S10 Simply, Add, Subtract Radicals

A square root radical is simplified, or in its simplest form when the radicand has no square factors.

A cube root radical is simplified, or in its simplest form when the radicand has no cubed factors.

A fourth root radical is simplified when the radicand has no factors that are raised to the 4^{th} power, and so on.

8 is not a perfect square so $\sqrt{8}$ can't be simplified into an integer. However, 8 has a perfect square factor (4) that CAN be simplified into an integer.

For example: $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$

Use your calculator to verify that $\sqrt{8} = 2\sqrt{2}$.

Note that $2\sqrt{2}$ is EXACTLY equal to $\sqrt{8}$ so it's more accurate than using a calculator to get a decimal approximation.

When simplifying radicals, it's helpful to easily recognize numbers that are perfect squares and perfect cubes. The most common perfect square factors you will use when simplifying square roots are: 4, 8, 16, and 25. The most common perfect cube factors are 8 and 27 (and 125 for simplifying really large radicands.)

Example 1: Simplify $\sqrt{150}$.

$$\sqrt{150} = \sqrt{25 \cdot 6} = \sqrt{\mathbf{25}} \cdot \sqrt{6} = \mathbf{5}\sqrt{6}$$

Notice that there are other ways of factoring 150 but the idea is to rewrite 150 using the largest, perfect square factor of that number. A calculator can be helpful for dividing radicands by 4, 8, 16, or 25 to see if the number is divisible by these perfect square factors.

There are no perfect square factors left under the radical so $5\sqrt{6}$ is simplified.

Example 2: Simplify $\sqrt[3]{32}$.

$$\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{\mathbf{8}} \cdot \sqrt[3]{4} = \mathbf{2}\sqrt[3]{4}$$

Notice that there are other ways of factoring 32 (like $16 \cdot 2$) but the idea is to rewrite the number using a perfect cube factor -in this case, 8. When simplifying a cube root, start by seeing if the number is divisible by 8 or 27

There are no perfect cube factors left under the radical so $2\sqrt[3]{4}$ is simplified.

1. Explain why x^2, x^4, x^6, x^8, or any other variable with even exponents are perfect square factors. How might this idea help you simplify $\sqrt{x^9}$?

2. Explain why x^3, x^6, x^9, or any other variables with exponents that are multiples of 3 are perfect cube factors. How might this idea help simplify $\sqrt{x^7}$?