Math-3
Lesson 8-2
Quadratic Formula

Find the Zeroes

$$y = x^2 - 6x + 8$$

$$0 = (x-4)(x-2)$$

$$\boxed{x = 4, 2}$$

$$y = x^2 - 6x + 2$$

$$0 = (x-3)^2 - 7$$

$$\boxed{x = 3 \pm \sqrt{7}}$$

$$y = 3x^2 - 5x - 12$$

$$0 = 3x^2 - 9x + 4x - 12$$

Factor by box

$$0 = (3x+4)(x-3)$$

$$\boxed{x = -\frac{4}{3}, 3}$$

How do we find the zeroes of non-factorable standard form quadratic equation?

$$y = x^2 - 6x + 2$$

1) Set y = 0
2) Convert to “vertex form”
3) Solve for ‘x’ by “isolate the square, undo the square”

$$0 = (x - 3)^2 - 7$$

$$\boxed{x = 3 \pm \sqrt{7}}$$

Quadratic Formula: gives the solutions (x-intercepts) to ANY quadratic equation in standard form.

$$y = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

How did we get the quadratic formula?

1) We set y = 0

$$y = ax^2 + bx + c$$

$$0 = ax^2 + bx + c$$

2) Convert to vertex form (remember the VSF = 'a')

x-coordinate of vertex: x = -b/2a

The y-coordinate of the vertex is: f(-b/2a)

$$f\left(-\frac{b}{2a}\right) = a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c$$

$$f\left(-\frac{b}{2a}\right) = \frac{ab^2}{4a^2} - \frac{b^2}{2a} + c$$

The x-coordinate of the vertex is: x = -b/2a

The y-coordinate of the vertex is: f(-b/2a)

$$f\left(-\frac{b}{2a}\right)=\frac{ab^2}{4a^2}-\frac{b^2}{2a}+c \qquad =\frac{b^2}{4a}-\frac{b^2}{2a}+c$$

Obtain common denominators then Add fractions

$$f\left(-\frac{b}{2a}\right)=\frac{b^2}{4a}-\frac{2b^2}{4a}+\frac{4ac}{4a} \qquad =\frac{-b^2+4ac}{4a}$$

Write in Vertex form:

$$0=a\left(x+\frac{b}{2a}\right)^2-\left(\frac{-b^2+4ac}{4a}\right)$$

Vertex form: $$0 = a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2-4ac}{4a}\right)$$

3) Solve for ‘x’ by “Isolate the square, undo the square”

$$\frac{b^2 - 4ac}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

$$\pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

$$\pm\frac{\sqrt{b^2 - 4ac}}{2a} = x + \frac{b}{2a}$$

$$\boxed{x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}}$$

Your turn: $y = ax^2 + bx + c$

Identify ‘a’ ‘b’ and ‘c’ for each of these standard form quadratic equations.

$$y = x^2 - 4x + 3$$

$$y = -2x^2 + 3x - 7$$

$$y = 3x^2 - 10$$

$$y = -x^2 + 3x$$

What if it's not in standard form?

$$2x = 3x^2 - 5$$

Use math properties to get it into standard form (same thing left/right, combine like terms, etc.)

$$0 = 3x^2 - 2x - 5$$ a = 3 b = -2 c = -5

a = ? b = ? c = ?

Determine the following values: a = ?, b = ?, c = ?

$$y = 3 - 12x^2 - 4x$$

$$5x = 3x^2 - 5x + 1$$

Solve using the Quadratic formula.

$$y = ax^2 + bx + c$$

$$y = x^2 - 6x + 4$$

a = 1 b = -6 c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2}$$

$$x = \frac{6 \pm \sqrt{4}\sqrt{5}}{2}$$

$$x = \frac{6 \pm 2\sqrt{5}}{2}$$

$$x = \frac{6}{2} \pm \frac{2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$

If the quadratic CANNOT be factored, the solutions are "ugly."

Can you "plug" 'a', 'b', and 'c' into the Quadratic formula?

$$y = ax^2 + bx + c$$

Identify 'a', 'b', and 'c' in the <u>standard form </u>equation.

$$y = x^2 + 3x + 2$$

a = 1 b = 3 c = 2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Replace the 'a', 'b', and 'c' with parentheses!!!

$$x = \frac{-(\ \ ) \pm \sqrt{(\ \ )^2 - 4(\ \ )(\ \ )}}{2(\ \ )}$$

Put the numerical values of 'a', 'b', 'c' into the parentheses.

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(2)}}{2(1)}$$

$$y = ax^2 + bx + c$$

$$y = x^2 + x - 1$$

a = 1 b = 1 c = -1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

“Gotcha” parts of the formula.

$$y = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = x^2 - 15x - 1$$

a = 1

b = -15

c = -1

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-1)}}{2(1)}$$

If you don’t use the parentheses, it would be easy to write this as -15

With the parentheses, -(-15) simplifies to +15!!!

If you don’t use the parentheses, it would be easy to write this as -15^2 instead of $(-15)^2$

$$-15^2 = -225$$

$$(-15)^2 = +225$$

“Gotcha” parts of the formula.

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-1)}}{2(1)}$$

It is easy to make mistakes with the negatives.

If you use parentheses, then you can type the expression under the radical into your calculator and it will be correct. $(-15)^2 - 4(1)(-1) = 229$

$$x = \frac{15 \pm \sqrt{229}}{2}$$

Plug in and simplify

$$y = ax^2 + bx + c$$

$$y = x^2 + x - 1$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1}{2} \pm \frac{\sqrt{5}}{2}$$

$$x = \frac{-1}{2} + \frac{\sqrt{5}}{2}$$

$$x = \frac{-1}{2} - \frac{\sqrt{5}}{2}$$

Another one (be careful of the “gotcha’s”)

$$y = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = x^2 - 4x + 3$$

a = 1

b = -4

c = 3

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

Can you simplify the result?

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(3)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-12}}{2}$$

$$x=\frac{4\pm\sqrt{4}}{2}$$

$$x=\frac{4\pm 2}{2}$$

$$x=\frac{4}{2}\pm\frac{2}{2}$$

$$x=2\pm 1$$

$$x=1,3$$

Find the Zeroes using the quadratic formula.

$$x = -\frac{(\ )}{2(\ )} \pm \frac{\sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$$

$$y = x^2 - 2x + 8$$

$$x = \frac{-(-2)}{2(1)} \pm \frac{\sqrt{(-2)^2 - 4(1)(8)}}{2(1)}$$

$$= 1 \pm \frac{\sqrt{4 - 32}}{2}$$

$$= 1 \pm \frac{\sqrt{-28}}{2} = 1 \pm \frac{2i\sqrt{7}}{2}$$

$$= 1 \pm i\sqrt{7}$$

Imaginary zeros ***always*** come in conjugate pairs.

Find the Zeroes using the quadratic formula.

$$x = -\frac{(\ )}{2(\ )} \pm \frac{\sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$$

$$y = x^2 - 6x + 2$$

$$x = \frac{-(-6)}{2(1)} \pm \frac{\sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$$

$$= 3 \pm \frac{\sqrt{36 - 8}}{2} \qquad = 3 \pm \frac{\sqrt{28}}{2}$$

$$= 3 \pm \frac{\sqrt{4 * 7}}{2} \qquad = 3 \pm \frac{2\sqrt{7}}{2} \qquad \boxed{x = 3 \pm \sqrt{7}}$$

Irrational zeros ***always*** come in conjugate pairs.

Find the Zeroes using the quadratic formula.

$$x = -\frac{(\ )}{2(\ )} \pm \frac{\sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$$

$$y = 3x^2 - 5x - 12$$

$$x = \frac{-(-5)}{2(3)} \pm \frac{\sqrt{(-5)^2 - 4(3)(-12)}}{2(3)}$$

$$x = \frac{5}{6} \pm \frac{\sqrt{25 + 144}}{6} \qquad = \frac{5}{6} \pm \frac{13}{6} \qquad = \frac{18}{6}, \frac{-8}{6}$$

$$x = \frac{5}{6} \pm \frac{\sqrt{169}}{6}$$

$$\boxed{x = 3, -\frac{4}{3}}$$

“Nice zeroes” means it could have been factored.

Fundament Theorem of Algebra: A second degree polynomial as two zeros. $y = ax^2 + bx + c$

2nd Degree polynomial zeros come in 3 "flavors"

Rational number zeros (numbers that can be written as a ratio of integers)

$$\boxed{x = 3, -\frac{4}{3}}$$

Irrational number zeros (always come in conjugate pairs

$$\boxed{x = 3 \pm \sqrt{7}}$$

Imaginary number zeros
(always come in conjugate pairs

$$\boxed{x = 1 \pm i\sqrt{7}}$$

Rational number zeros $\boxed{x = 3, -\frac{4}{3}}$

Irrational number zeros $\boxed{x = 3 \pm \sqrt{7}}$

Imaginary number zeros $\boxed{x = 1 \pm i\sqrt{7}}$

How can the quadratic formula allow you to *predict* what kind of zeros you'll get?

$$y = ax^2 + bx + c \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rational number zeros $\boxed{x = 3, -\frac{4}{3}}$

$$y = ax^2 + bx + c$$

Irrational number zeros $\boxed{x = 3 \pm \sqrt{7}}$

Imaginary number zeros $\boxed{x = 1 \pm i\sqrt{7}}$

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$$

How can the quadratic formula allow you to *predict* what kind of zeros you'll get?

1) If the radicand is a perfect square → rational # zeros.

2) If the radicand is a positive number but not a perfect square → irrational # zeros.

3) If the radicand is a negative number → imaginary # zeros.

"Discriminant" the radicand of the quadratic formula → it allows us to predict what kind of zeros we will get.

Rational number zeros $\boxed{x = 3, -\frac{4}{3}}$

$$y = ax^2 + bx + c$$

Irrational number zeros $\boxed{x = 3 \pm \sqrt{7}}$

Imaginary number zeros $\boxed{x = 1 \pm i\sqrt{7}}$

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$$

What kind of zeros will we get for this equation?

$$y = x^2 - 3x + 8$$

$$\boxed{b^2 - 4ac} = (-3)^2 - 4(1)8 \quad = -23$$

Imaginary zeros

$$y = x^2 - 5x + 5$$

$$\boxed{b^2 - 4ac} = (-5)^2 - 4(1)5 \quad = 5$$

Irrational zeros

$$y = x^2 - 4x + 3$$

$$\boxed{b^2 - 4ac} = (-4)^2 - 4(1)3 \quad = 4$$

Rational zeros