Math-3

Lesson 8-1

3rd Degree Polynomials

And Polynomial Division

Find the zeroes of the following 3rd degree Polynomial

$$y = x^3 + 5x^2 + 4x$$ Set y = 0

$$0 = x^3 + 5x^2 + 4x$$ Factor out the common factor.

$$0 = x(x^2 + 5x + 4)$$ Factor the quadratic

$$0 = x(x+1)(x+4)$$ Identify the zeroes

0, -1, -4

Factor the following "nice" 3rd degree polynomials then find the "zeroes" of the polynomial.

$$y = x^3 + 5x^2 - 14x$$

$$0 = x^3 + 5x^2 - 14x$$

$$0 = x(x^2 + 5x - 14)$$

$$0 = x(x+7)(x-2)$$

0, -7, 2

$$y = 3x^3 - 24x^2 + 6$$

$$0 = 3x(x^2 - 8x + 2)$$

x = 0

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)}$$

$$x = 4$$

$$y = f(4) = (4)^2 - 8(4) + 2$$

$$y = -14 \qquad 0 = (x+4)^2 - 14$$

$$x = -4 \pm \sqrt{14}$$

Another "Nice" 3rd Degree Polynomial

$$y = ax^3 + bx^2 + cx + d$$

This has the constant term, but it has a very useful feature:

$$y = 1x^3 + 2x^2 + 2x + 4$$

What pattern do you see?

$$3rd \big/ 1st = 2 \big/ 1 \qquad 4th \big/ 2nd = 4 \big/ 2 = 2 \big/ 1$$

An easy method is “box factoring” if it has this nice pattern.

$$y = 1x^3 + 2x^2 + 2x + 4$$

These 4 terms are the *numbers in the box*.

Find the *common factor* of the 1st row.

Fill in the rest of the box.

	x	2
x^2	x^3	$2x^2$
2	$2x$	4

Rewrite in intercept form.

$$y = 1x^3 + 2x^2 + 2x + 4$$

$$y = (x^2 + 2)(x + 2)$$

Find the “zeroes.”

$$0 = (x^2 + 2)(x + 2)$$

$$0 = x^2 + 2 \qquad 0 = x + 2$$

$$-2 = x^2 \qquad x = -2$$

$$x = \pm i\sqrt{2}$$

Find the zeroes using "box factoring"

$$y = 4x^3 - 5x^2 + 12x - 15$$

	$4x$	-5
x^2	$4x^3$	$-5x^2$
3	$12x$	-15

$$0 = (x^2 + 3)(4x - 5)$$

$$0 = x^2 + 3 \qquad 4x - 5 = 0$$

$$-3 = x^2 \qquad 4x = 5$$

$$x = \pm i\sqrt{3} \qquad x = {}^5/_4$$

$$x = \ \mathrm{i}\sqrt{3},\ -\mathrm{i}\sqrt{3},\ {}^5/_4$$

What have we learned so far?

"Nice" Common Factor 3rd degree polynomial:

$$y = x^3 + 3x^2 + 2x \qquad = x(x^2 + 3x + 2)$$

$$= x(x+1)(x+2)$$

"Nice" Factor by box 3rd degree polynomial:

$$y = 2x^3 + 3x^2 + 4x + 6$$

“Nice” Difference of Squares (of higher degree):

$$y = x^4 - 81$$

Use “m” substitution

Let $m^2 = x^4$

Then $m = x^2$

$$y = m^2 - 81$$

$$y = (m + 9)(m - 9)$$

Use “m” substitution

$$y = (x^2 + 9)(x^2 - 9)$$

$$y = (x + 3)(x - 3)(x + 3i)(x - 3i)$$

Find the *zeroes*. $x =$ -3, 3, -3i, 3i

Zeroes of Polynomials ←→ factors of Polynomials

Zeros come from linear factor of the polynomial

$x =$ -3, 4, -5, 6

$$y = (x+3)(x-4)(x+5)(x-6)$$

Imaginary Zeros ***always*** come in conjugate pairs.

$x =$ *3, -2i, 2i*

$$y = (x-3)(x+2i)(x-2i)$$

Irrational Zeros ***always*** come in conjugate pairs.

$$x = 0, \sqrt{5}, -\sqrt{5}$$

$$y = x(x-\sqrt{5})(x+\sqrt{5})$$

Fundamental Theorem of Algebra: If a polynomial has a degree of "n", then the polynomial has "n" zeroes (provided that repeat zeroes, called "multiplicities" are counted separately).

$$y = 6x^4 + 42x^3 + 96x^2 + 28x + 48$$

"4th Degree" → 4 zeroes   x = -4, -3, 2i, -2i

Linear Factorization Theorem: If a polynomial has a degree of "n", then the polynomial can be factored into "n" linear factors.

$$y = 6(x+4)(x+3)(x-2i)(x+2i)$$

Since each linear factor has one zero, these two theorems are almost saying the same thing.

Polynomial Long division: One method used to divide polynomials similar to long division for numbers.

$$\frac{x^3 + 3x^2 + 14x - 18}{(x-1)} = ax^2 + bx + c$$

Divide Evenly: A divisor divides evenly if there is a zero for the remainder.

The Factor Theorem: If a linear factor divides the polynomial evenly, then it is a factor of the polynomial and the zeros of the factor are also zeros of the polynomial.

The purpose for long division: is to "rapidly" determine the zeros of "***not nice***" polynomials.

What are the zeroes? $0 = (2x - 5)(3x + 7)$ $x = \frac{5}{2}, \frac{-7}{3}$

Multiply the two binomials (convert to standard form)

$$0 = 6x^2 - x - 35$$

What do you notice about the first and last terms and the zeroes?

$$0 = 6x^2 - x - 35$$

$$x = \frac{5}{2}, \frac{-7}{3}$$

5(-7) = -35

2(3) = 6

The Rational Zeroes Theorem: the possible rational zeroes of a polynomial are factors of the constant divided by factors of the lead coefficient.

$$0 = 6x^2 - x - 35$$

$$x = \pm \frac{1, 5, 7, 35}{1, 2, 3, 6} \qquad x = \frac{5}{2}, \quad \frac{-7}{3}$$

$$x = \pm 1, 5, 7, 35, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{5}{2}, \frac{7}{2}, \frac{7}{3}, \ldots, \frac{35}{3}, \frac{35}{6}$$

What are the zeroes? $0 = (x-3)(2x+1)(x-4)$

$$x = 3, \quad \frac{-1}{2}, \quad 4$$

If we convert to standard form, we have:

$$0 = 2x^3 - 13x^2 + 17x + 12$$

What do you notice about the first and last terms and the zeroes?

$$0 = 2x^3 - 13x^2 + 17x + 12$$

$$x = 3, \quad \frac{-1}{2}, \quad 4$$

$$\pm\frac{1, 2, 3, 4, 6, 12}{1, 2} \qquad \pm\frac{1, 2, 3, 4, 6, 12}{1, 2} \qquad \pm\frac{1, 2, 3, 4, 6, 12}{1, 2}$$

Polynomial Long Division

$$x^2$$

$$x-1 \overline{) \; x^3 + 3x^2 + 14x - 18}$$

1) Look at left-most numbers

2) What # times "left" = "left"?

$$x^3 / x = ? = x^2$$

3) Multiply

$$x^2(x-1) = x^3 - x^2$$

4) Subtract

$$-(x^3 - x^2)$$

Polynomial Long Division

$$
\begin{array}{r}
x^2 \\
x-1 \overline{) \; x^3 + 3x^2 + 14x - 18} \\
-(x^3 - x^2) \\
\hline
4x^2 + 14x - 18
\end{array}
$$

4) Subtract

Careful with the negatives!

5) Bring down.

Polynomial Long Division

$$x^2 + 4x$$

$$x - 1 \,\big)\, x^3 + 3x^2 + 14x - 18$$

$$-(x^3 + x^2)$$

$$4x^2 + 14x - 18$$

$$-(4x^2 - 4x)$$

$$18x$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$$\frac{4x^2}{x} = ? \quad = 4x$$

3) Multiply

$$4x(x-1) = 4x^2 - 4x$$

4) Subtract

$$-(4x^2 - 4x)$$

Polynomial Long Division

$$\begin{array}{r} x^2 + 4x \\ x-1 \overline{) x^3 + 3x^2 + 14x - 18} \\ -(x^3 + x^2) \\ \hline 4x^2 + 14x - 18 \\ -(4x^2 - 4x) \\ \hline 18x - 18 \end{array}$$

4) Subtract

Careful of the negatives

5) Bring down.

Polynomial Long Division

$$\begin{array}{r} x^2 + 4x + 18 \\ x-1 \overline{) x^3 + 3x^2 + 14x - 18} \\ -(x^3 + x^2) \\ \hline 4x^2 + 14x - 18 \\ -(4x^2 - 4x) \\ \hline 18x - 18 \\ -(18x - 18) \\ \hline 0 \end{array}$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$$\frac{18x}{x} = 18$$

3) Multiply

$$18(x-1) = 18x - 18$$

4) Subtract

$$-(18x - 18)$$

$$
\begin{array}{r|l}
 & x^2 + 4x - 18 \\
x-1 & x^3 + 3x^2 + 14x - 18 \\
 & -(x^3 + x^2) \\
\hline
 & 4x^2 + 14x - 18 \\
 & -(4x^2 - 4x) \\
\hline
 & 18x - 18 \\
 & -(-18x + 18) \\
\hline
 & 0
\end{array}
$$

How do we find the zeroes of the quadratic factor?

$$x^3 + 3x^2 + 14x - 18 = (x - 1)(x^2 + 4x - 18)$$

1) Factor, or
2) Quadratic Formula, or
3) Convert to vertex form and take square roots.

Synthetic Division

$$x-1\overline{)\,x^3-4x^2-15x+18}$$

1)	1	-4	-15	18
	1			

1st step: Write the polynomial with only its coefficients.

2nd step: Write the "zero" of the linear divisor.

3rd step: Bring down the lead coefficient

$$x-1\overline{)\;x^3-4x^2-15x+18}$$

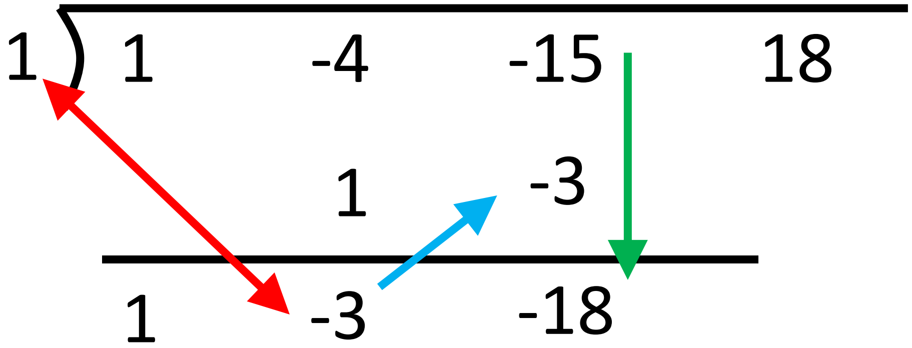
1	1	-4	-15	18
		1		
	1	-3		

4th step: Multiply the “zero” by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

$$x-1\overline{)\,x^3-4x^2-15x+18}$$

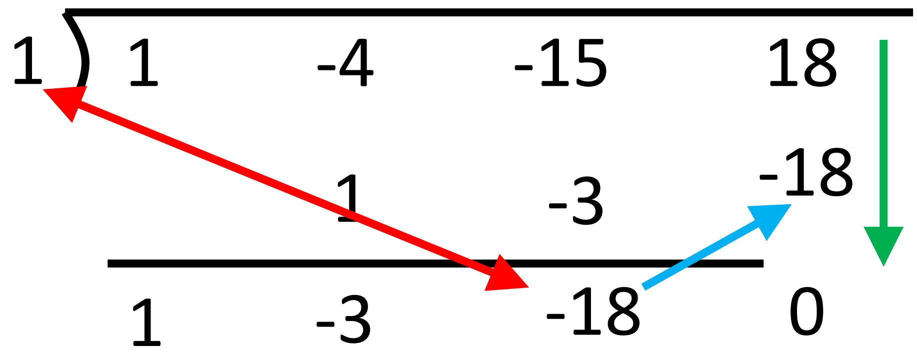


7th step: Multiply the “zero” by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

$$x - 1 \overline{\big) \; x^3 - 4x^2 - 15x + 18}$$



10th step: Multiply the “zero” by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

$$x-1 \overline{) \; x^3 - 4x^2 - 15x + 18} = x^2 - 3x - 18$$

1)	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

This last number is the remainder when you divide:

$$x^3 - 4x^2 - 15x + 18$$

by

$$x - 1$$

Because the remainder = 0, then (x – 1) is a factor AND x = 1 is a zero of the original polynomial!

<u>The Sum of cubes</u>: factors as the cubed root of each term multiplied by a 2nd degree polynomial.

$$y = x^3 + 64$$

Find the 1st zero

$$0 = x^3 + 64 \qquad -64 = x^3$$

$$\sqrt[3]{-64} = x \qquad x = -4 \qquad y = (x+4)(ax^2 + bx + c)$$

Use polynomial division to find the quadratic factor.

$$x + 4 \overline{\big)\, x^3 + 64} \qquad y = (x+4)(x^2 - 4x + 16)$$

-4	1	0	0	64
		-4	16	-64
	1	-4	16	**0**

Find Zeros the Quadratic Factor

1) Factor (not possible)
2) Quadratic Formula or
3) Convert to vertex form and take square roots.