College Algebra Math 1050

Sample Final Exam 3

Name: _____ School: _____

Instructor: _____ Period: _____

Scientific (not graphing) calculators are allowed. Time limit is 120 minutes. The point value of each problem is written next to the problem. **You must show your work to receive any credit, except on problems 1–29.** Work neatly.

Fill in the blank or circle the correct answer.

1. (2 points) Multiply: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix} =$ _____.

2. (2 points) The graph of the function $f(x) = 4x^7 - 3x^4 - x^3 + 13$ has at most _____ turning points.

3. (3 points) Nino wants to triple her initial investment of \$2000. Her account offers 6.5% annual interest, **compounded quarterly**. Set up an equation that enables her to calculate how many years it will take for her investment to triple. **Do not solve.**

_____.

4. (3 points) Convert the logarithmic equation: $\log_a 3 = 5$ into exponential form. _____.

5. (3 points) If $\log_b x^3 = 8$, evaluate $\log_b x$. _____.

For problems 6-8, consider the function $f(x) = \dfrac{x^2 + 6x - 7}{x^2 - 7x + 12}$.

6. (3 points) The domain of the function $f(x)$ is _____. **Write your answer in any form.**

7. (3 points) The $x-$intercept(s) of the function $f(x)$ is (are) _____. **Write your answer(s) as ordered pair(s) and leave in exact form.**

8. (3 points) The $y-$intercept(s) of the function $f(x)$ is (are) _____. **Write your answer(s) as ordered pair(s).**

For problems 9-10, consider the function $g(x) = \dfrac{2x^2 + 5}{x^2 + 3x - 10}$.

9. (3 points) The vertical asymptote(s), if any, of the function $g(x)$ is (are) _____.
Write your answer(s) as equation(s).

10. (3 points) The non-vertical asymptote(s), if any, of the function $g(x)$ is (are) _____.
Write your answer(s) as equation(s).

11. (3 points) Write an expression to find the term containing a^4 in the expansion of $(2a - 3)^{33}$.

Do not simplify. _____.

12. (3 points) Consider the system of linear equations $\begin{cases} 2x+y-z &= 3 \\ 3y-x+2z &= 4 \\ x+z-y &= 0 \\ x+2z+4y &= 7 \end{cases}$

Which student is proposing a valid first step to find the solution of the given system of linear equations using matrices (row operations)? Circle all that apply.

(a) Maka says that the system does not have a solution because there are more equations than the number of unknowns.

(b) Beqa's first step to find a solution of the system is: $\left[\begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ 3 & -1 & 2 & 4 \\ 1 & 1 & -1 & 0 \\ 1 & 2 & 4 & 7 \end{array}\right]$.

(c) Maya's first step to find a solution of the system is: $\left[\begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ -1 & 3 & 2 & 4 \\ 1 & -1 & 1 & 0 \\ 1 & 4 & 2 & 7 \end{array}\right]$.

(d) None of the students are proposing a valid first step to find a solution of the system of equations.

13. (3 points) If $f(x) = \dfrac{2x+1}{x-1}$ and then $f^{-1}(x) = \dfrac{x+1}{x-2}$. The **range** of f is _____.
Write your answer in any form.

14. (3 points) f is a one-to-one function defined by: $\{(5, \frac{1}{2}), (-2, 4), (2, -1), (-3, 2), (\frac{1}{2}, 3)\}$.

Find $f^{-1}(2)$ _____.

15. (3 points) Let $f(x) = x^3 + 2$. Which of the following is (are) the inverse f^{-1}? Circle all that apply.

(a) $(f^{-1})(x) = -x^3 - 2$.

(b) $(f^{-1})(x) = \dfrac{1}{x^3 + 2}$.

(c) $(f^{-1})(x) = \sqrt[3]{x - 2}$.

(d) $(f^{-1})(x) = \sqrt[3]{x + 2}$.

16. (3 points) Given the table:

x	-3	-2	-1	0	1	2	3
$f(x)$	-7	-5	-3	-1	3	5	7
$g(x)$	8	3	0	-1	0	3	8

Evaluate $(\mathbf{g} \circ \mathbf{f})(\mathbf{0})$ _____.

17. (3 points) Consider the equation: $\log_4(3x - 1) - \log_4(x + 1) = 6$. Which student is proposing a valid first step for solving the equation? Circle all that apply.

(a) Giorgi's first step to solve the equation is: $\log_4 \dfrac{3x - 1}{x + 1} = 6$.

(b) Lado's first step to solve the equation is: $\dfrac{\log_4(3x - 1)}{\log_4(x + 1)} = 6$.

(c) Devi's first step to solve the equation is: $\log_4 \dfrac{x + 1}{3x - 1} = 6$.

(d) None are proposing a valid first step to solve the equation.

18. (3 points) The **maximum value** of the function $f(x) = -x^2 - 8x + 5$ is: $y =$ _____.

19. (3 points) Let $c_1 = -2$ and $c_n = 3c_{n-1} - 1$. Then $c_3 =$ _____.

20. (3 points) How long will it take an initial investment of \$2000 to be worth \$3500 if the investment **compounds continuously** at an annual interest rate of 6%?

Round your answer to the nearest tenth of a year. _____.

21. (3 points) Consider the equation: $4 \cdot 5^{3x} = 12$. Which student is proposing a valid first step to solve the equation? Circle all that apply.

(a) Dato's first step to solve the equation is: $3x \ln 4 \ln 5 = \ln 12$.

(b) Vaja's first step to solve the equation is: $20^{3x} = 12$.

(c) Maka's first step to solve the equation is: $5^{3x} = 3$.

(d) All three of the students are proposing a valid first step to solve the equation.

(e) None of the students are proposing a valid first step to solve the equation.

22. (4 points) The form of the partial fraction decomposition of the rational function

$f(x) = \dfrac{2x+1}{x(x+1)^2}$ is _____.

23. (4 points) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & -4 & 3 \\ 2 & 2 & 1 & -1 \end{bmatrix}$. Some row operation(s) have been applied to A to obtain $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & -4 & 3 \\ -1 & -4 & \mathbf{x} & \mathbf{y} \end{bmatrix}$. What are the values of $\mathbf{x}$ and $\mathbf{y}$? _____.

24. (4 points) Find the **remainder**: $\dfrac{3x^2 - x + 5}{x - 2}$ _____.

25. (3 points) Consider the inequality: $\dfrac{2}{x+4} \geq 5$. Which student is proposing a valid first step to solve the inequality? Circle all that apply.

(a) Levan's first step to solve the inequality is: $2 \geq 5(x + 4)$.

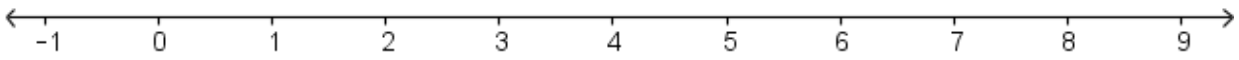
(b) Ladi's first step to solve the inequality is: $\dfrac{2}{x+4} - 5 \geq 0$.

(c) Both Levan and Ladi are proposing a valid first step to solve the inequality.

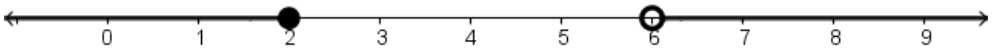
(d) Neither Levan nor Ladi is proposing a valid first step to solve the inequality.

26. (3 points) The rational expression $\dfrac{8-x}{x^2-6x+8}$ has critical numbers at $x=2$, $x=4$ and $x=8$. Find the solution to the inequality $\dfrac{8-x}{x^2-6x+8} \leq 0$.

Graph solution on number line.



27. (2 points) Circle **all** of the correct ways to write the intervals marked on the number line below.



(a) $(-\infty, 2] \cap (6, \infty)$ **(c)** $(-\infty, 2]$ or $(6, \infty)$

(b) $(-\infty, 2] \cup (6, \infty)$ **(d)** $(-\infty, 2]$ and $(6, \infty)$

28. (4 points) Consider the inequality: $3x^2 + 5x - 2 > 0$. Which student is proposing a valid first step to solve the inequality? Circle all that apply.

(a) Elene's first step to solve the inequality is: $(3x-1)(x+2) > 0$.

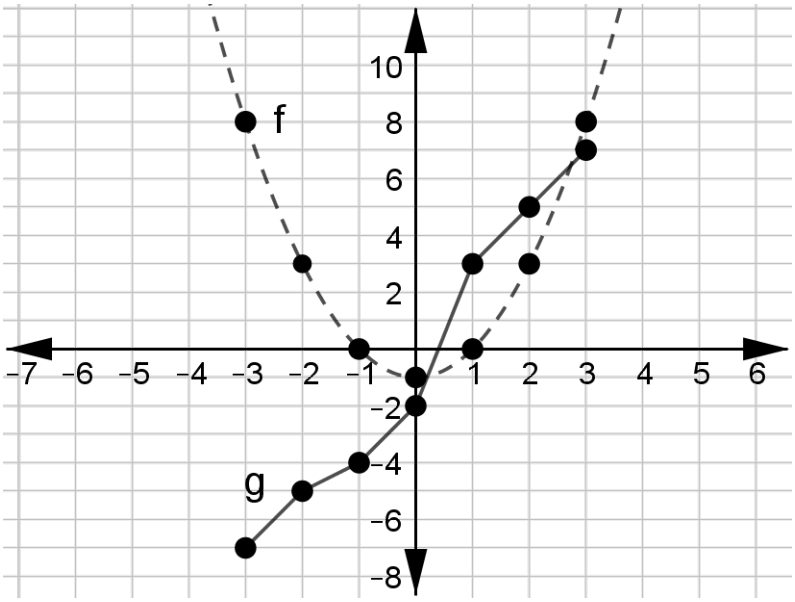
(b) Keti's first step to solve the inequality is: $8x^3 - 2 > 0$.

(c) Masho's first step to solve the inequality is: $3x^2 + 3x > 0$.

(d) All three students are proposing a valid first step to solve the inequality.

(e) None of the students are proposing a valid first step to solve the inequality.

29. (4 points) $f(x)$ is graphed below in a dashed line and $g(x)$ is graphed below in a solid line. Use the graphs to evaluate $(f+g)(1)$. _____.



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30. (4 points) Consider the function: $f(x) = \log_4(x-1)$.

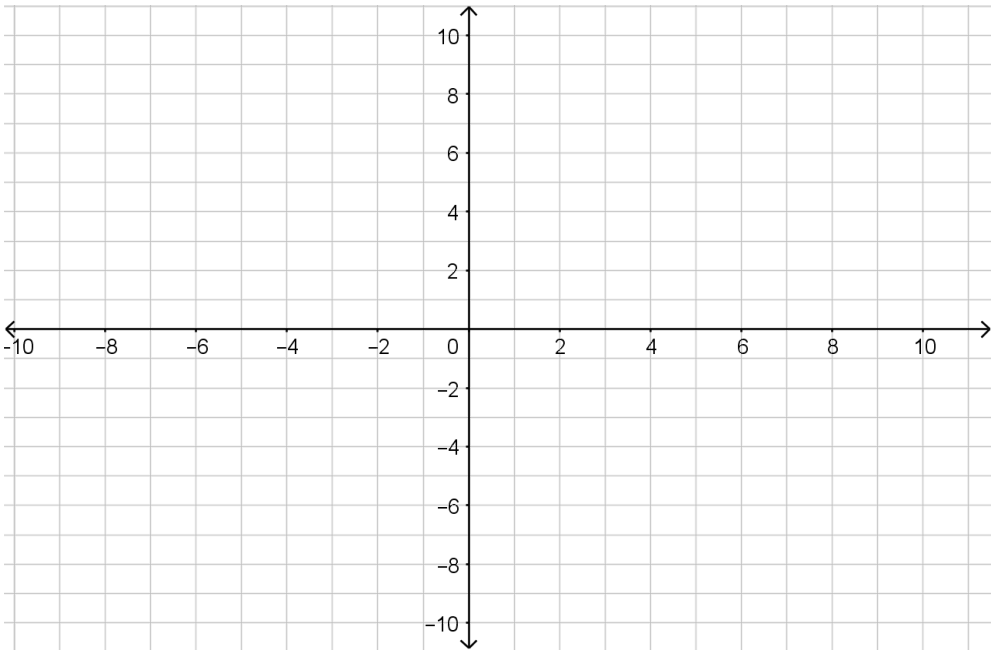
(a) (2 points) What is the domain of f? _____. **Write your answer in any form.**

(b) (2 points) What is the intercept of f? _____. **Write your answer as an ordered pair.**

31. (7 points) Assume that the following sequence $2, -1, -4, -7, ...$ is arithmetic. Find the sum using appropriate formulas: $2 - 1 - 4 - 7 - ... - 169$.

31. Answer _____.

32. (9 points) Graph the rational function $f(x) = \dfrac{x^2}{x-2}$. Your graph should clearly show and label all $x-$ and $y-$intercept(s) and all asymptotes.



33. (7 points) Consider the system of linear equations $\begin{cases} x & +y & & = -1 \\ & -y & +2z & = 0 \\ 3x & & -2z & = -11 \end{cases}$

Solve the system of equations using matrices (row operations). If the system has no solution, say that it is inconsistent.

33. Answer: _____.

34. (7 points) Solve $\log_{15} x + \log_{15}(x-2) = 1$. **No points will be awarded if the solution is found by trial and error.**

34. Answer_____.

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35. (7 points) A certain type of bacteria, given a favorite growth medium, doubles in population every 6 hours. ($N(t) = N_0 e^{kt}$).

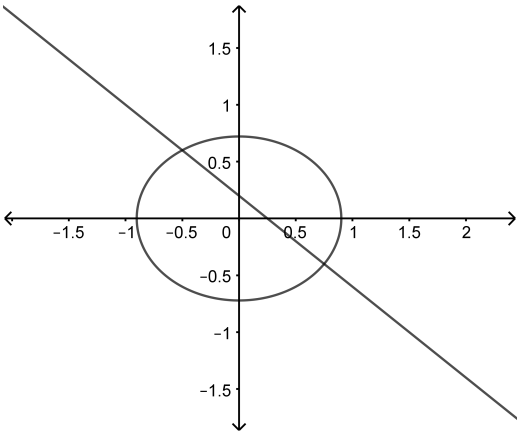
Given that there were 200 bacteria to start with, how many bacteria will there be in 40 hours?

35. Answer_____.

36. (7 points) Consider the system of nonlinear equations:

$$4x + 5y = 1$$
$$16x^2 + 25y^2 = 13$$

(a) (2 points) Use the graph of the equations below to estimate the real solution(s).



36. (a) Answer_____.

(b) (5 points) Algebraically solve the system of equations given above. Keep the solutions as fractions, not decimals. **No points will be awarded if the soluntion is found by trial and error.**

36. (b) Answer_____.

37. (5 points) Let $f(x) = \dfrac{x+1}{3x-2}$. Find the inverse of f and **simplify completely.**

37. Answer_____.

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38. (8 points) Solve the inequality: $\dfrac{4x+3}{x-1} \geq 3$. **Write the solution in interval notation.**

38. Answer_____.