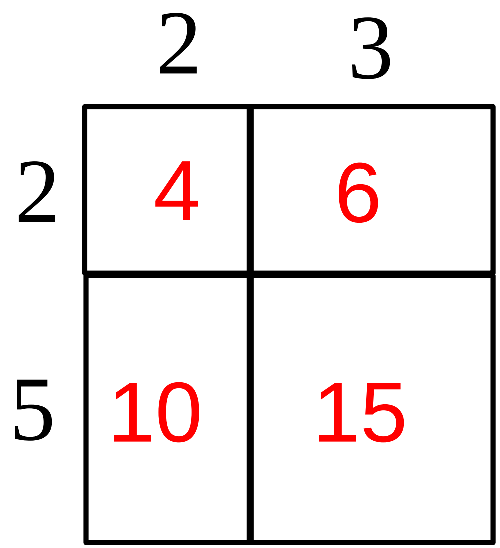
Math 1010

Lesson 2-3
Multiply and Divide Polynomials

The "Box Method" of multiplying Polynomials

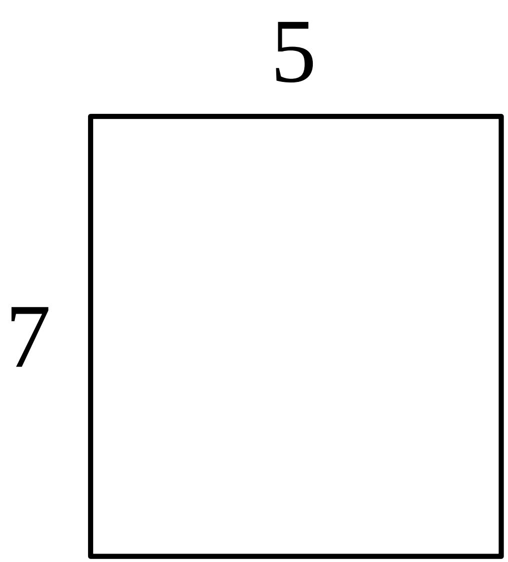
Widths and lengths of the rectangles are shown.



Find the area of each rectangle

Find the total area of the 4 rectangles.

$$4 + 6 + 10 + 15 = 35$$



Combine the side lengths of the smaller rectangles to find the side lengths of the outer rectangle

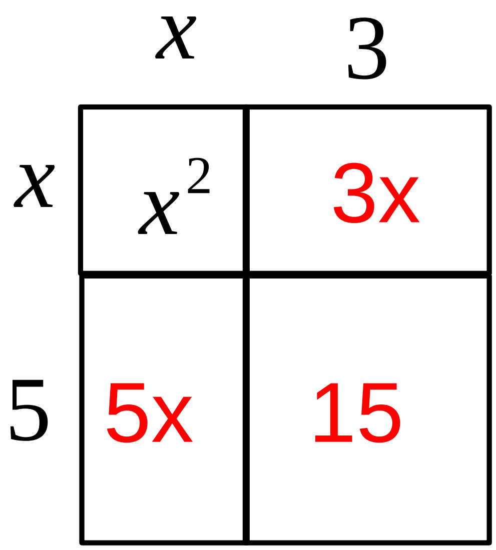
Find the area of the outer rectangle.

$$5 * 7 = 35$$

Explain the result to your neighbor.

The "Box Method" of multiplying Polynomials

Widths and lengths of the rectangles are shown.

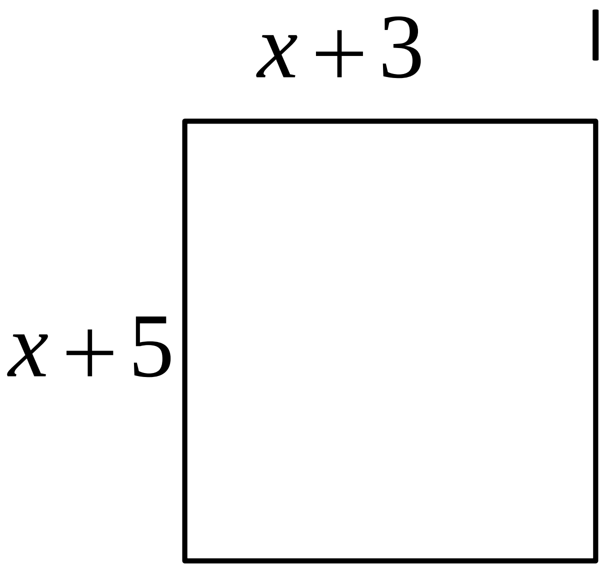


Find the area of each rectangle

Find the total area of the 4 rectangles.

$$x^2 + 3x + 5x + 15 \Rightarrow x^2 + 8x + 15$$

Combine the side lengths of the smaller rectangles to find the side lengths of the outer rectangle

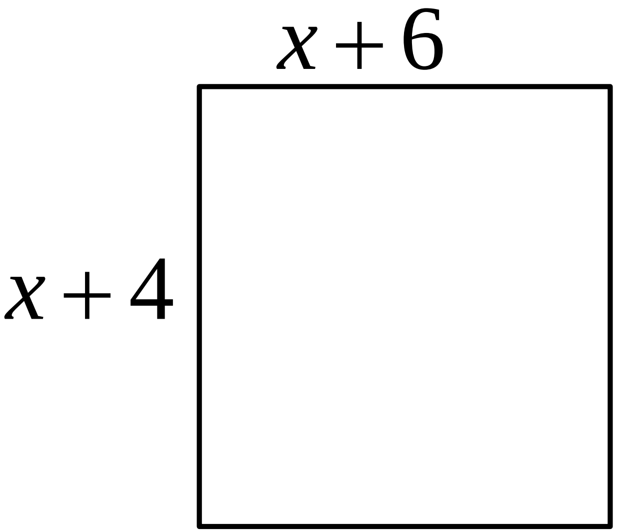


Find the area of the outer rectangle.

$$(x+3)(x+5) = x^2 + 8x + 15$$

The "Box Method" of multiplying Polynomials

We start with the combined side lengths



We break the large rectangle into four smaller rectangles.

We break the combined side lengths to label the smaller rectangles

	x	6
x	x^2	6x
4	4x	24

Find the area of each small rectangle.

Combine the areas to find the total area.

$$x^2 + 6x + 4x + 24 \Rightarrow x^2 + 10x + 24$$

$$(x+6)(x+4) = x^2 + 10x + 24$$

Use the “Box Method” to multiply Polynomial $(x+1)(x+7)$

	x	1
x	x^2	x
7	7x	7

$$x^2 + x + 7x + 7$$

$$\Rightarrow x^2 + 8x + 7$$

$$(x+1)(x+7) = x^2 + 8x + 7$$

Use the “Box Method” to multiply Polynomial $(x-3)(x-6)$

	x	-3
x	x^2	-3x
-6	-6x	18

$$x^2 - 3x - 6x + 18$$

$$\Rightarrow x^2 - 9x + 18$$

$$(x-3)(x-6) = x^2 - 9x + 18$$

$$(x-3)(x^2+2x-4)$$

Distributive property
(twice)

$$x(x^2+2x-4) \quad -3(x^2+2x-4)$$

$$x^3+2x^2-4x \quad -3x^2-6x \quad +12$$

simplify

$$x^3-x^2 \quad -10x \quad +12$$

"Box Method"

"break apart" into individual terms (small rectangle lengths and widths)

$$(3x-2)(2x^2+x-3)$$

	$2x^2$	x	(-3)
$3x$			
(-2)			

“multiply rows and columns”

$$(3x-2)(2x^2+x-3)$$

	$2x^2$	x	(-3)
$3x$	$6x^3$	$3x^2$	$-9x$
(-2)			

“multiply rows and columns”

$$(3x-2)(2x^2+x-3)=?$$

	$2x^2$	x	(-3)
$3x$	$6x^3$	$3x^2$	$-9x$
(-2)	$-4x^2$	$-2x$	6

add (combine “like terms”)

$$(3x-2)(2x^2+x-3)$$

	$6x^3$	$3x^2$	$-9x$
	$-4x^2$	$-2x$	6

$6x^3 \quad - x^2 \quad -11x \quad +6$

Diagonals have “like terms”

Simplify

(Hint: use the box method) $(3x^2+2x-1)^2$

$$(3x^2+2x-1)(3x^2+2x-1)$$

	$3x^2$	$2x$	(-1)
$3x^2$	$9x^4$	$6x^3$	$-3x^2$
$2x$	$6x^3$	$4x^2$	$-2x$
(-1)	$-3x^2$	$-2x$	1

$$(3x^2 + 2x - 1)(3x^2 + 2x - 1)$$

	$9x^4$	$6x^3$	$-3x^2$
	$6x^3$	$4x^2$	$-2x$
	$-3x^2$	$-2x$	1

$$9x^4 \quad +12x^3 - 2x^2 \quad - 4x \quad +1$$

add (combine “like terms”)

$$(x^2-1)(x^2-1) \quad \Rightarrow x^4-2x^2+1$$

$$(4x^2+2)(4x^2-2) \Rightarrow 16x^4-4$$

$$(x-1)(x^3-3x+4) \Rightarrow x^4-x^3-3x^2+7x-4$$

Division of Polynomials   Box Method

$$(2x^3 + 15x^2 + 27x + 5) \div (2x + 5)$$

Only the upper left and bottom right boxes are known.

	x^2	$5x$	1
$2x$	$2x^3$	$10x^2$	$2x$
5	$5x^2$	$25x$	5

$15x^2$ $27x$

Diagonals have "like terms"

$$(2x^3 + 15x^2 + 27x + 5) \div (2x + 5) = x^2 + 5x + 1$$

Division of Polynomials   Box Method

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1)$$

Only the upper left and bottom right boxes are known.

Diagonals have “like terms”

	x^3	$4x^2$	$4x$	2
$3x^2$	$3x^5$	$12x^4$	$12x^3$	$6x^2$
$0x$	$0x^4$	$0x^3$	$0x^2$	$0x$
-1	$-x^3$	$-4x^2$	$-4x$	-2

$12x^4 \quad 11x^3 \quad 2x^2 \quad -4x \quad -2$

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1) = x^3 + 4x^2 + 4x + 2$$

Division with remainders

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1)$$

Only the upper left and bottom right boxes are known.

Diagonals have "like terms"

	x^3	$4x^2$	$4x$	2
$3x^2$	$3x^5$	$12x^4$	$12x^3$	$6x^2$
$0x$	$0x^4$	$0x^3$	$0x^2$	$0x$
-1	$-x^3$	$-4x^2$	$-4x$	-2

$12x^4$ $11x^3$ $2x^2$ $-4x$ -2

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1) = x^3 + 4x^2 + 4x + 2$$

Division with remainders

$$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$$

Only the upper left and bottom right boxes are known.

	$-x^3$	$-2x^2$	$4x$	-8
x	$-x^4$	$-2x^3$	$4x^2$	$-8x$
3	$-3x^3$	$-6x^2$	$12x$	-24

Remainder

25

$$-x^4 \quad -5x^3 \quad -2x^2 \quad +4x \quad +1$$

$$\frac{25}{x+3}$$

Diagonals have “like terms”

$$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$$

$$= (-x^3 - 2x^2 + 4x - 8 + \frac{25}{x+3})$$

Vocabulary

Polynomial Long division: One method used to divide polynomials similar to long division for numbers.

$$\frac{x^3 - 4x^2 - 15x + 18}{(x-1)} = ax^2 + bx + c$$

Divide Evenly: A divisor divides evenly if there is a zero for the remainder.

Polynomial Long Division

$$
\begin{array}{r|l}
 & x^2 \\
x-1 & x^3-4x^2-15x+18
\end{array}
$$

1) Look at left-most numbers

2) What # times "left" = "left"?

$$x^3/x = ? = x^2$$

3) Multiply

$$x^2(x-1) = x^3 - x^2$$

4) Subtract

$$-(x^3 - x^2)$$

Polynomial Long Division

$$\begin{array}{r} x^2 \\ x-1 \overline{\big)\; x^3-4x^2-15x+18} \\ -(x^3-x^2) \\ \hline -3x^2\;-15x+18 \end{array}$$

4) Subtract

Careful with the negatives!

5) Bring down.

Polynomial Long Division

$$
\begin{array}{r|l}
 & x^2 - 3x \\
\hline
x - 1 & x^3 - 4x^2 - 15x + 18 \\
 & -(x^3 + x^2) \\
\hline
 & -3x^2 - 15x + 18
\end{array}
$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$$\frac{-3x^2}{x} = ? = -3x$$

3) Multiply

$$-3x(x-1) = -3x^2 + 3x$$

4) Subtract

$$-(-3x^2 + 3x)$$

Polynomial Long Division

$$
\begin{array}{r}
x^2 - 3x \\
x - 1 \;\big)\; x^3 - 4x^2 - 15x + 18 \\
-(x^3 + x^2) \\
\hline
-3x^2 - 15x + 18 \\
-(-3x^2 + 3x) \\
\hline
-18x
\end{array}
$$

4) Subtract

Careful of the negatives

5) Bring down.

Polynomial Long Division

$$\begin{array}{r} x^2 - 3x \\ x-1 \enclose{longdiv}{x^3 - 4x^2 - 15x + 18} \\ -(x^3 + x^2) \\ \hline -3x^2 - 15x + 18 \\ -(-3x^2 + 3x) \\ \hline -18x + 18 \end{array}$$

5) Bring down.

Polynomial Long Division

$$x^2 - 3x - 18$$

$$x - 1 \overline{) x^3 - 4x^2 - 15x + 18}$$

$$-(x^3 + x^2)$$

$$-3x^2 - 15x + 18$$

$$-(-3x^2 + 3x)$$

$$-18x + 18$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$$\frac{-18x}{x} = ? = -18$$

3) Multiply

$$-18(x - 1) = -18x + 18$$

4) Subtract

$$-(-18x + 18)$$

Polynomial Long Division

$$x^2 - 3x \quad -18$$

$$x - 1 \;\big)\; x^3 - 4x^2 - 15x + 18$$

$$-(x^3 + x^2)$$

$$-3x^2 - 15x + 18$$

$$-(-3x^2 + 3x)$$

$$-18x + 18$$

$$-(-18x + 18)$$

$$0$$

4) Subtract

$$-(-18x + 18)$$

Remainder = 0

(x – 1) divides evenly.

(x – 1) is a factor

$$x^3 - 4x^2 - 15x + 18 = (x-1)(x^2 - 3x - 18)$$

Synthetic Division

$$f(x) = -x^4 - 5x^3 - 2x^2 + 4x + 1$$

$(x+3)$?

-3	-1	-5	-2	4	1
		3	6	-12	24
	-1	-2	4	-8	25

(Since (x+3) has a remainder of 25, then (x + 3) is not a factor).

$$= (-x^3 - 2x^2 + 4x - 8 + \frac{25}{x+3})$$