Math-2

Lesson 5-5

Convert Standard Form Quadratic Equations to Vertex Form

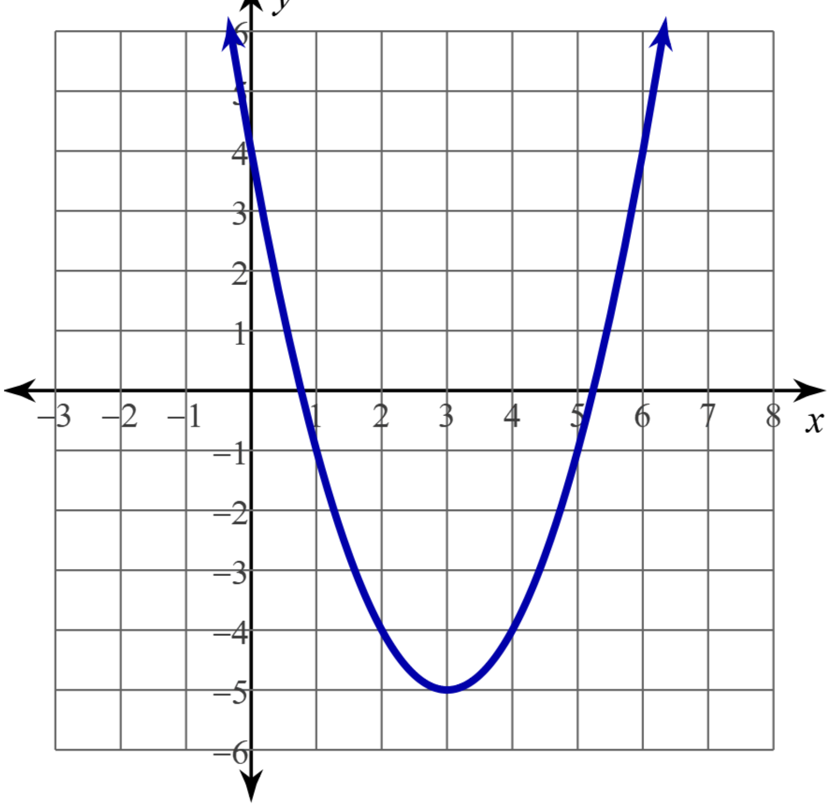
$$y = x^2 - 6x + 4$$

Can this be factored?

The x-intercepts are *"ugly"*

What is the vertex form equation?

$$y = (x-3)^2 - 5$$



Has anyone been taught the Quadratic Formula?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

I like this version more.

What is the purpose of the formula?

The ‘x’ in the formula are the x-intercepts of the *standard form equation.* $y = ax^2 + bx + c$

Our goal for this lesson is NOT learning the quadratic formula.

We’ll learn that latter, but I want to show you an example.

$$y = ax^2 + bx + c$$

$$y = x^2 - 6x + 4$$

a = 1 b = -6 c = 4

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(\quad)}{2(\quad)} \pm \frac{\sqrt{(\quad)^2 - [4(\quad)(\quad)]}}{2(\quad)}$$

$$x = \frac{-(-6)}{2(1)} \pm \frac{\sqrt{(-6)^2 - [4(1)(4)]}}{2(1)}$$

$$x = \frac{6}{2} \pm \frac{\sqrt{36 - 16}}{2}$$

$$x = 3 \pm \frac{\sqrt{20}}{2}$$

$$x = 3 \pm \frac{\sqrt{4}\sqrt{5}}{2}$$

$$x = 3 \pm \frac{2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$

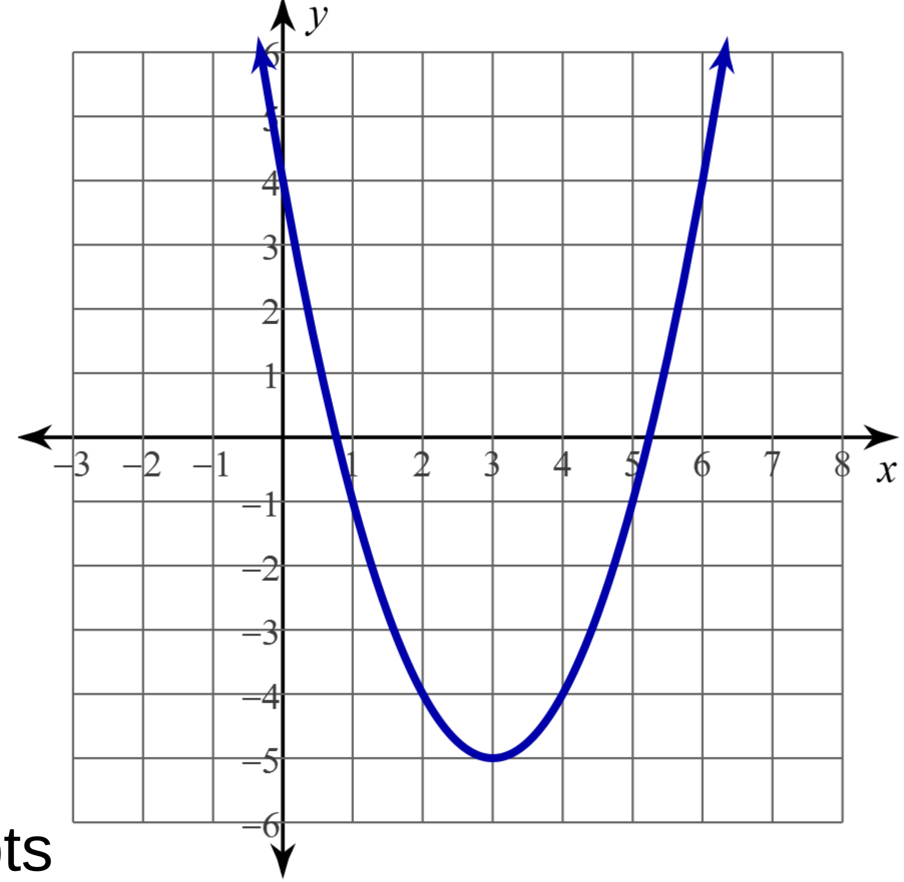
This formula often-times results in "*ugly*" calculations where students make mistakes.

Standard Form Equation

$$y = x^2 - 6x + 4$$

Vertex Form Equation

$$y = (x - 3)^2 - 5$$



The x-intercepts that came from the quadratic formula were:

$$x = 3 \pm \sqrt{5}$$

How could you get the x-intercepts from the *vertex form equation*?

Set 'y' to zero. Isolate the square, "undo" the square.

$$0 = (x - 3)^2 - 5$$

$$5 = (x - 3)^2$$

$$\pm\sqrt{5} = x - 3$$

$$x = 3 \pm \sqrt{5}$$

Find the X-intercepts from the Vertex Form Equations

$y = -2(x-3)^2 + 4$ Set y = 0 (y-value of an x-int. is 0)

$0 = -2(x-3)^2 + 4$ Subtract 4 (left/right)

$-4 = -2(x-3)^2$ Divide by -2 (left/right)

$2 = (x-3)^2$

$2 = (\_\_\_\_)^2$ What number, squared, equals 2?

$2 = \left(\sqrt{2}\right)^2 \quad 2 = \left(-\sqrt{2}\right)^2$

$\pm\sqrt{2} = x - 3$ Add 3 (left/right)

$x = 3 \pm \sqrt{2}$

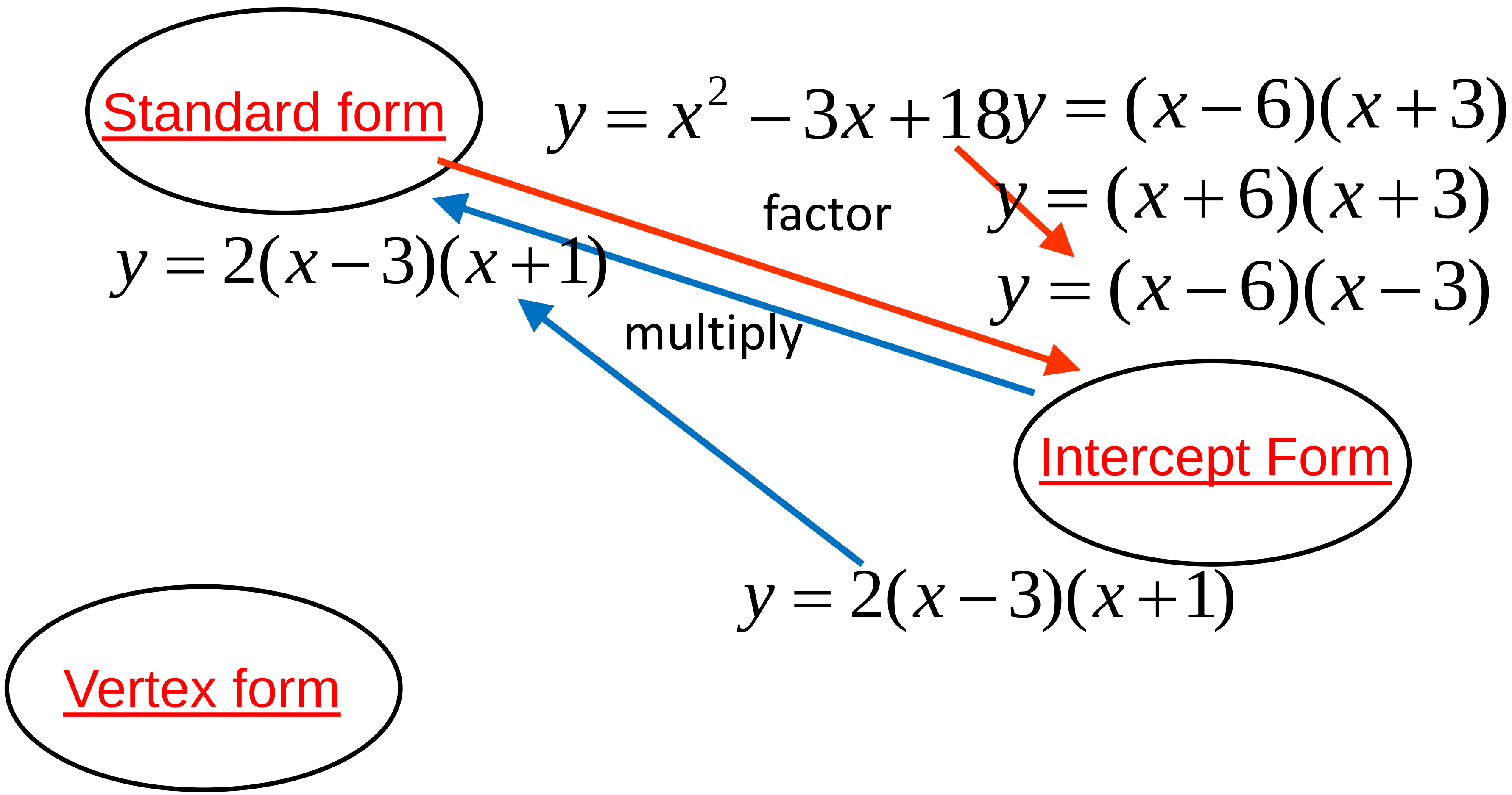
Find the X-intercepts from the Vertex Form Equations

$$y = (x - 5)^2$$

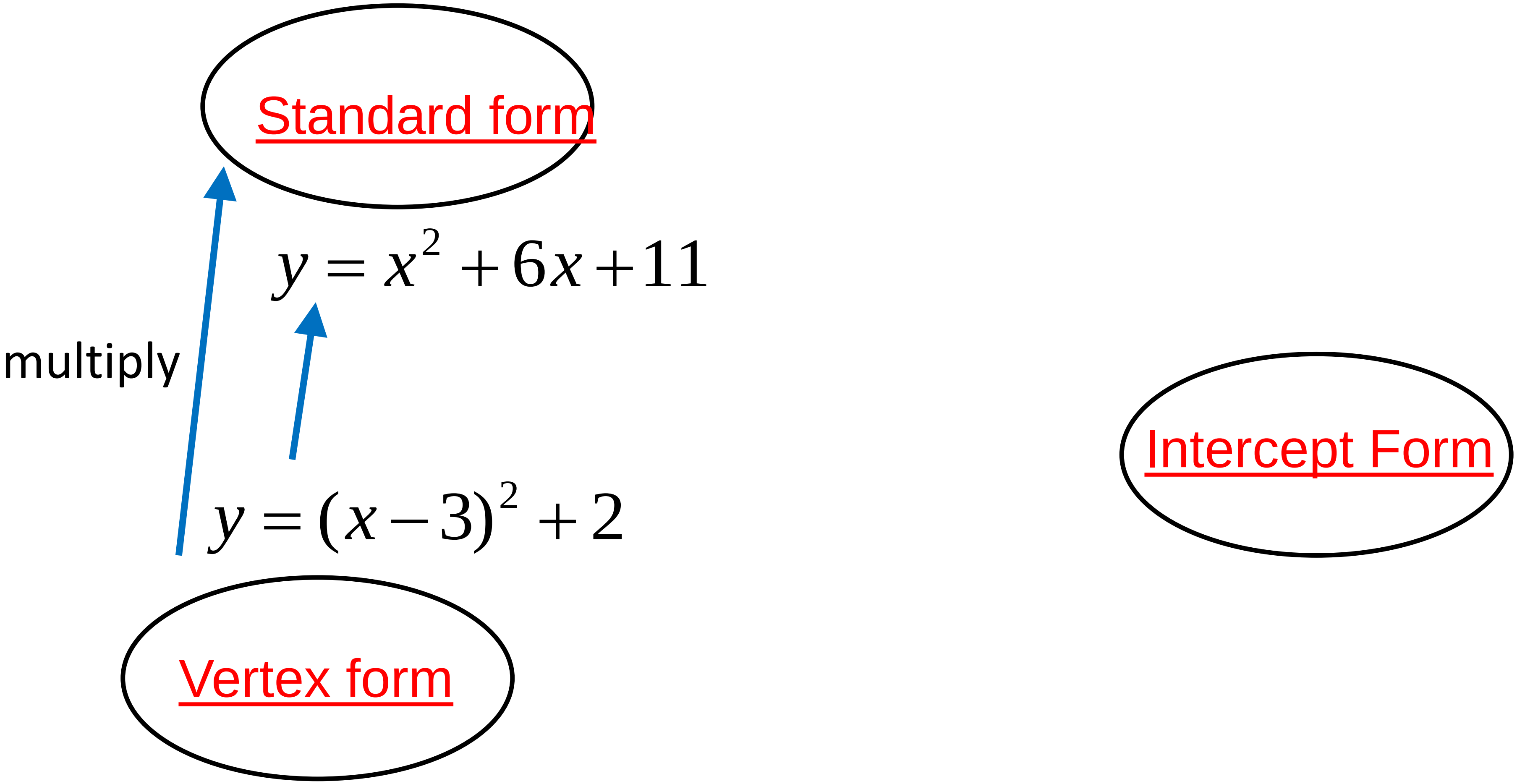
$$y = -2(x-3)^2 + 4$$

$$y = -(x+2)^2 + 5$$

What process would you use to ….?



What process would you use to ….?



What have we learned?

1. The quadratic formula can give us x-intercepts (only if you have the *standard form equation*).

There are a lot of numbers and calculations. You can easily make a mistake.

2. If you "isolate the square, undo the square" on the *vertex form equation*, you can also find x-intercepts.

You have to know how to simplify square roots.

$$y = (x-2)^2 - 12$$

$$0 = (x-2)^2 - 12$$

$$12 = (x-2)^2$$

$$x = 2 \pm \sqrt{12}$$

$$x = 2 \pm \sqrt{4 * 3}$$

$$x = 2 \pm \sqrt{4}\sqrt{3}$$

$$x = 2 \pm 2\sqrt{3}$$

3. You can convert *standard form quadratic equations* into *intercept form quadratic equations* by: *factoring*

$$y = 2x^2 + 16x + 24 \quad \rightarrow y = 2(x+6)(x+2)$$

3. You can convert *intercept form quadratic equations* into *vertex form quadratic equations* by:

a) Finding the x-coordinate of the vertex (half way between x-intercepts) $x = -6, -2$ Vertex: $(-4, \_\_\_)$

b) Substituting the x-value into the equation to find the y-coordinate of the vertex. $y = 2(-4+6)(-4+2)$

$y = 2(2)(-2) = -8$ Vertex: $(-4, -8)$

c) Using the VSF and the vertex to write the vertex form equation.

VSF = 2, Vertex: $(-4, -8)$ $y = 2(x+4)^2 - 8$

How can we convert *Standard Form Quadratic Equations* directly into *Vertex form*? (without converting to *Intercept Form* first?)

Remember the quadratic formula gave us these x-intercepts.

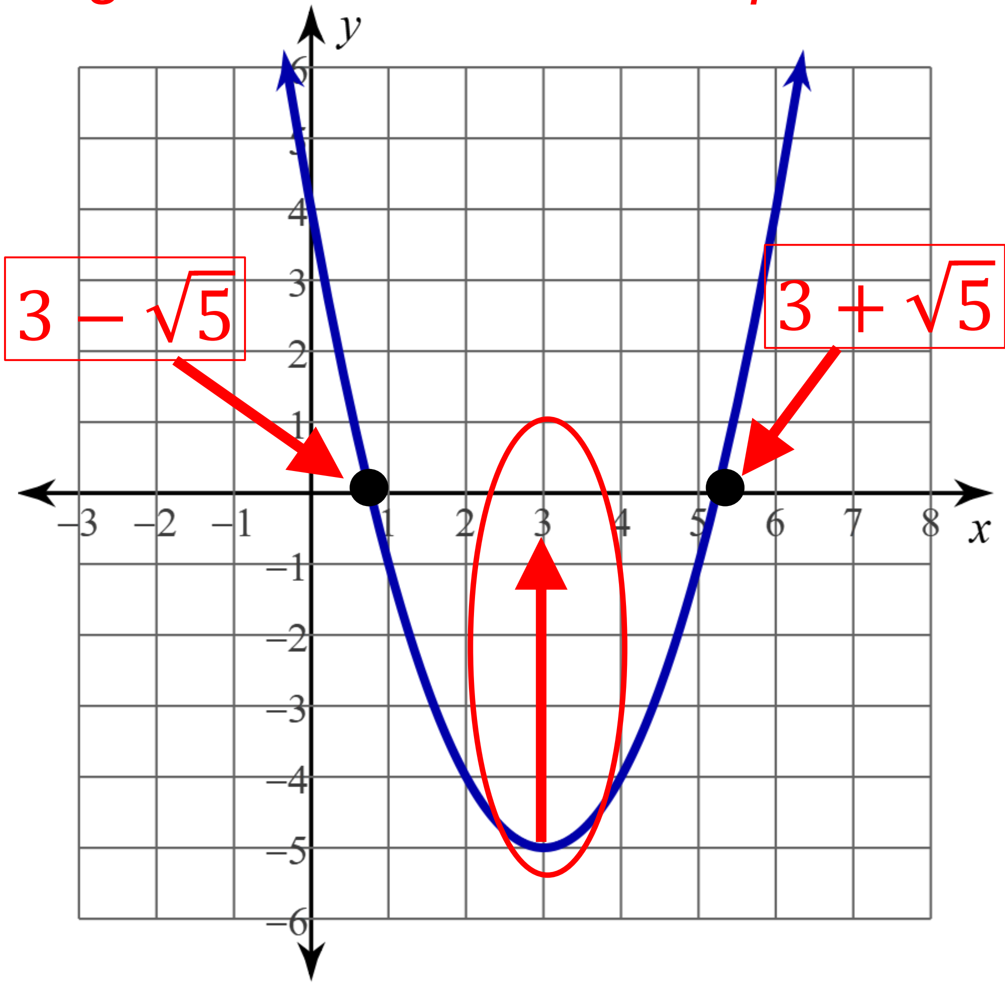
$$y = x^2 - 6x + 4$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = 3 \pm \sqrt{5}$$

The x-coordinate of the vertex is 3.

x-coord. of vertex = $\frac{-b}{2a}$



What process would you use to ….?

Standard form

$$y = ax^2 + bx + c$$

$$h = {}^{-b}\!/_{2a}$$

$$k = f\left({}^{-b}\!/_{2a}\right)$$

$$f(x) = a(x-h)^2 + k$$

Vertex form

$$y = x^2 - 8x - 10$$

$$y = (x-4)^2 - 26$$

Intercept Form

What is the x-coordinate of the vertex?

$$y = 2x^2 + 16x + 24$$

a = 2   b = 16

x-coord. of vertex = $\frac{-b}{2a}$

$$\frac{-b}{2a} = \frac{-16}{2(2)} = -4$$

Vertex: $(-4, f(-4))$

What is the y-coordinate of the vertex?

$$f(-4) = 2(-4)^2 + 16(-4) + 24$$

$$f(-4) = -8$$

Vertex: $(-4, -8)$

What is the Vertex form equation?

$\text{VSF} = 2, \text{vertex} = (-4, -8)$

$$y = 2(x + 4)^2 - 8$$

What is the x-coordinate of the vertex?

$$y = x^2 - 6x + 13$$

a = 1   b = -6

x-coord. of vertex = $\frac{-b}{2a}$

$$\frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$$

Vertex: $(3, f(3))$

What is the y-coordinate of the vertex?

$$f(3) = (3)^2 - 6(3) + 13$$

$$f(3) = 4$$

Vertex: $(3, 4)$

What is the Vertex form equation?

$\mathrm{VSF} = 1, \mathrm{vertex} = (3, 5)$

$$y = (x - 3)^2 + 4$$

What is the x-coordinate of the vertex?

$$y = 3x^2 + 6x - 12$$

a = 3   b = -4

x-coord. of vertex = $\frac{-b}{2a}$

$$\frac{-b}{2a} = \frac{-(6)}{2(3)} = -1$$

Vertex: $(-1, f(-1))$

What is the y-coordinate of the vertex?

$$f(-1) = 3(-1)^2 + 6(-1) - 12$$

$$f(-1) = -15$$

Vertex: $(-1, -15)$

What is the Vertex form equation?

$\text{VSF} = 3, \text{vertex} = (-1, -15)$

$$y = 3(x + 1)^2 - 15$$

We have converted the following standard form equations into vertex form. What are the x-intercepts of the following equations?

$y = 2x^2 + 16x + 24$ → $y = 2(x+4)^2 - 8$

$y = x^2 - 6x + 13$ → $y = (x-3)^2 + 4$

$y = 3x^2 - 6x - 12$ $y = 3(x+1)^2 - 15$

Convert the following *non-factorable* standard form equations into vertex form. Find the x-intercepts.

$$y = x^2 - 2x - 12$$

$$y = x^2 + 20x + 99$$

$$y = x^2 - 14x + 50$$