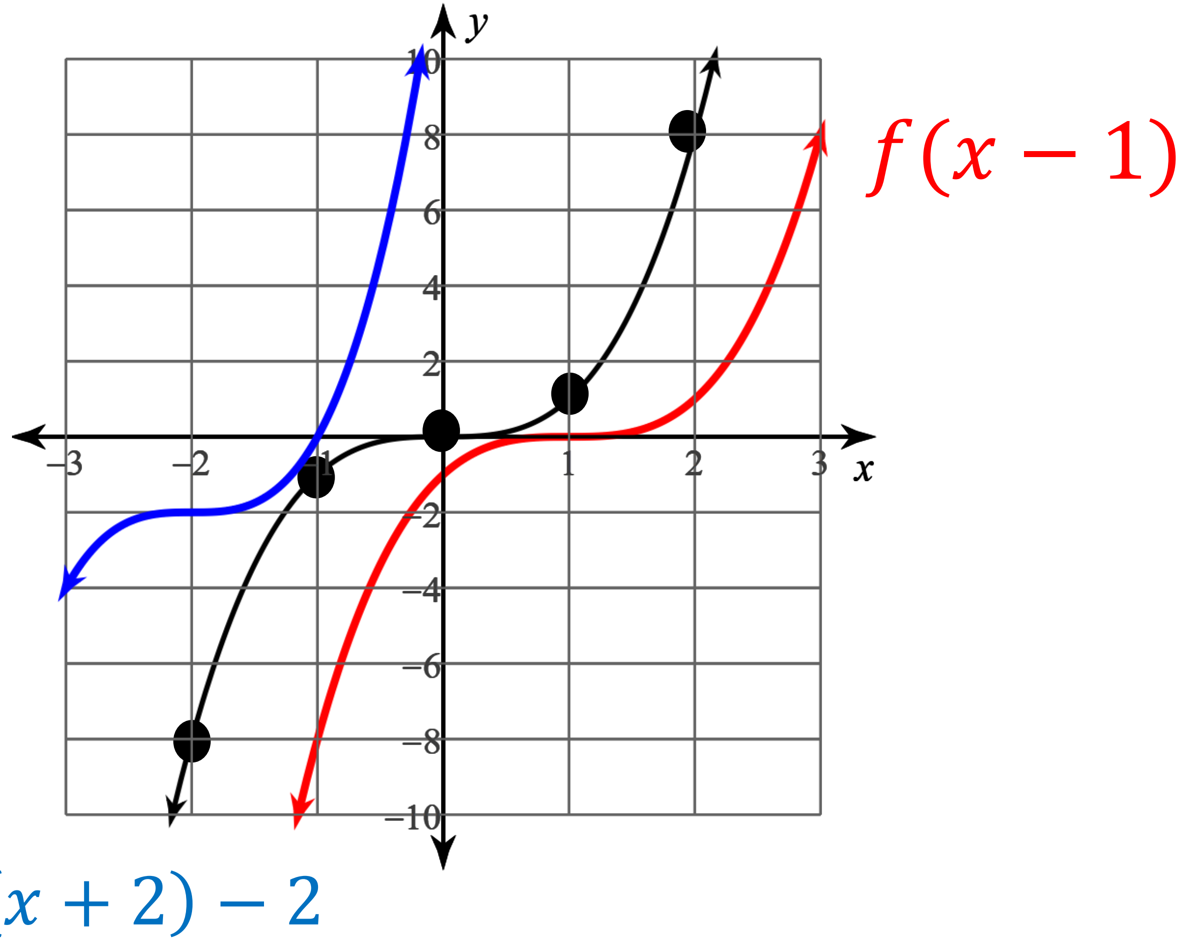
Math-1050
Session #11
(Textbook 5.1: Polynomials)

The Cube Function

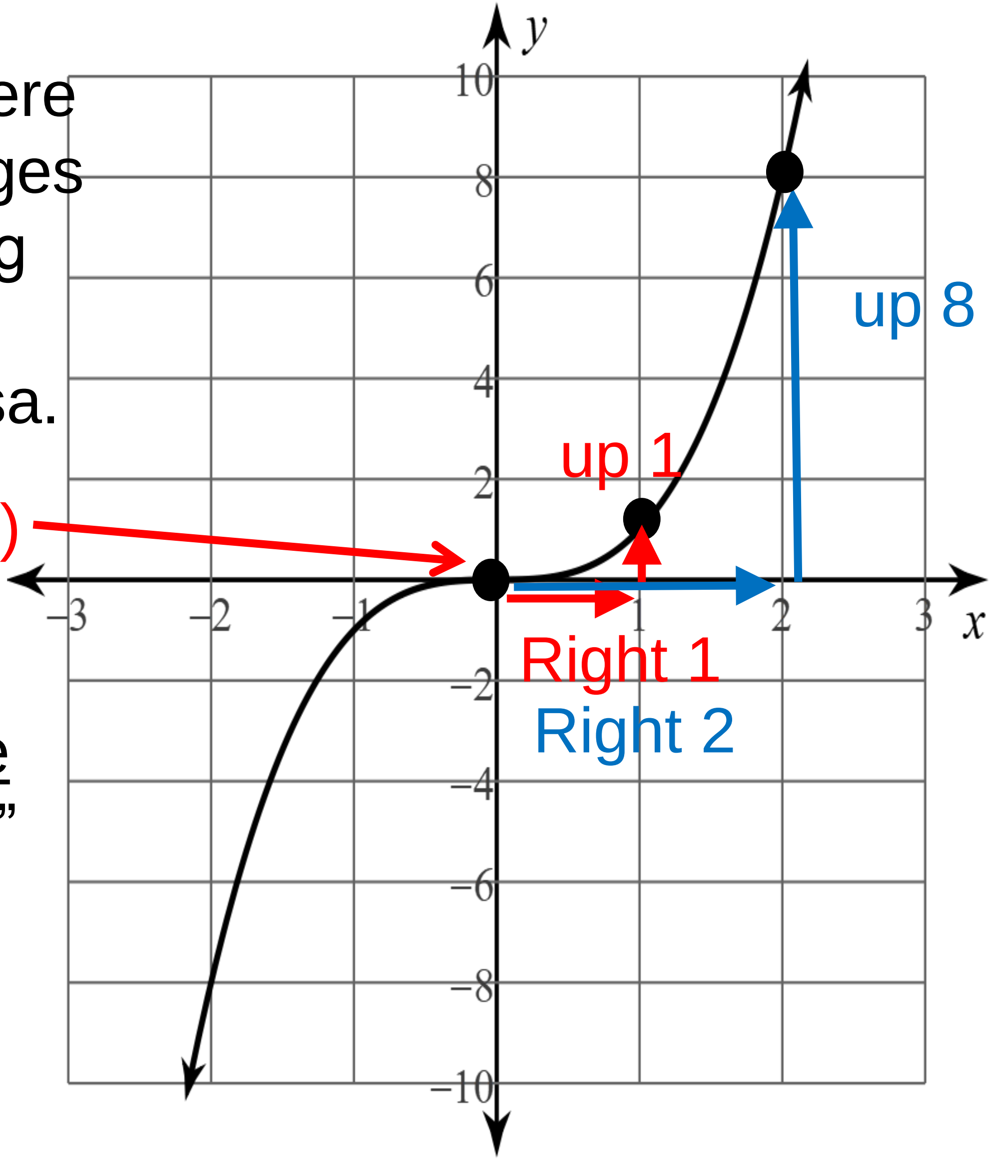
$$f(x) = x^3$$

x	y
-2	-8
-1	-1
0	0
1	1
2	8



$f(x) = x^3$

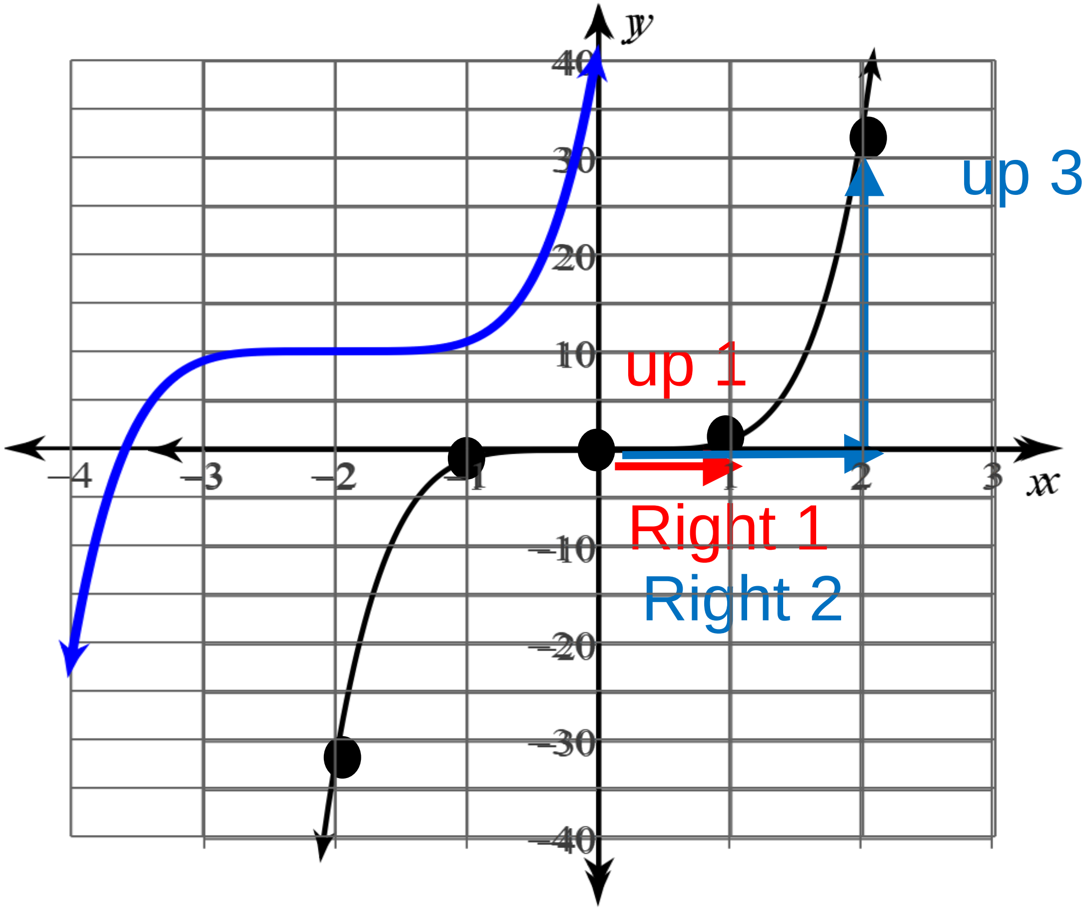
<u>Inflection Point</u>: the point where the shape of the graph changes from “concave down” (curving downward) to “concave up” (curving upward) or vice versa.



<u>Shape of the graph:</u> Not vertically stretched: <u>from the inflection point</u> “right 1, up 1” And “right 2, up 8"

$f(x) = x^5$

x	y
-2	-32
-1	-1
0	0
1	1
2	32



$f(x+2)+1$

Polynomial: An equation (or an expression) with same-base powers being added that are raised to a *natural number* exponent.

Example: $y = 8x^5 + 5x^4 + 9x^3 + x^2 + 2x + 3$

Not a polynomial $y = x^{0.5} + 3x^{2/3} + 6\sqrt{x}$

Lead coefficient: the coefficient of the largest power.

$$y = -8x^5 + 5x^4 + 9x^3 + x^2 + 2x + 3$$

Degree: the largest exponent of the polynomial.

Standard Form Polynomial A polynomial ordered so that the exponents get smaller from the left-most term to the right-most term.

$$y = \underline{8x^5} + \underline{5x^4} + \underline{9x^3} + \underline{x^2} + \underline{2x} + \underline{3}$$

Term: powers (or the constant) separated by either a '+' or '-' symbol.

Number of terms: If all terms are present, a 2nd degree polynomial as 3 terms in standard form.

$$y = 2x^2 - 4x + 5$$

If you include the number *zero* as a possible coefficient, an "n-th degree polynomial has n+1 terms (i.e., a 3rd degree has 4 terms).

$$y = 4x^3 + 0x^2 - 4x + 5$$

Intercept Form Polynomial A polynomial that has been factored into *linear factors*, from which you can identify the input values that make the output value equal to zero.

Example: $$y = 6(x+4)(x+3)(x-2i)(x+2i)$$

Linear factors: the exponent of the power is a '1'.

Why do we call these linear factors?

$y = mx + b$ Is a linear equation so (x + 2) is a linear factor

Fundamental Theorem of Algebra: If a polynomial has a degree of "n", then the polynomial has "n" zeroes (provided that repeat zeroes, called "multiplicities" are counted separately).

$$y = 6x^4 + 42x^3 + 96x^2 + 28x + 48$$

"4th Degree" → 4 zeroes x = -4, -3, 2i, -2i

Linear Factorization Theorem: If a polynomial has a degree of "n", then the polynomial can be factored into "n" linear factors.

$$y = 6(x+4)(x+3)(x-2i)(x+2i)$$

Since each linear factor has one zero, these two theorems are almost saying the same thing.

Zeroes of Polynomials: come from linear factors.

$$y = 6x^4 + 42x^3 + 96x^2 + 28x + 48$$

$$0 = 6(x+4)(x+3)(x-2i)(x+2i)$$

$$x = -4 \quad x = -3 \quad x = 2i \quad x = -2i$$

If the polynomial is already in intercept form: just find the zeroes.

$$0 = (x+5)(x-2)(x-\sqrt{3})(x+\sqrt{3})$$

$$x = -5 \quad x = 2 \quad x = \sqrt{3} \quad x = -\sqrt{3}$$

Given the zeroes, write the equation of the Polynomial:

$x = -2 \quad x = 3 \quad x = 6$

$y = (x + 2)(x - 3)(x - 6)$

Polynomial: An equation (or an expression) with same-base powers being added that are raised to a *non-negative integer exponents.* → Convert to "standard form."

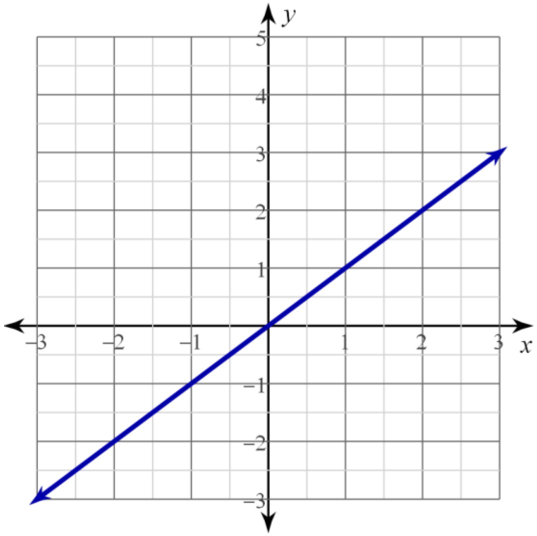
$y = a(x + 2)(x - 3)(x - 6)$ → Assume VSF = 1

$y = (x + 2)(x^2 - 9x + 18)$

	x^2	$-9x$	18
x	x^3	$-9x^2$	$18x$
2	$2x^2$	$-18x$	36

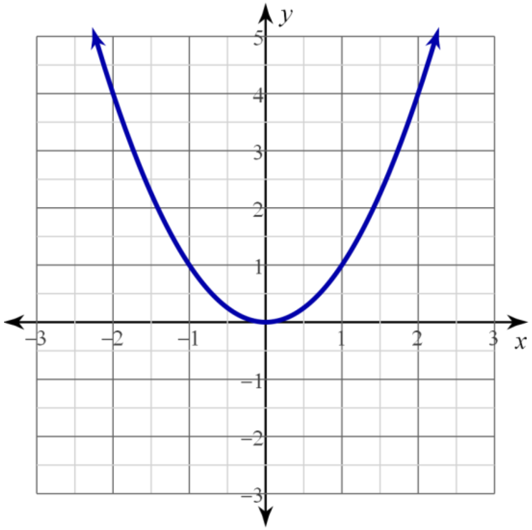
$y = x^3 - 7x^2 + 36$

What is the pattern between degree and end behavior?



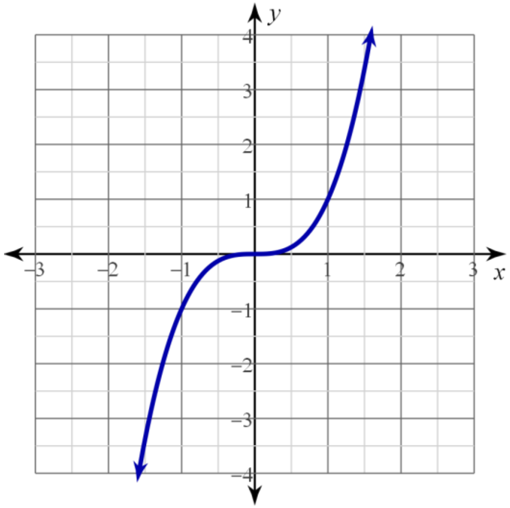
$y = x$

↓ left ↑ right



$y = x^2$

↑ left ↑ right



$y = x^3$

↓ left ↑ right

Odd degree: ↓ left ↑ right even degree ↑ left ↑ right

What is the pattern between degree and # turns?

An (n-degree) polynomial turns (n – 1) times.

Degree vs. End Behavior

$$y = x^5 + x^4 + x^3 + x^2 + x + 1$$

Pick a very large input value: 1,000,000 = 10^6 then compare each term.

$$(10^6)^5 = 10^{30}$$

$$(10^6)^4 = 10^{24}$$

$$(10^6)^3 = 10^{18}$$

$$(10^6)^2 = 10^{12}$$

$$(10^6 \quad = 10^6$$

$$1 = 10^0$$

Compare the largest two powers.

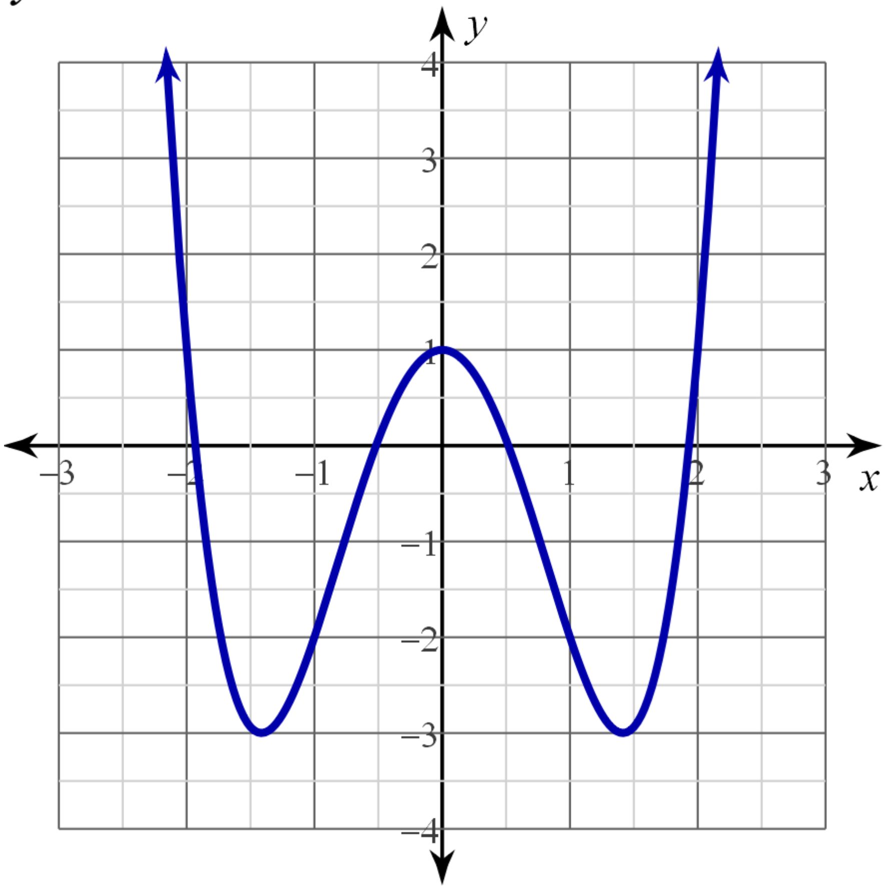
$$10^{30} = 10^{24} * 10^6$$

Input = 1,000,000 → output coming from the 1st term is 1,000,000 times as big as the output coming from the second term

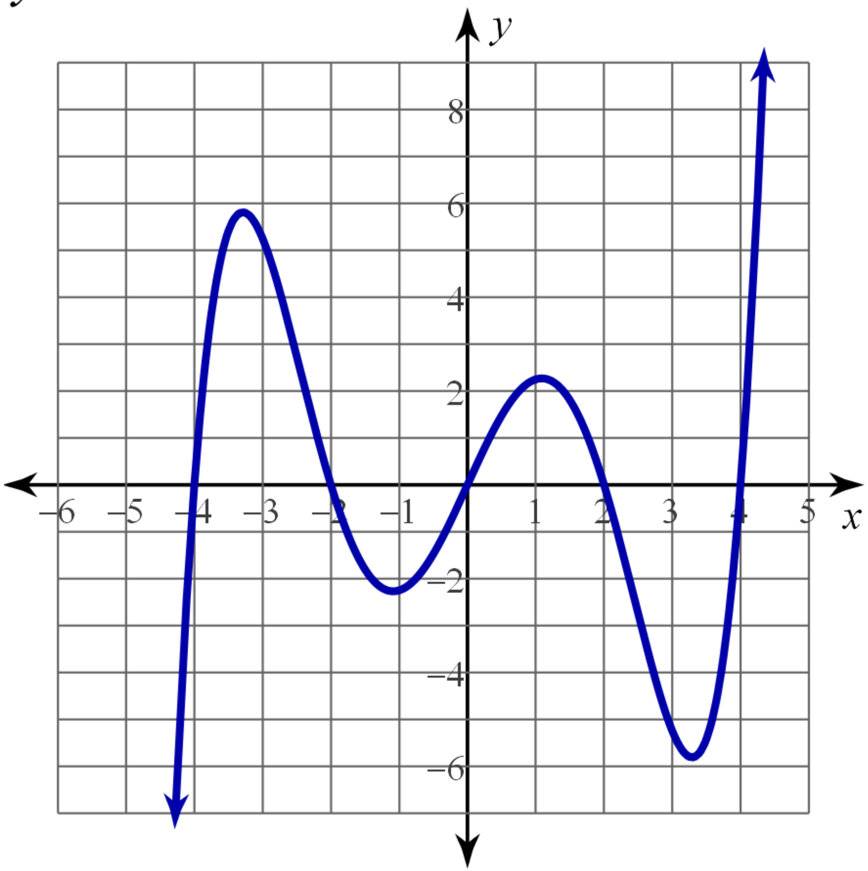
The term with the largest exponent **dominates** the output of the function.

Max number of x-intercepts?

$y = x^4 - 4x^2 + 1$



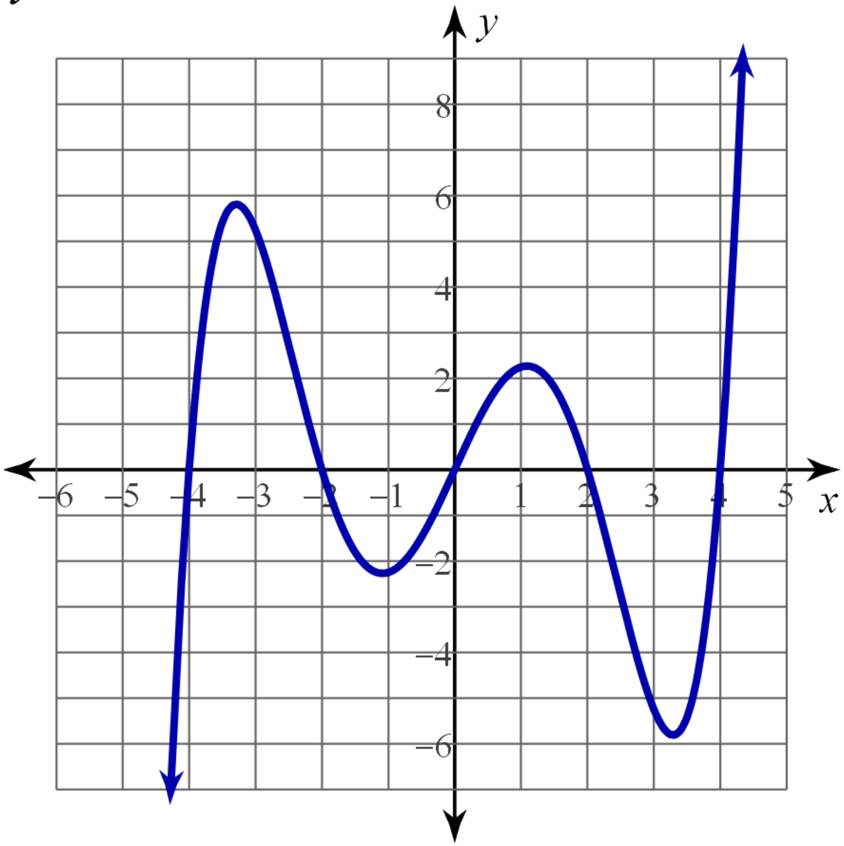
$y = x^5 - x^4 - 4x^2 + 1$



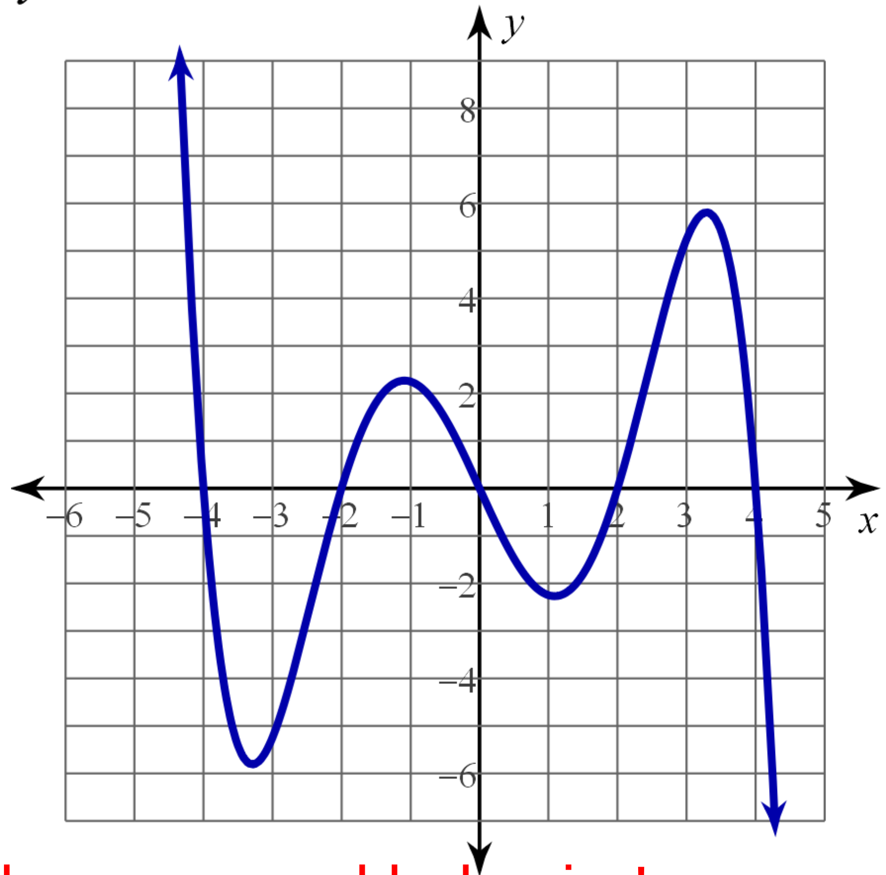
The degree of the polynomial equals the number of zeroes AND gives you the max number of x-intercepts (real number zeroes).

Lead Coefficient & Degree→ End Behavior?

$y = x^5 - x^4 - 4x^2 + 1$



$y = -x^5 - x^4 - 4x^2 + 1$



All odd degree polynomials have the same end behavior!

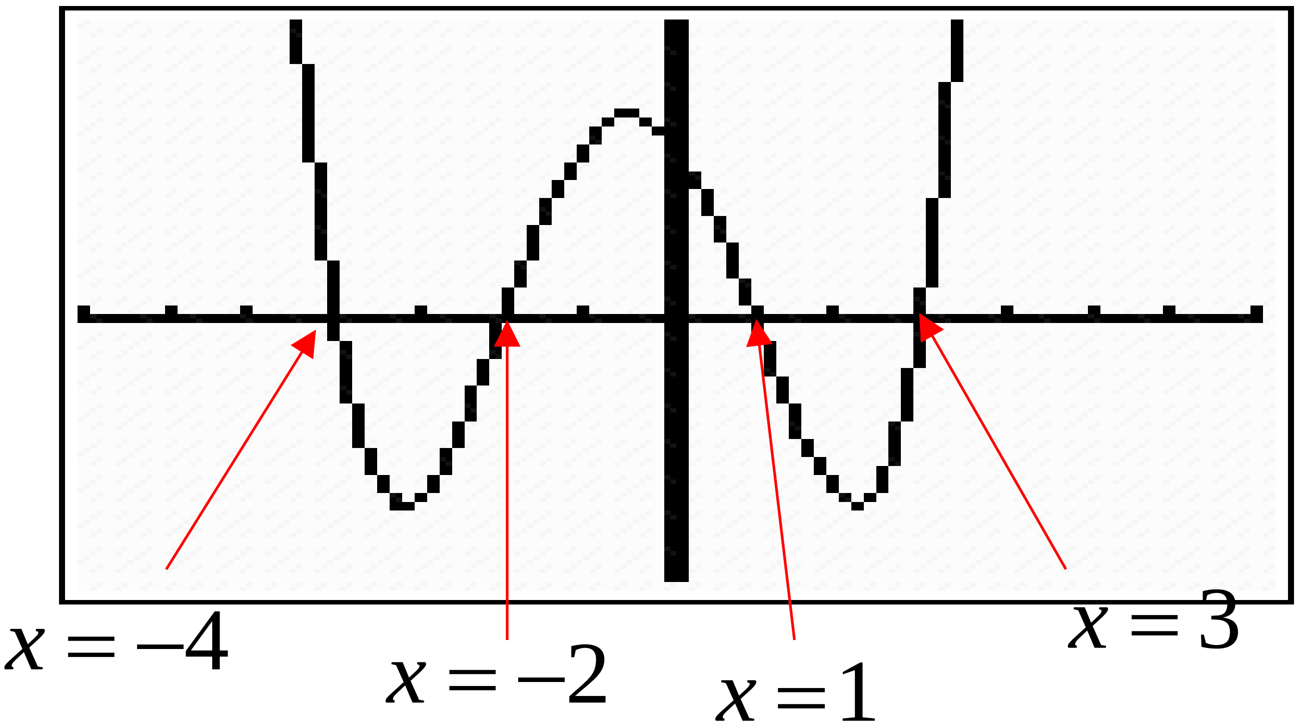
negative lead coefficient: reflection across the x-axis, all negative-odd polynomials have the same end behavior!

4^{th} Degree Polynomial (even degree)

$$y = (2x+8)(3x+6)(x-1)(x-3)$$

"left * left * left * left" = ? $= 6x^4$

$$y = 6x^4 + \ldots\text{(other terms)}$$



Fundamental Theorem of Algebra: If the polynomial has degree 'n', then polynomial has 'n' number of "zeroes" (provided that we count "repeated" x-intercepts separately)

2nd degree polynomial → "two zeroes"

$$y = (x-1)^2 = (x-1)(x-1)$$

x = 1 is a "zero" of the polynomial twice.

We say that x = 1 is a zero of the polynomial with multiplicity 2

Multiplicity: the number of times a zero is repeated for a polynomial.

How can you tell if there are zeroes that are multiplicities?

$$y = 3(x-2)^3(x+4)^2(x-\sqrt{5})(x+\sqrt{5})(x-3i)(x+3i)$$

Zeroes: x = 2 (multiplicity 3), -4 (mult. 2), etc.

What is the degree of this polynomial?

(3rd degree) *(2nd degree)*(1st)(1st)(1st)(1st) = 9th degree

Multiply powers property of exponents!!!

Complex Conjugates Theorem

If f(x) is a function with real coefficients and if (a + bi) is a zero of f(x), then its complex conjugate (a - bi) is also a zero of f(x).

Example: $x = 2i$, $x = -2i$

Example: $x = 3 - 2i$, $x = 3 + 2i$

Irrational Roots Theorem

If f(x) is a function with real coefficients and if

$$a - \sqrt{b}$$

is a zero of f(x), then its irrational conjugate

$$a + \sqrt{b}$$

is also a zero of f(x).

Example: $\sqrt{3}, \quad -\sqrt{3}$

Example: $4 - \sqrt{2}, \ 4 + \sqrt{2}$

Describe the end behavior

$$f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$$

Positive lead coefficient, even degree

Up on left/right

$$f(x) = -7x^3 - 8x^2 + 14x + 8$$

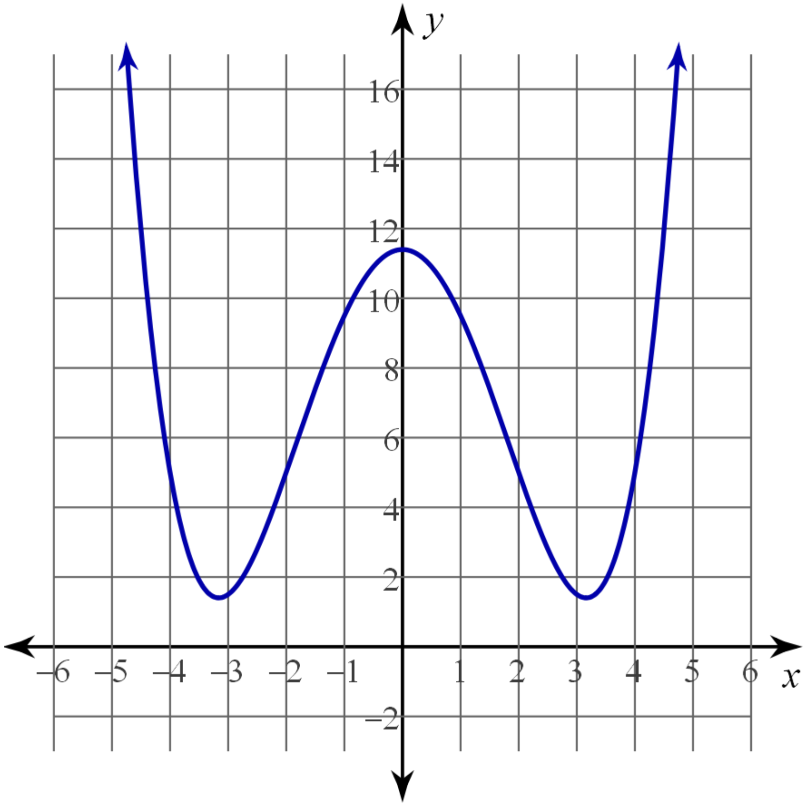
negative lead coefficient, odd degree

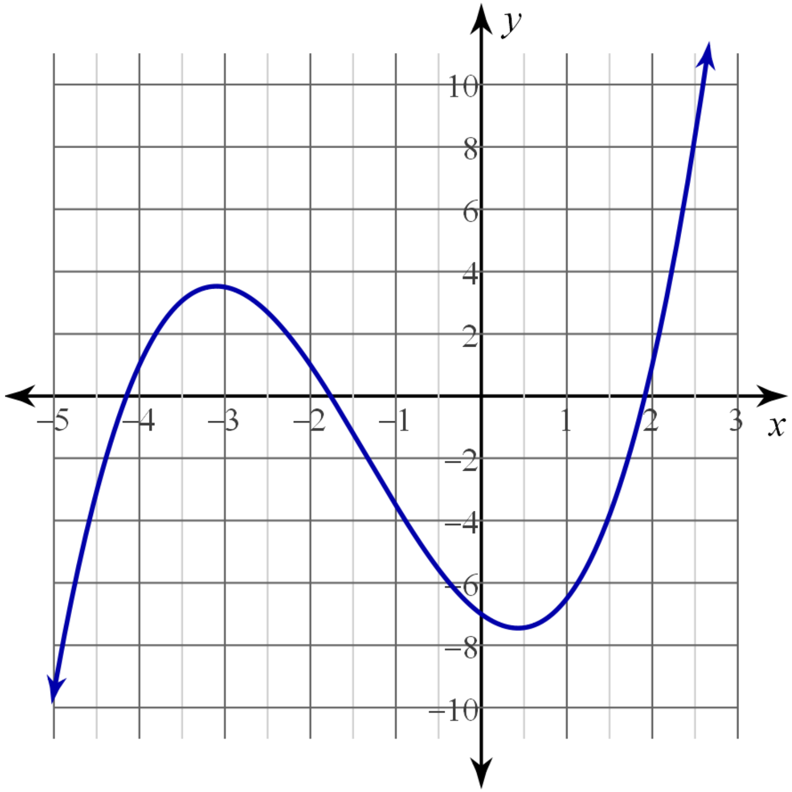
Up on left, down on right

How many real zeroes will the polynomial have?

Does an even degree polynomial necessarily cross the x-axis?

Does an odd degree polynomial necessarily cross the x-axis?





Make a table of the possible zeroes by category

Degree	Real zeroes	Imaginary Zeroes
2	2	0
	0	2

Make a table of the possible zeroes by category

Degree	Real zeroes	Imaginary Zeroes
3	0	3
	1	2
	2	1
	3	0

Not possible

Not possible

The General Shape of the Graph of a Polynomial

$$f(x) = (x-2)(x-3)(x+4)$$

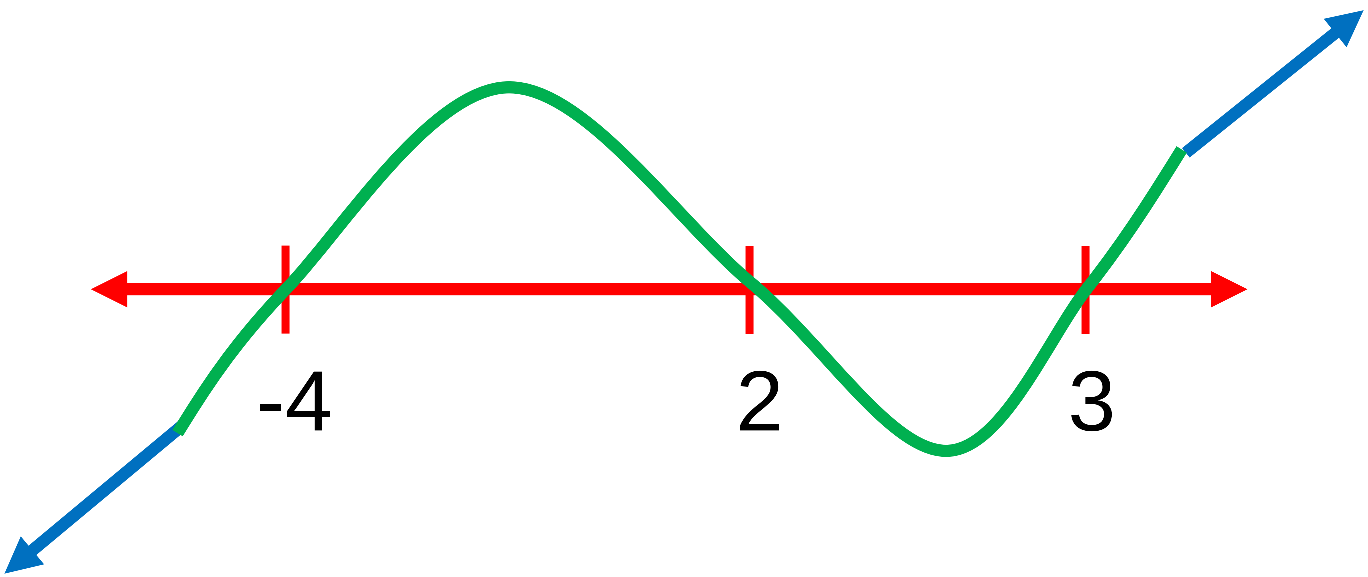
zeroes: x = 2, 3, and -4.

All are real numbers.

All are x-intercepts.

positive lead coefficient and an odd degree.

The end behavior is **Up on right, down on left**?



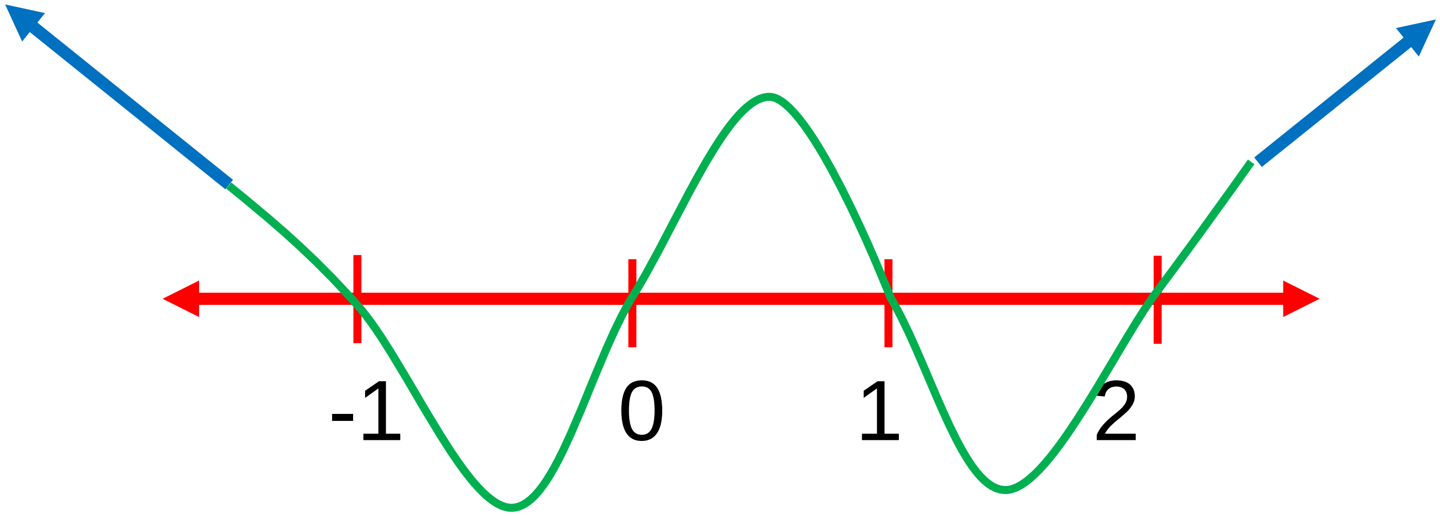
$$f(x) = x(x+1)(x-1)(x-2)$$

zeroes: x = 0, -1, 1, and 2.

All are real numbers. All are x-intercepts.

positive lead coefficient and an even degree.

The end behavior is **Up on right, up on left**?



$$f(x)=(x+1)^2(x+3)(x-4)$$

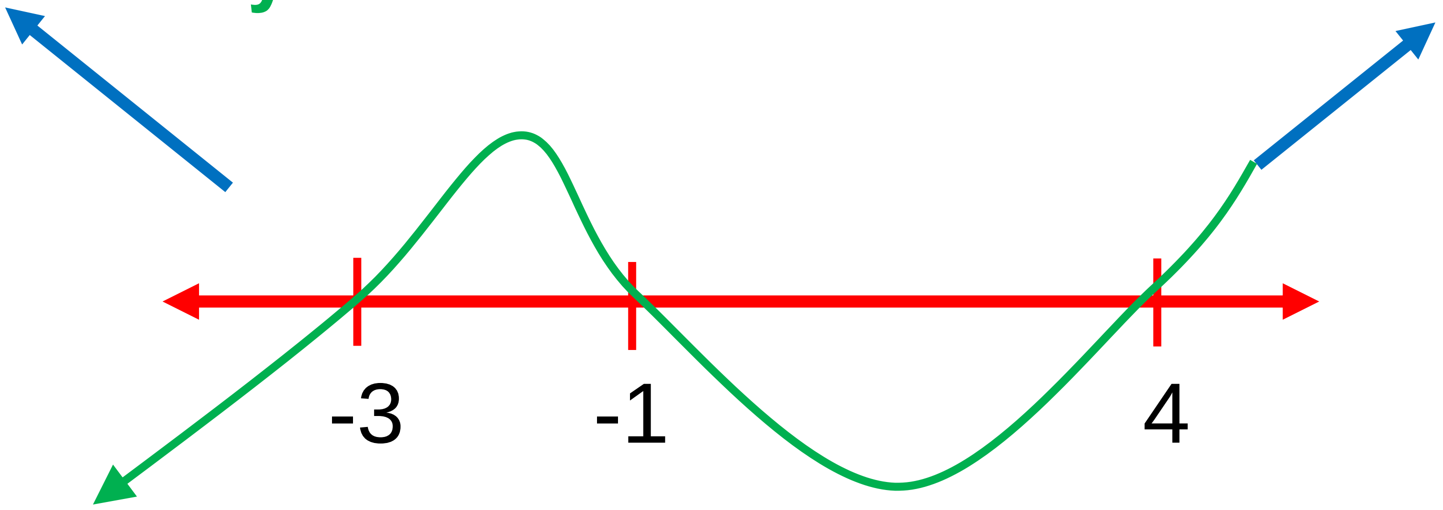
zeroes: x = -1, -1, -3, and 4.

All are <u>real numbers.</u> All are <u>x-intercepts.</u>

<u>positive lead coefficient</u> and an <u>even degree.</u>

The <u>end behavior</u> is **Up on right, up on left**?

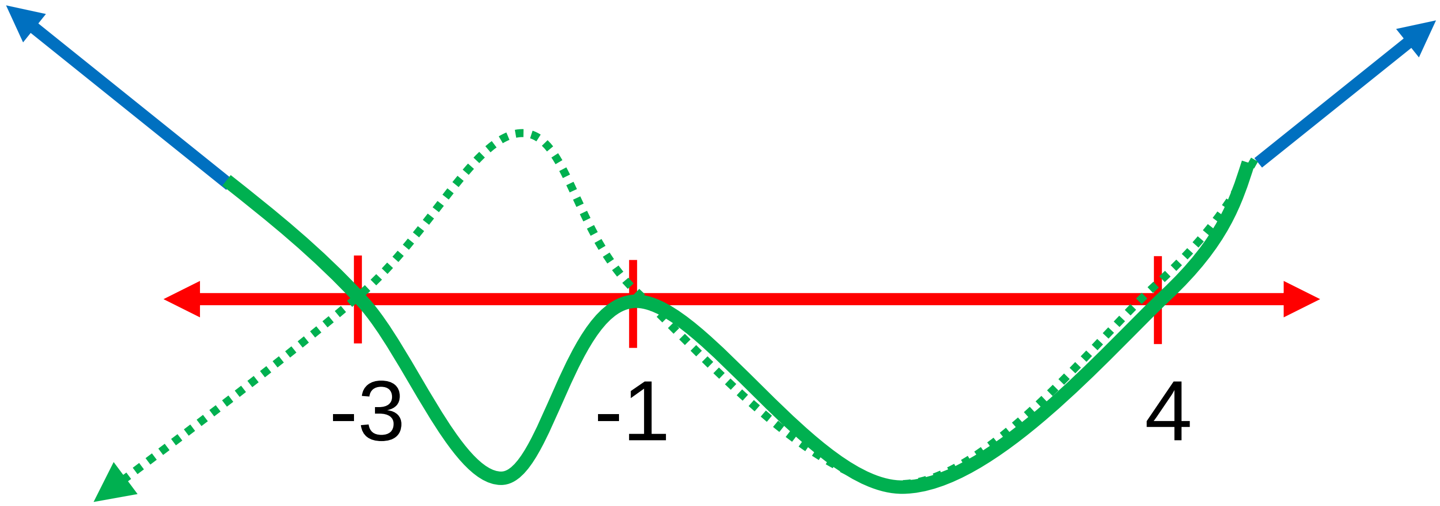
Why doesn't the "end behavior" line up?



$$f(x) = (x+1)^2(x+3)(x-4)$$

It has the following zeroes: x = -1, -1, -3, and 4.

The zero with an EVEN "multiplicity will just "kiss" the x-axis. Remember $y = x^2$?



$$f(x) = (x+2i)(x-2i)(x-4)^2(x+2)$$

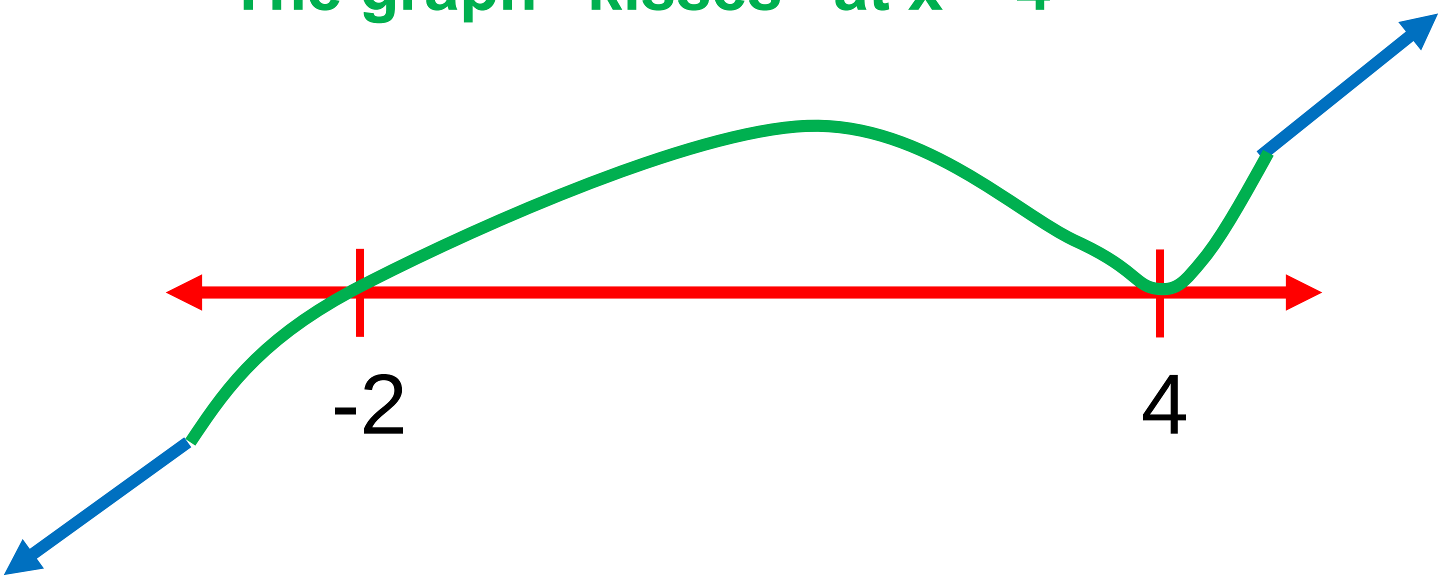
It has the following zeroes: x = 2i, -2i, 4, 4, and -2.

Only 4 and -2 are real numbers. These are x-intercepts.

positive lead coefficient and an odd degree.

The end behavior is **Up on right, down on left** ?

The graph “kisses” at x = 4



How does the graph *"behave"* near the zero: x = 1

$$f(x) = x(x+1)(x-1)(x-2)$$

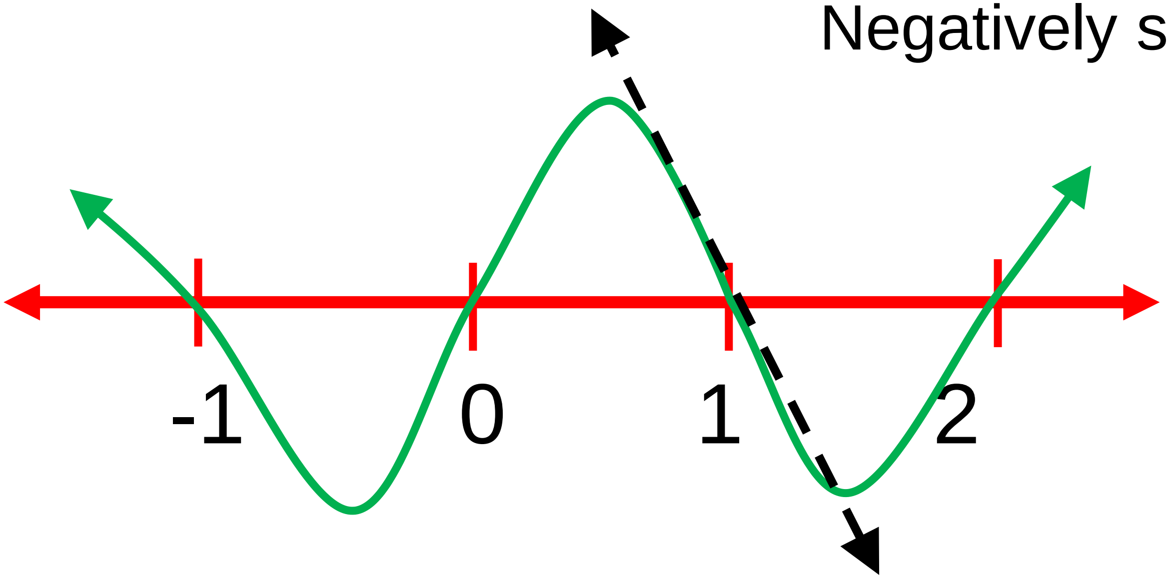
Substitute x = 1 into every linear factor *except the one causing the zero* of the function.

$$f(x) = 1(1+1)(x-1)(1-2)$$

$$f(x) = 1(2)(x-1)(-1)$$

$$f(x) = -2(x-1) \quad f(x) = -2x+2$$

Negatively sloped line



$$f(x) = (x+1)^2(x+3)(x-4)$$

How does the graph ***"behave"*** near the zero: x = -1

Substitute x = -1 into every linear factor ***except the one causing the zero*** of the function.

$$f(x) = (x+1)^2(-1+3)(-1-4)$$

$$f(x) = (x+1)^2(2)(-5)$$

$$f(x) = -10(x+1)^2$$

Downward opening parabola with vertex at (0, -1)