

Math-1010

Lesson 2-6

Rational Exponents

$$\sqrt[4]{x^{13}}$$

We can simplify this in two ways

1. $\sqrt[4]{x * x * x * x * x * x * x * x * x * x * x * x * x}$

x

x

x

$$\sqrt[4]{x^{13}} \rightarrow x * x * x * \sqrt[4]{x} \rightarrow x^3 \sqrt[4]{x}$$

2. $\sqrt[4]{x^{12} * x^1} \rightarrow x^{12/4} * \sqrt[4]{x} \rightarrow x^3 \sqrt[4]{x}$

We can write radical as powers!!

$$\sqrt[4]{x^{13}} \rightarrow x^{13/4}$$

Are radicals related to powers?

$$x = \sqrt[3]{5}$$

$$x^3 = 5$$

Applying the property of equality,
I can take the “1/3” power of each side.

$$(x^3)^{1/3} = 5^{1/3}$$

$$x = 5^{1/3}$$

$$5^{1/3} = \sqrt[3]{5}$$

Another way to think about it.

$$9^{1/2} \rightarrow (3 * 3)^{1/2} \rightarrow 3$$

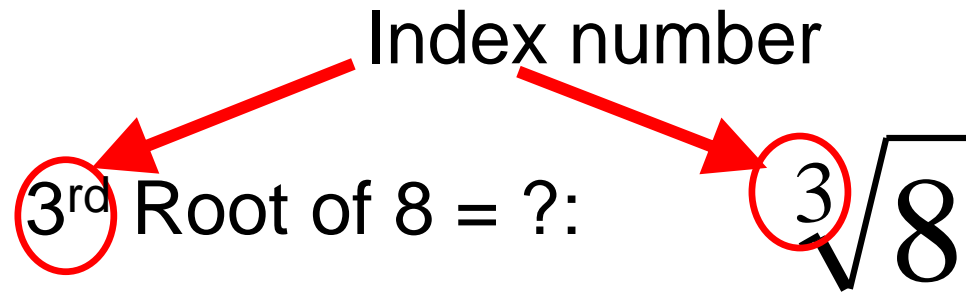
Factor 9 Take $\frac{1}{2}$ of each pair of factors

$$27^{1/2} \rightarrow (3 * 3 * 3)^{1/2} \rightarrow 3(3)^{1/2}$$

Index number

3^{rd} Root of 8 = ?:

$\sqrt[3]{8}$

A diagram with the text "Index number" at the top. Two red arrows originate from this text. The left arrow points to the "3" in "3rd" of the expression "3rd Root of 8 = ?:". The right arrow points to the "3" in the index of the cube root symbol in the expression "∛8".

N^{th} Root of 5 = ?:

$\sqrt[n]{5}$

Your turn:

Write the following in “radical form”

$$5^{\text{th}} \text{ Root of } 18 = \sqrt[5]{18}$$

$$4^{\text{th}} \text{ Root of } 25 = \sqrt[4]{25}$$

What type of number does 5^{th} sound like?

$$\frac{1}{5}$$

Corresponding Parts of the 2 forms

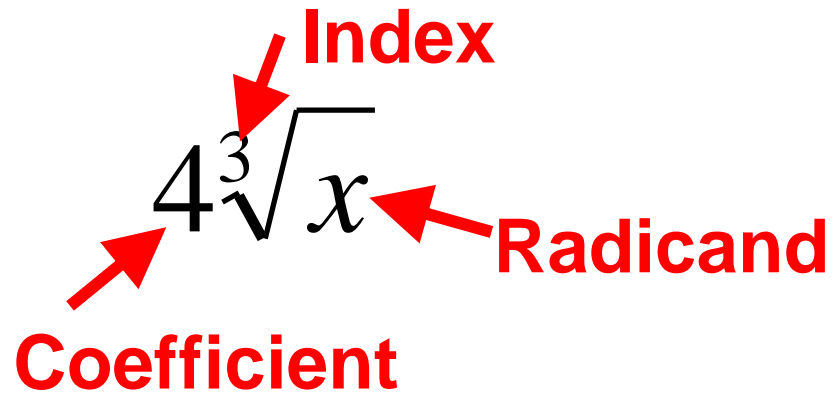


Diagram illustrating the parts of the radical form $4\sqrt[3]{x}$. Red arrows point from labels to the corresponding parts: **Index** points to the 3, **Radicand** points to the x , and **Coefficient** points to the 4.

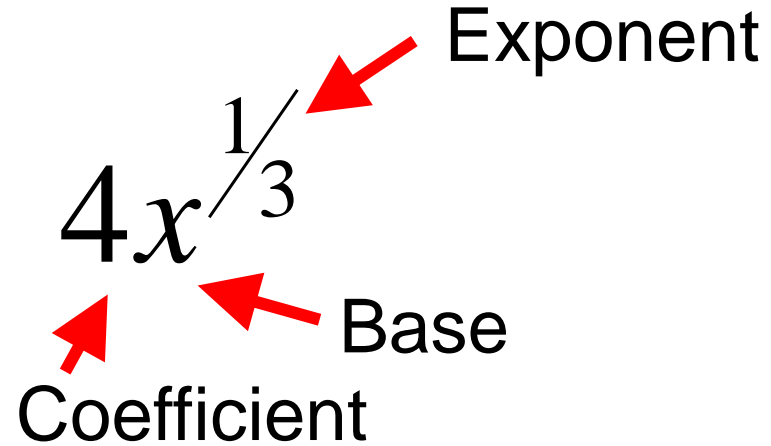


Diagram illustrating the parts of the exponential form $4x^{1/3}$. Red arrows point from labels to the corresponding parts: **Exponent** points to the 1/3, **Base** points to the x , and **Coefficient** points to the 4.

Coefficient \longrightarrow Coefficient

Radicand \longrightarrow Base

Index \longrightarrow Denominator of the Exponent

The index number is the denominator of the exponent.

Writing Radicals in “Exponent Form”

Index # (of radical): same as the denominator of an exponent.

$$\sqrt[2]{x} = x^{1/2}$$

$$\sqrt[3]{7} = 7^{1/3}$$

None of these have coefficients!

Index # (of radical): same as the denominator of an exponent.

$$3\sqrt[2]{y} = 3y^{1/2}$$

$$5\sqrt[3]{7} = 5(7)^{1/3}$$

$$\textcircled{2}\sqrt[4]{5} = \textcircled{2}(5)^{1/4}$$

Where did the coefficient of the radical end up when re-written in “power form”? A Coefficient remains a coefficient.

Notice that there was only a single term as the radicand.

rewrite in “exponent form”

$$\sqrt[5]{w} \rightarrow w^{1/5}$$

$$2x\sqrt[3]{10} \rightarrow 2x(10)^{1/3}$$

$$3\sqrt[4]{11} \rightarrow 3(11)^{1/4}$$

rewrite in “radical form”

$$x^{1/4} \rightarrow \sqrt[4]{x}$$

$$2(32)^{1/6} \rightarrow 2\sqrt[6]{32}$$

$$2(xy^3)^{1/3} \rightarrow 2y\sqrt[3]{x}$$

What happens if there is a product under the radical?

$$2\sqrt{xy} = (xy)^{1/2}$$

$$5\sqrt[3]{3x} = 5(3x)^{1/3}$$

$$2\sqrt[4]{21mn} = 2(21mn)^{1/4}$$

How did we show that the index number applied to the entire product (radicand) when re-written in “power form”?

Power of a product → product inside parentheses with an exponent.

What happens if there is a power under the radical?

$$\sqrt[5]{x^2 y} = (x^2 y)^{1/5} = x^{2/5} y^{1/5}$$

$$6\sqrt[3]{3m^2} = 6(3m^2)^{1/3} = 6(3^{1/3})m^{2/3}$$

How did we show that the index number applied to the entire product (including the power) when re-written in “power form”?

Power of a product → product inside parentheses with an exponent.

“Exponential Form” that has both a numerator and denominator

The exponent can be written as a rational number.

$$x^{\frac{5}{2}}$$

Numerator:
Exponent of the base.

$$\sqrt[2]{x^5}$$

Denominator:
Root of the base.

$$\sqrt[3]{2^2}$$

Radical Form

$$\rightarrow 2^{\frac{2}{3}}$$

Exponential Form

What form should you use?

Usually, the problem will tell you.

rewrite in “radical form”

$$5(y)^{2/3} \rightarrow 5\sqrt[3]{y^2}$$

rewrite in “exponent form”

$$2\sqrt[5]{x^3 y^2} \rightarrow 2x^{3/5} y^{2/5}$$

Product of Powers Property

$$y^2 * y^3 \rightarrow y^{2+3} \rightarrow y^5$$

When multiplying “same based powers” add the exponents.

$$x^{2/3} x^{3/4} \rightarrow x^{\frac{2}{3} + \frac{3}{4}} \rightarrow x^{\left(\frac{2*4}{3*4}\right) + \left(\frac{3*3}{4*3}\right)} \rightarrow x^{\frac{8}{12} + \frac{9}{12}} \rightarrow x^{\frac{17}{12}}$$

Exponent of a Power Property

When multiplying “same based powers” add the exponents.

$$(y^2)^3 \rightarrow y^{2*3} \rightarrow y^6$$

$$\left(y^{1/2}\right)^{2/3} \rightarrow y^{\frac{1}{2} * \frac{2}{3}} \rightarrow y^{\frac{2}{6}} \rightarrow y^{\frac{1}{3}}$$

$$\left(\frac{x^2}{y^{3/2}}\right)^{2/3} \rightarrow \frac{x^{\frac{2}{3} * \frac{2}{3}}}{y^{\frac{3}{2} * \frac{2}{3}}} \rightarrow \frac{x^{\frac{4}{9}}}{y^1}$$

Negative Exponent Property

We don't want negative exponents in our answers

$$\frac{x^2 y^{2/3}}{y^{-1/2}} \rightarrow x^2 y^{2/3} y^{1/2} \rightarrow x^2 y^{\frac{2}{3} + \frac{1}{2}} \rightarrow x^2 y^{\frac{4}{6} + \frac{3}{6}} \rightarrow x^2 y^{\frac{7}{6}}$$

Rational Exponents in the Denominator

Rational exponent in the denominator means irrational denominator, which we rationalize

$$\frac{1}{y^{1/2}} \rightarrow \frac{1}{\sqrt{y}} * \frac{\sqrt{y}}{\sqrt{y}} \rightarrow \frac{\sqrt{y}}{y}$$
$$\frac{1}{y^{1/2}} * \frac{y^{1/2}}{y^{1/2}} \rightarrow \frac{y^{1/2}}{y}$$