Math-1010

Lesson 2-2
Properties of Exponents

Properties of Exponents

What is a power?

Power: An expression formed by repeated multiplication of the base.

$$3x^4$$

Exponent

Coefficient

Base

The exponent applies to the number or variable immediately to its left, not to the coefficient !!!

No Exponent? $3x = 3^1 x^1$

Usually, we don't write the exponent '1' (saves ink).

No Coefficient? $x^3 = 1 * x^3 = 1^1 * x^3$

Usually, we don't write the coefficient '1' (saves ink).

Negative? $-x^2 = (-1) * x^2 = (-1)^1 * x^2$

Usually, we don't write the coefficient '-1', we just put the "negative symbol" (saves ink).

Factor: a number that is being multiplied.

x^4 means "base x used as a factor 4 times"

Base   Exponent

$$x^4 = x * x * x * x$$

Product: the result ("answer") when multiplying.

Power: is repeated multiplication

$$x^4 = x * x * x * x$$

multiplication: is repeated addition

$$3x = x + x + x$$

How can you tell which definition to use?

Use the “context” of the problem

$$3x + 4x$$ (<u>adding</u> two terms)

$$= (x + x + x) + (x + x + x + x)$$

$$3x + 4x = 7x$$

How can you tell which definition to use?

Use the "context" of the problem

$$2x^2 + 3x^2$$ (<u>adding</u> two terms)

$$= (x^2 + x^2) + (x^2 + x^2 + x^2)$$

$$2x^2 + 3x^2 = 5x^2$$

How can you tell which definition to use?

Use the "context" of the problem

$$x^2 * x^3$$ (multiplying two terms)

$$= (x * x)(x * x * x)$$

$$x^2 * x^3 = x^5$$

Simplify

$$(4y)^2$$

$$\rightarrow 16y^2$$

$$\left(\frac{-2}{x}\right)^4 \rightarrow \frac{16}{x^4}$$

$$2(5x)^2$$

$$\rightarrow 50x^2$$

$$\left(\frac{x}{2}\right)^3 \rightarrow \frac{x^3}{8}$$

Product of Powers Property

$$(x^2)(x^3) = (x * x)(x * x * x)$$

This is 'x' used as a factor how many times?

$(x^2)(x^3) =$ **'x' used as a factor <u>five</u> times** $= x^5$

$$x^2 x^3 = x^{2+3}$$

When you multiply powers having the same base, you add the exponents.

Power of a Power Property $(x^2)^3$

$$(x^2)^3 = (x*x)(x*x)(x*x)$$

This is 'x' used as a factor how many times?

$(x^2)^3 =$ **'x' used as a factor <u>six</u> times** $= x^6$

$$(x^2)^3 = x^{2*3} = x^6$$

When you raise a power to another power
("power of a power")
you <u>multiply</u> the exponents.

Power of a Product Property $(xy)^2$

$$=(xy)(xy) \quad = x*y*x*y \quad = x*x*y*y$$

$$= x^2y^2 \qquad (xy)^m = x^m y^m$$

This makes it seem like you can "distribute" in the exponent. This only works with the power of a product

$$(x-y)^2 \neq x^2 - y^2$$

You must "FOIL" the power of a sum!!

$$(x-y)^2 = (x-y)(x-y)$$

$$= x^2 - 2xy + y^2$$

$$(3x^3y^4)^2 = (3^1x^3y^4)^2$$

Constants (integer, etc.) usually have an exponent of '1'.

$$= 3^2x^6y^8$$

'x' is a number, we just don't know what it is. You treat all numbers the same (whether they are variables or constants).

$$(3^a x^b y^c)^m = 3^{am}x^{bm}y^{cm}$$

$$3x^2(4x^3) = ?$$

$$= 3*4*(x^2)(x^3)$$

You can re-arrange the order of multiplication.

$$\rightarrow 12x^5$$

<u>Coefficients</u> of the powers are handled separately from the base and the exponent.

Simplify:

$$(5x^2)(2x^3) \rightarrow 10x^5$$

$$(2x)\left(\frac{1}{2}x^3\right) \rightarrow x^4$$

$$5(x)^3 * -2x^4 \rightarrow -10x^7$$

$$(2x^2)^5 \rightarrow 32x^{10}$$

$$(-2y^5)^3 \rightarrow -8y^{15}$$

What is the difference between?

$$(x)^4 \, and \; \mathrm{x}^4$$

$$(x^2)^3 \text{ and } (x^3)^2$$

$$x^4x^3 \text{ and } x^3x^4$$

$$(x+1)^2 \text{ and } (x+1)(x+1)$$

Watch the negatives! $(-x^3y^4)^2$

$$=((-1)^1x^3y^4)^2$$ **Turn negative signs into multiplication by -1.**

$$=(-1)^2x^6y^8$$ **This way you will be able to tell if the simplified version is positive or negative.**

$$=x^6y^8$$

$$(-2x^2y^6)^3$$ **Negative coefficients have an exponent of '1'.**

$$=((-2)^1x^2y^6)^3$$

$$=(-2)^3x^6y^{18}$$ **A negative number raised to an odd exponent remains negative.**

$$=-8x^6y^{18}$$

Your Turn: simplify

$$(-2x^2y^4z)^3 \quad \rightarrow -8x^6y^{12}z^3$$

$$2(-m^4x^3)^5 \quad \rightarrow -2w^{20}x^{15}$$

$$-3(-2x^2yz^3)^4 \quad \rightarrow -48x^{12}$$

Negative Exponent Property

"Grab and drag"

$$x^{-2} = \frac{1 * x^{-2}}{1} = \frac{1}{x^2}$$

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

Why do we call this the negative exponent property?

Examples:

$$x^2 y^{-2} = \frac{x^2}{y^2}$$

$$\left(\frac{1}{x^3}\right)^{-2} = \frac{1}{x^{-6}} = \frac{1}{x^{-6}} = x^6$$

Negative Exponent Property

Possible errors

$$4x^{-2} = \frac{4 * x^{-2}}{1} = \frac{4}{x^{2}}$$

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

DO NOT GRAB the coefficient!

$$\frac{4 * x^{-2}}{1} \neq \frac{1}{4x^{2}}$$

<u>Zero Exponent Property</u>

<u>Any</u> <u>base</u> raised to the <u>zero power</u> simplifies to <u>one.</u>

$$10^3 = 1000$$

$$2^0 = 1$$

$$10^2 = 100$$

$$(2x)^0 = 1$$

$$10^1 = 10$$

$$2x^0 = 2*1 = 2$$

$$10^0 = 1$$

Combination: (1) Negative Exponent, (2) Product of Powers, (3) Power of a Power, (4) Power of a Quotient

$$\left(\frac{3x^{2}}{2x^{-4}y}\right)^{2} = \left(\frac{3x^{2}x^{4}}{2y}\right)^{2} = \left(\frac{3x^{6}}{2y}\right)^{2} = \left(\frac{3^{1}x^{6}}{2^{1}y^{1}}\right)^{2}$$

$$= \frac{3^{1*2}x^{6*2}}{2^{1*2}y^{1*2}} = \frac{3^{2}x^{12}}{2^{2}y^{2}} = \frac{9x^{12}}{4y^{2}}$$

Your Turn:

$$(w^{-2})^5 \rightarrow \frac{1}{w^{10}}$$

$$\frac{1}{2}\left(3x^{-3}\right)^2 \rightarrow \frac{9}{2x^6}$$

$$\left(\frac{2x^2}{3y^{-2}z^3}\right)^{-2} \rightarrow \frac{9z^6}{4x^4y^4}$$

$$\left(\frac{5x^4}{3y^{-2}}\right)^{-1} \rightarrow \frac{3}{5x^4y^2}$$

Examples:

$$\frac{32x^{10}}{x^{2}y^{17}} = \frac{32x^{10}x^{-2}}{y^{17}} = \frac{32x^{10-2}}{y^{17}}$$

"Grab and drag"

Product of powers: add the exponents of <u>same based powers</u>

$$\boxed{= \frac{32x^{8}}{y^{17}}}$$

Your Turn:

$$\frac{2x^3}{4x^5} = \frac{1}{2x^2}$$

$$\frac{(x^{-2})^4}{2x^{-3}} = \frac{1}{2x^5}$$

$$\frac{9(2x)^4}{2x} = 72x^3$$

$$\frac{(-2y^2x^{-3})^4}{2yx^{-3}} = \frac{8y^7}{x^9}$$

Do you "grab and drag (<u>up</u> or <u>down</u>)??

<u>It doesn't matter!!!!</u>

$$\frac{3x^2}{2x^{-4}y} = \frac{3x^2x^4}{2y} = \frac{3x^{2+4}}{2y} \boxed{= \frac{3x^6}{2y}}$$

<u>Product of powers property</u>: add the exponents of like-based powers

$$\frac{3x^2}{2x^{-4}y} = \frac{3}{2x^{-4}x^{-2}y} = \frac{3}{2x^{-4-2}y} = \frac{3}{2x^{-6}y} \boxed{= \frac{3x^6}{2y}}$$

<u>Product of powers property</u>: add the exponents of like-based powers

<u>Make sure</u> when you're all done, there are <u>NO NEGATIVE EXPONENTS</u> remaining.