S9 Rational Exponents

Name: ___________________________

The properties of exponents can be expanded to include **rational** exponents.

For example, given 4^3, the exponent 3 means there are 3 factors of 4.

1. Assuming the use of an exponent doesn't change from one scenario to another, what does this mean about $4^{1/2}$

Example 1: Write $4^{1/2}$ without an exponent.

In the case of $4^{1/2}$, start by factoring 4: $4 \rightarrow 2 \times 2$, and then take half of those factors.
$4^{1/2} = 2$

Example 2: Evaluate $9^{1/2}$.

Taking half of the factors of 9 means taking one of its two factors of 3.
$9^{1/2} = 3$.

2. Find $27^{1/3}$ by listing the factors of 27 and taking 1/3 of those factors listed.

3. Use the same method to find $27^{2/3}$.

You may have noticed that evaluating $9^{1/2}$ is similar to evaluating $\sqrt{9}$. It is the same.

> Let *a* represent a nonnegative real number, symbolically written as *a* $a \geq 0$. The principal **square root** of *a*, denoted by $\sqrt{a}$, is defined as the nonnegative number that, when squared, produces *a*.

Note that taking the square root of a number is the inverse operation of squaring a number AND the same thing as taking half of the factors of the squared number.

$$(\sqrt{9})^2 = 9, \ \sqrt{9^2} = 9$$

AND $$(9^{1/2})^2 = 9, \quad (9^2)^{1/2} = 9$$

In general, since squaring and square rooting are inverse operations, $(\sqrt{a}\,)^2 = a \ if \ a \geq 0$.

4. Explain why $\sqrt{a} = not\ a\ real\ number$ if *a* is negative.

5. Prove algebraically that $\sqrt{a} = a^{1/2}$. In other words, solve for the exponent m if

$$a^m = \sqrt{a}\,, \qquad for\ a \geq 0.$$

6. Evaluate the following.

a. $36^{1/2}$ b. $-49^{1/2}$ c. $(-49)^{1/2}$ d. $0^{1/2}$

7. If the volume of a cube is 64 cubic inches, determine the length of one side of the cube.

> The **cube root** of any real number *a*, denoted by $\sqrt[3]{a}$, is defined as the number that when cubed, gives *a*.

8. To find the exponent, *m*, that's equivalent to taking the cube root of a number, solve

$$a^m = \sqrt[3]{a}, \qquad for\ any\ number\ a$$

9. Evaluate the following.

a. $\sqrt[3]{8}$ b. $\sqrt[3]{125}$ c. $\sqrt[3]{-1000}$ d. $\sqrt[3]{0}$

e. $\sqrt[3]{-8}$ f. $\sqrt[3]{100}$ (nearest tenth) d. $-\sqrt[3]{27}$ e. $-\sqrt[3]{-125}$

> In general, $\sqrt[n]{a} = a^{1/n}$, the *n*th root of *a.* The number *a*, called the **radicand**, must be nonnegative if *n,* called the **index**, is even*.*

10. Calculate each of the following, and then verify your answer using your calculator.

a. $\sqrt[4]{81}$ b. $32^{1/5}$ c. $\sqrt[5]{-32}$ d. $-225^{1/4}$ (calculator)

11. Try to compute $\sqrt[4]{-81}$ on your calculator. What happens and why?

12. Yachts that compete in the America's Cup must satisfy the International America's Cup Class rule that requires $L + 1.25\sqrt{S} - 9.8\sqrt[3]{D} \leq 16.296$ meters.

    Where L represents the yacht's length in meters,
    S represents the rated sail area, in square meters, and
    D represents the water displacement, in cubic meters.

    a. Is a yacht with length 21.85 meters, sail area 305.5 square meters, and displacement 21.85 cubic meters eligible to compete? Explain.

    b. Explain why the units of your numerical answer in part (a) are meters.

The properties of exponents can be expanded to include rational exponents where the numerator is different from one.

For example:

$$\begin{aligned} 8^{2/3} &= 8^{2\cdot(1/3)} \\ &= (8^2)^{1/3} \\ &= \sqrt[3]{8^2} \\ &= \sqrt[3]{64} \\ &= 4 \end{aligned}$$

In this example, the cube root was taken after the squaring was done. An equivalent answer can be found by taking the cube root of 8 first, then squaring the result.

In general, $a^{p/q} = \sqrt[q]{a^p}$ *or* $a^{p/q} = (\sqrt[q]{a}\,)^p$, where $a \geq 0$ if *q* is even and *p* and *q* are integers.

13. Compute each of the following. Show each step of the computation. Then verify the answer using your calculator.

    a. $25^{3/2}$ b. $(-8)^{2/3}$ c. $32^{4/5}$ d. $-16^{3/4}$

    e. $243^{2/5}$ f. $(-16)^{3/4}$ g. $-25^{3/2}$ h. $4^{-3/2}$

14. Compute $7^{2/3}$ on your calculator, and explain how you might reverse the operation to check the answer.

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Rational Exponents

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Simplify.

1) $\left(n^{4}\right)^{\frac{1}{2}}$

2) $\left(64x^{4}\right)^{\frac{1}{2}}$

3) $\left(r^{6}\right)^{\frac{4}{3}}$

4) $\left(100000r^{5}\right)^{\frac{3}{5}}$

5) $\left(16x^{6}\right)^{\frac{1}{2}}$

6) $\left(81r^{2}\right)^{\frac{1}{2}}$

7) $\left(x^{8}\right)^{-\frac{5}{4}}$

8) $\left(36n^{4}\right)^{\frac{1}{2}}$

9) $\left(x^{4}\right)^{\frac{3}{2}}$

10) $\left(64k^{2}\right)^{\frac{3}{2}}$

Write each expression in radical form.

11) $\left(2x\right)^{-\frac{3}{4}}$

12) $x^{\frac{2}{5}}$

13) $n^{\frac{3}{4}}$

14) $n^{\frac{3}{2}}$

Write each expression in exponential form.

15) $\left(\sqrt[3]{5a}\right)^{5}$

16) $\left(\sqrt[3]{7x}\right)^{4}$

17) $\sqrt[4]{r}$

18) $\left(\sqrt{6k}\right)^{5}$

19) $\left(\sqrt[4]{x}\right)^{7}$

20) $\dfrac{1}{\sqrt{10n}}$