Math-1010

Lesson 2-6

Rational Exponents

$\sqrt[4]{x^{13}}$ We can simplify this in two ways

1. $\sqrt[4]{x * x * x * x * x * x * x * x * x * x * x * x * x}$

x

x

x

$$\sqrt[4]{x^{13}} \rightarrow x * x * x * \sqrt[4]{x} \rightarrow x^{3}\sqrt[4]{x}$$

2. $\sqrt[4]{x^{12} * x^{1}} \rightarrow x^{12/4} * \sqrt[4]{x} \rightarrow x^{3}\sqrt[4]{x}$

We can write radical as powers!! $\sqrt[4]{x^{13}} \rightarrow x^{13/4}$

Are radicals related to powers?

$$x = \sqrt[3]{5}$$

$$x^3 = 5$$

Applying the property of equality,
I can take the "1/3" power of each side.

$$\left(x^3\right)^{1/3} = 5^{1/3}$$

$$x = 5^{1/3}$$

$$5^{1/3} = \sqrt[3]{5}$$

Another way to think about it.

$$9^{1/2} \rightarrow (3*3)^{1/2} \rightarrow 3$$

Factor 9 Take ½ of each <u>pair</u> of factors

$$27^{1/2} \rightarrow (3*3*3)^{1/2} \rightarrow 3(3)^{1/2}$$

Index number

3rd Root of 8 = ?: $\sqrt[3]{8}$

Nth Root of 5 = ?: $\sqrt[n]{5}$

Your turn:

Write the following in "radical form"

5th Root of 18 $= \sqrt[5]{18}$

4th Root of 25 $= \sqrt[4]{25}$

What type of number does 5th sound like?

$$\frac{1}{5}$$

Corresponding Parts of the 2 forms

$$4\sqrt[3]{x}$$

Index, Radicand, Coefficient

$$4x^{1/3}$$

Exponent, Base, Coefficient

Coefficient → Coefficient

Radicand → Base

Index → Denominator of the Exponent

The <u>index number</u> is the <u>denominator of the exponent</u>.

Writing Radicals in "Exponent Form"

Index # (of radical): same as the denominator of an exponent.

$$\sqrt[2]{x} = x^{1/2}$$

$$\sqrt[3]{7} = 7^{1/3}$$

None of these have coefficients!

Index # (of radical): same as the denominator of an exponent.

$$3\sqrt[2]{y} = 3y^{1/2}$$

$$5\sqrt[3]{7} = 5(7)^{1/3}$$

$$2\sqrt[4]{5} = 2(5)^{1/4}$$

Where did the coefficient of the radical end up when re-written in "power form"? **A Coefficient remains a coefficient.**

Notice that there was only a <u>single term</u> as the <u>radicand.</u>

rewrite in "exponent form"

$$\sqrt[5]{w} \rightarrow w^{1/5}$$

$$2x\sqrt[3]{10} \rightarrow 2x(10)^{1/3}$$

$$3\sqrt[4]{11} \rightarrow 3(11)^{1/4}$$

rewrite in "radical form"

$$x^{1/4} \rightarrow \sqrt[4]{x}$$

$$2(32)^{1/6} \rightarrow 2\sqrt[6]{32}$$

$$2(xy^3)^{1/3} \rightarrow 2y\sqrt[3]{x}$$

What happens if there is a <u>product</u> under the radical?

$$\sqrt[2]{xy} = (xy)^{1/2}$$

$$5\sqrt[3]{3x} = 5(3x)^{1/3}$$

$$2\sqrt[4]{21mn} = 2(21mn)^{1/4}$$

How did we show that the <u>index number</u> applied to the <u>entire product (radicand)</u> when re-written in "power form"?

Power of a product → product <u>inside parentheses</u> with an exponent.

What happens if there is a power under the radical?

$$\sqrt[5]{x^2 y} = (x^2 y)^{1/5} = x^{2/5} y^{1/5}$$

$$6\sqrt[3]{3m^2} = 6(3m^2)^{1/3} = 6(3^{1/3})m^{2/3}$$

How did we show that the index number applied to the entire product (including the power) when re-written in “power form”?

Power of a product → product inside parentheses with an exponent.

"Exponential Form" that has both a numerator and denominator

The exponent can be written as a rational number.

$$x^{5/2} \rightarrow \sqrt[2]{x^5}$$

Numerator:
Exponent of the base.

Denominator:
Root of the base.

$$\sqrt[3]{2^2} \rightarrow 2^{2/3}$$

Radical Form

Exponential Form

What form should you use? Usually, the problem will tell you.

rewrite in "radical form"

$$5(y)^{2/3} \rightarrow 5\sqrt[3]{y^2}$$

rewrite in "exponent form"

$$2\sqrt[5]{x^3y^2} \rightarrow 2x^{3/5}y^{2/5}$$

Product of Powers Property

$$y^2 * y^3 \rightarrow y^{2+3} \rightarrow y^5$$

When multiplying "same based powers" add the exponents.

$$x^{2/3}x^{3/4} \rightarrow x^{\frac{2}{3}+\frac{3}{4}} \rightarrow x^{\left(\frac{2}{3}*\frac{4}{4}\right)+\left(\frac{3}{4}*\frac{3}{3}\right)} \rightarrow x^{\frac{8}{12}+\frac{9}{12}} \rightarrow x^{\frac{17}{12}}$$

Exponent of a Power Property

When multiplying "same based powers" add the exponents.

$$(y^2)^3 \rightarrow y^{2*3} \rightarrow y^6$$

$$\left(y^{1/2}\right)^{2/3} \rightarrow y^{\frac{1}{2}*\frac{2}{3}} \rightarrow y^{\frac{2}{6}} \rightarrow y^{\frac{1}{3}}$$

$$\left(\frac{x^2}{y^{3/2}}\right)^{2/3} \rightarrow \frac{x^{\frac{2}{1}*\frac{2}{3}}}{y^{\frac{3}{2}*\frac{2}{3}}} \rightarrow \frac{x^{\frac{4}{3}}}{y^1}$$

Negative Exponent Property

We don’t want negative exponents in our answers

$$\frac{x^2 y^{2/3}}{y^{-1/2}} \rightarrow x^2 y^{2/3} y^{1/2} \rightarrow x^2 y^{\frac{2}{3}+\frac{1}{2}} \rightarrow x^2 y^{\frac{4}{6}+\frac{3}{6}} \rightarrow x^2 y^{\frac{7}{6}}$$

Rational Exponents in the Denominator

Rational exponent in the denominator means <u>irrational denominator</u>, which we <u>rationalize</u>

$$\frac{1}{y^{1/2}} \rightarrow \frac{1}{\sqrt{y}} * \frac{\sqrt{y}}{\sqrt{y}} \rightarrow \frac{\sqrt{y}}{y}$$

$$\frac{1}{y^{1/2}} * \frac{y^{1/2}}{y^{1/2}} \rightarrow \frac{y^{1/2}}{y}$$