Math-1050 Session #11 (Textbook 5.1: Polynomials)

Polynomial: An equation (or an expression) with same-base powers being added that are raised to a ***natural number exponent.***

Example: $y = 8x^5 + 5x^4 + 9x^3 + x^2 + 2x + 3$

Not a polynomial $y = x^{0.5} + 3x^{2/3} + 6\sqrt{x}$

Lead coefficient: the coefficient of the largest power.

$$y = -8x^5 + 5x^4 + 9x^3 + x^2 + 2x + 3$$

Degree: the largest exponent of the polynomial.

Standard Form Polynomial A polynomial ordered so that the exponents get smaller from the left-most term to the right-most term. $y = 8x^5 + 5x^4 + 9x^3 + x^2 + 2x + 3$

Term: powers (or the constant) separated by either a '+' or '-' symbol.

Number of terms: If all terms are present, a 2nd degree polynomial as 3 terms in standard form.

$$y = 2x^2 - 4x + 5$$

If you include the number *zero* as a possible coefficient, an "n-th degree polynomial has n+1 terms (i.e., a 3rd degree has 4 terms). $y = 4x^3 + 0x^2 - 4x + 5$

Intercept Form Polynomial A polynomial that has been factored into *linear factors*, from which you can identify the input values that make the output value equal to zero.

Example: $y = 6(x+4)(x+3)(x-2i)(x+2i)$

Linear factors: the exponent of the power is a ‘1’.

Why do we call these linear factors?

$y = mx + b$ The linear equation is a 1st degree polynomial so (x + 2) is a linear factor

Solve by factoring: If the equation has only one variable (‘y’ has already been set to zero), solve by factoring means to convert a standard form polynomial into intercept form (by factoring) and then identifying the zeroes of the polynomial. $y = 6x^4 + 42x^3 + 96x^2 + 28x + 48$

$$0 = 6(x+4)(x+3)(x-2i)(x+2i)$$

$$x = -4 \quad x = -3 \quad x = 2i \quad x = -2i$$

Find the zeroes: means that the equation has two variables. 1st step: set y = 0, then solve by factoring.

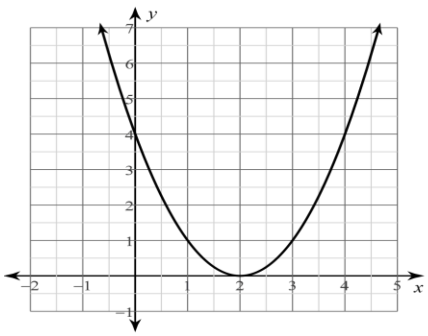
If the polynomial is already in intercept form: “solve by factoring “ means just find the zeroes.

$$0 = (x+5)(x-2)(x-\sqrt{3})(x+\sqrt{3})$$

$$x = -5 \quad x = 2 \quad x = \sqrt{3} \quad x = -\sqrt{3}$$

The “end behavior” of a function means: “on the right end of the graph is the y-value going UP or DOWN? And “on the left end of the graph, is the y-value going UP or DOWN?

$f(x) = (x-2)^2$



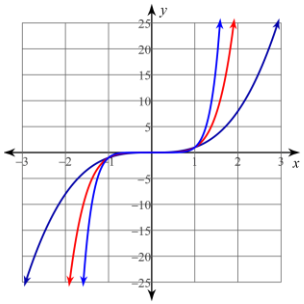
In English we could say: “up on right, up on left”

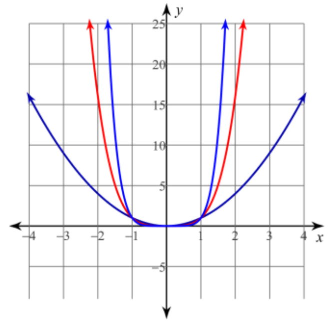
As ‘x’ gets bigger (right end) ‘y’ gets bigger (goes upward)

As ‘x’ gets smaller (left end), ‘y’ gets bigger (goes upward)

The "<u>end behavior</u>" of all *<u>odd-degree polynomials</u>* is the same. The graphs of all *<u>even-degree</u>* polynomials is the same. The graphs below are:

$y = x^3$
$y = x^5$
$y = x^7$

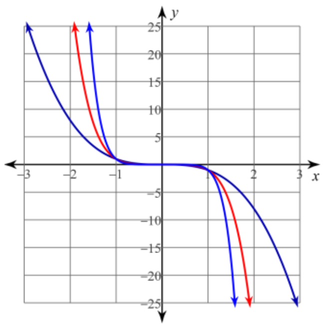


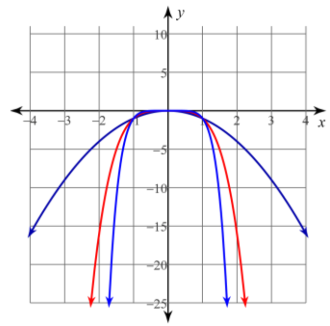


$y = x^2$
$y = x^4$
$y = x^6$

Reflection across the x-axis will change the end-behavior.

$y = -x^3$
$y = -x^5$
$y = -x^7$





$y = -x^2$
$y = -x^4$
$y = -x^6$