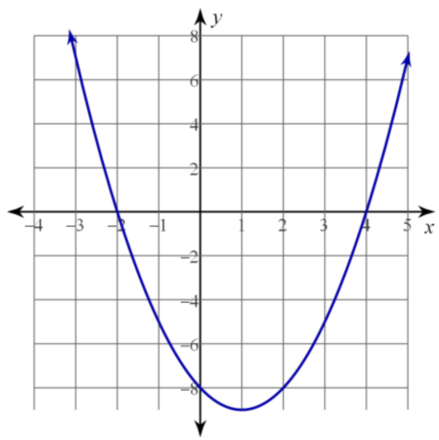
Math-2 Lesson 5-4

Finding X-intercepts of the Vertex Form Quadratic Equation

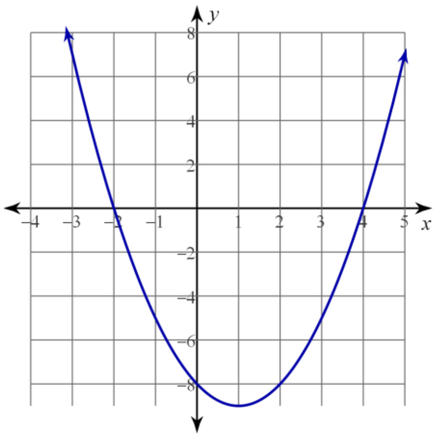


What is the name of the quadratic equation form?

Vertex Form quadratic equation (transformation form)

$$f(x) = a(x-h)^2 + k$$

$$f(x) = -3(x+1)^2 + 2$$



Standard Form quadratic equation

$$g(x) = ax^2 + bx + c$$

$$g(x) = 6x^2 + 7x + 2$$

Intercept Form quadratic equation

$$k(x) = a(mx-p)(nx-q)$$

$$k(x) = 2(3x+7)(x-2)$$

$$y = ax^2 + bx + c$$

In General, if the standard form quadratic has no x-term $(b = 0)$,

then we have a "nice" 2-term quadratic equation

(a ≠ 1, and b = 0) that is easily factored. $y = ax^2 + c$

Always factors into: $y = (\sqrt{ax^2} + \sqrt{c})(\sqrt{ax^2} - \sqrt{c})$

Check it with the box.

$$y = (9x + 5)(9x - 5)$$

$$y = 81x^2 - 25$$

	9x	-5
9x	$81x^2$	-45x
5	45x	-25

Check it with the box.

$$y = (7x + i)(7x - i)$$

$$y = 49x^2 + 1$$

	7X	i
7x	x^2	$7xi$
$-i$	$-7xi$	$-i^2$

Irrational Conjugates Theorem: IF an equation is of the form $y = x^2 - c$, where ‘a’ is not a perfect square, THEN it always factors into ***irrational conjugate***

$$y = (x + \sqrt{c})(x - \sqrt{c})$$

with the following zeroes: $x = -\sqrt{c}, \sqrt{c}$

Complex Conjugates Theorem: IF an equation is of the form $y = x^2 + c$, THEN it always factors into conjugate binomial pairs of *imaginary numbers*.

$$y = (x + i\sqrt{c})(x - i\sqrt{c})$$

with the following zeroes: $x = -i\sqrt{c}, i\sqrt{c}$

<u>Review</u>: simplify the following expressions

$$\frac{\sqrt{6}}{2} \rightarrow \frac{\sqrt{2*3}}{2}$$

No pairs, can't be simplified

$$\rightarrow \frac{\sqrt{6}}{2}$$

$$\frac{\sqrt{12}}{2} \rightarrow \frac{\sqrt{2*2*3}}{2} \rightarrow \frac{2\sqrt{3}}{2} \rightarrow \sqrt{3}$$

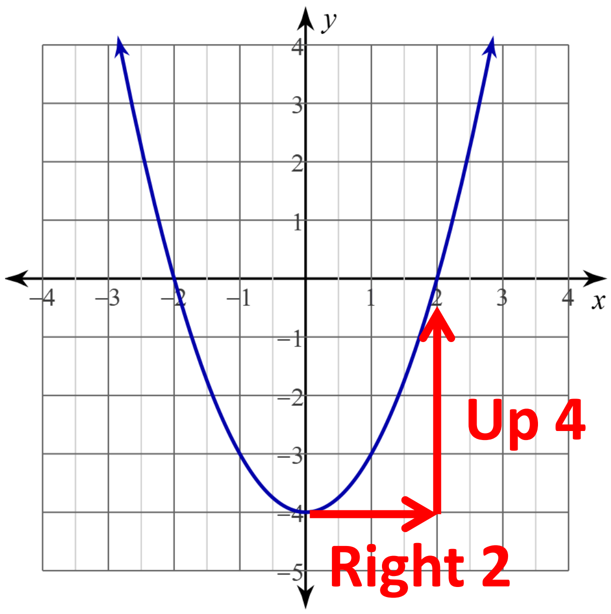
$$\frac{1}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}}*\frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

$$\frac{3}{\sqrt{5}} \rightarrow \frac{3}{\sqrt{5}}*\frac{\sqrt{5}}{\sqrt{5}} \rightarrow \frac{3\sqrt{5}}{5}$$

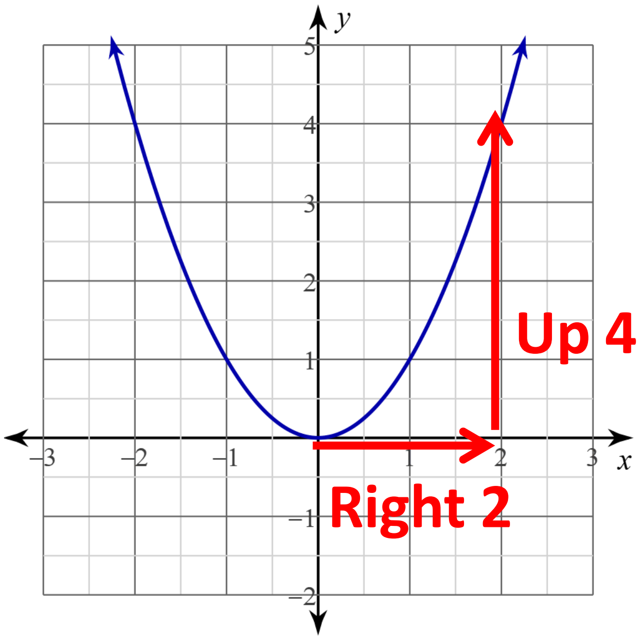
Describe the transformation to the parent function:

$$g(x) = x^2 - 4$$

$$f(x) = x^2$$



down 4



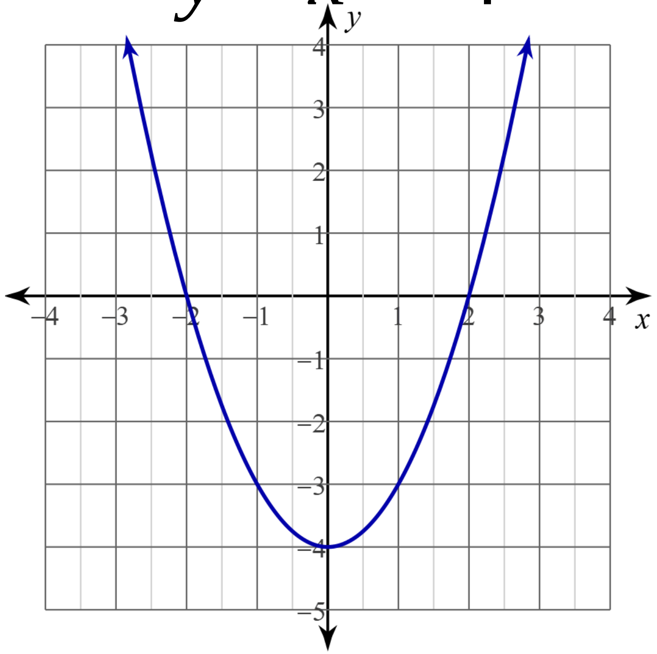
Notice the x-intercepts are (-2,0) and (2, 0)

<u>"Zero" of a 2-variable equation</u>: the input value of an equation that causes the output to equal zero.

"Zeroes" often (<u>but not always</u>) are the x-intercepts of the graph.

"Zero" of a 2-variable equation: the input value that causes the output to equal zero.

$$y = x^2 - 4$$



1) Find Zeroes by taking square roots

$$y = x^2 - 4$$ Set 'y = 0"

$$0 = x^2 - 4$$ Get the 'x' squared term by itself:

$$x^2 = 4$$ How do you "undo" a square function?

$$\sqrt{x^2} = \sqrt{4}$$

$$x^2 = 4$$

$$x = 2, -2$$

Why is it both "+" and "-" 2?

$$(2)^2 = 4$$ "+" number squared is positive

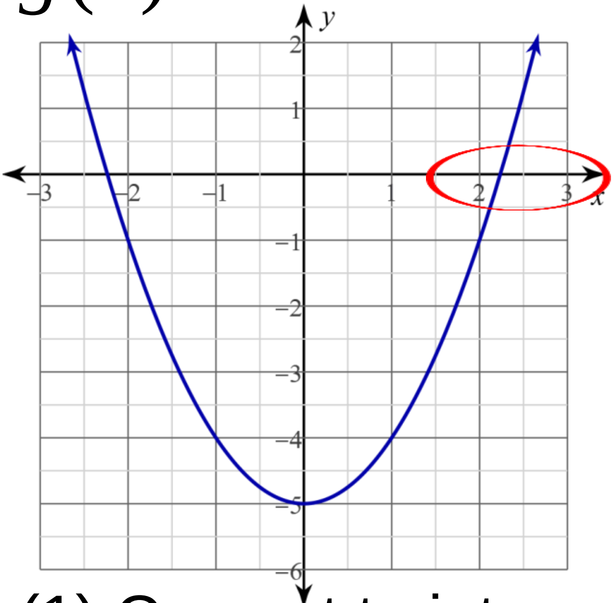
$$(-2)^2 = 4$$ "-" number squared is positive

Describe the transformation: down 5

$g(x) = x^2 - 5$

Would you consider this equation "vertex form"?

yes



Notice the positive x-intercept is between 2 and 3.

What are the 'x' intercepts (?, 0) and (?, 0)

Another method to find the zeroes: "Isolate the square, "undo" the square.

$$0 = x^2 - 5$$

$$x^2 = 5$$

$$\sqrt{x^2} = \sqrt{5}$$

$$x = -\sqrt{5}, \sqrt{5}$$

$$-\sqrt{5} \approx -2.24 \quad +\sqrt{5} \approx +2.24$$

(1) Convert to intercept form:

$$y = x^2 - 5$$

$$0 = (x - \sqrt{5})(x + \sqrt{5})$$

$$(-\sqrt{5}, 0) \text{ and } (\sqrt{5}, 0)$$

The "zeroes" are: $x = \pm\sqrt{5}$

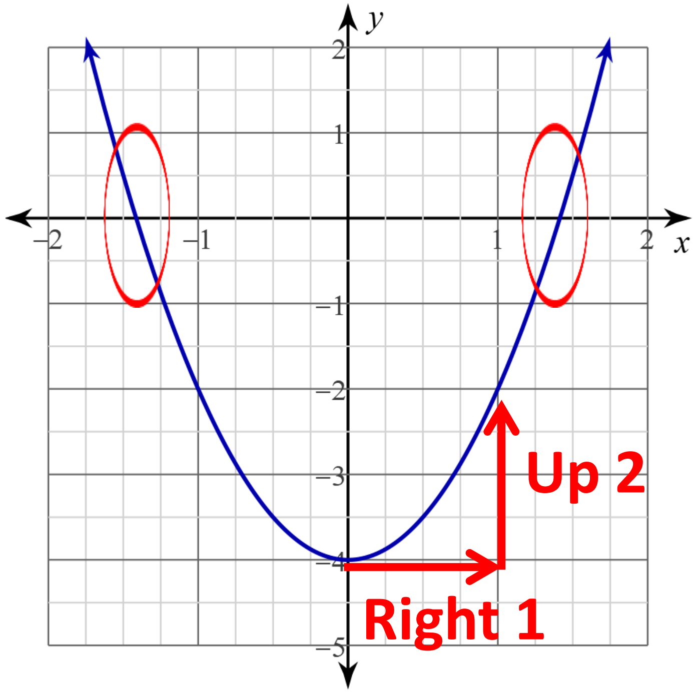
Find the zeroes of the following equations by taking square roots.

$y = x^2 - 1$ Set 'y = 0"

$0 = x^2 - 1$ Isolate the 'x' squared term

$x^2 = 1$ Take square roots

$\sqrt{x^2} = \sqrt{1}$ $x = \pm 1$



$y = 2x^2 - 4$ Describe the transformations

$0 = 2x^2 - 4$ VSF = 2, down 4

$4 = 2x^2$

$2 = x^2$

$\sqrt{x^2} = \sqrt{2}$ $x = \pm\sqrt{2}$ $x \approx \pm 1.41$

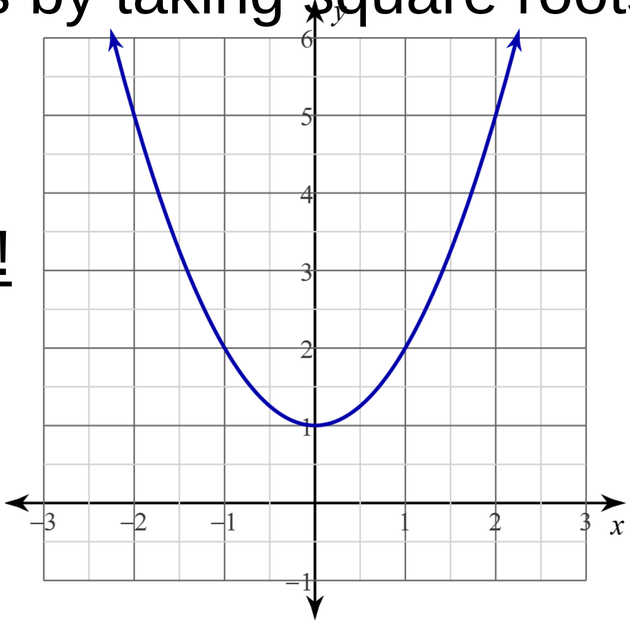
Find the zeroes of the following equations by taking square roots.

$$y = x^2 + 1$$

Describe the transformations

Up 1

There are no x-intercepts!



$$y = x^2 + 1$$

Find the zeroes by taking square roots.

$$0 = x^2 + 1$$

Set 'y = 0"

$$x^2 = -1$$

Isolate the 'x' squared term

$$\sqrt{x^2} = \sqrt{-1}$$

Take square roots

$$x = \pm i$$

The zeroes of the equation are imaginary numbers!

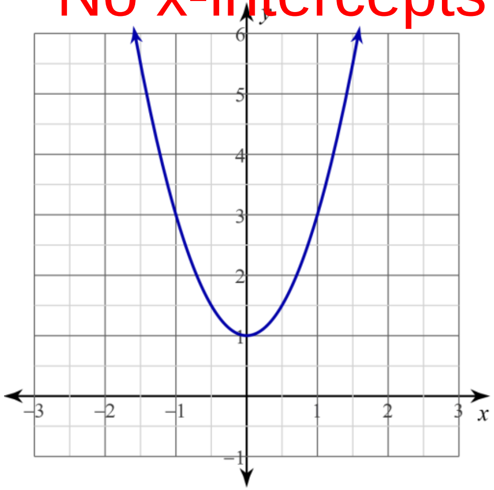
We cannot graph imaginary numbers on the "real plane"

There are no x-intercepts, but there are two zeroes of the equation!

Which of the following have x-intercepts?

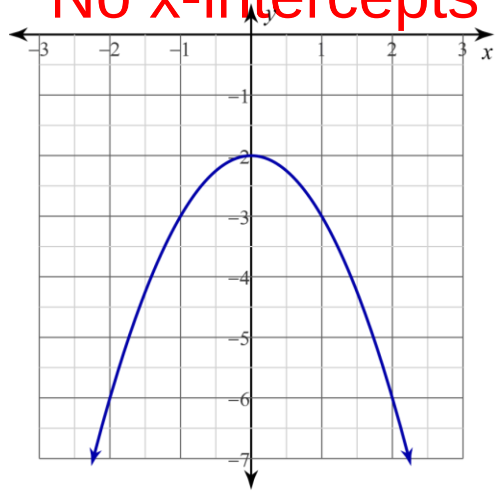
$$y = 2x^2 + 1$$

No x-intercepts



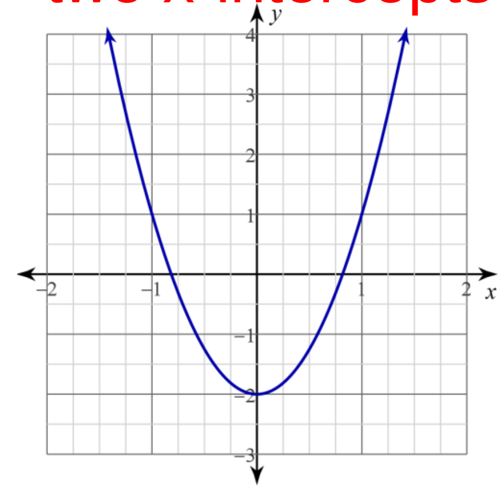
$$y = -x^2 - 2$$

No x-intercepts



$$y = 3x^2 - 2$$

two x-intercepts



What are the zeroes of the functions?

$$0 = 2x^2 + 1$$

$$-1 = 2x^2$$

$$\frac{-1}{2} = x^2 \qquad \pm\frac{i}{\sqrt{2}} = x$$

$$\pm\frac{\sqrt{-1}}{\sqrt{2}} = x \qquad \pm\frac{i\sqrt{2}}{2} = x$$

$$0 = -x^2 - 2$$

$$x^2 = -2$$

$$x = \pm\sqrt{-2}$$

$$x = \pm i\sqrt{2}$$

$$0 = 3x^2 - 2$$

$$2 = 3x^2$$

$$\frac{2}{3} = x^2$$

$$\pm\frac{\sqrt{2}}{\sqrt{3}} = x = \pm\frac{\sqrt{6}}{3}$$

$$y = (x-2)^2 - 9$$

1. Transformations of the parent function? Right 2, down 9

2. Which form of the quadratic is this? Vertex Form

3. What is the vertex? (2, -9)

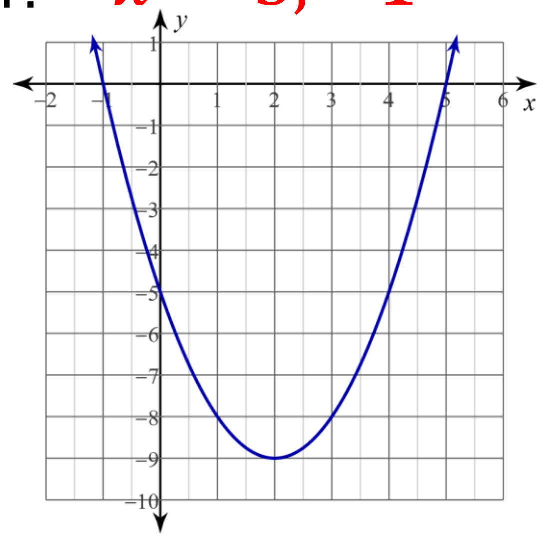
4. Convert the equation to standard form. $y = x^2 - 4x - 5$

5. Convert standard form equation into intercept form:

$$y = (x-5)(x+1)$$

6. What are the zeroes of the equation? $x = 5, -1$

7. Draw a graph of the function.



8. Are the zeroes real or imaginary?

The graph crosses the x-axis
→ it has real number zeroes.

Can we use "isolate the square, "undo" the square to find the zeroes of the equation?

$y = (x-2)^2 - 9$

x-coord of vertex

$y = a(x-h)^2 + k$

Let y = 0 $\quad 0 = (x-2)^2 - 9$

Zeroes: x-coord. of vertex +/- a number.

Isolate the squared term

$0 = (x-2)^2 - 9$

$+9 \qquad\qquad + 9$

$9 = (x-2)^2$

"take square roots"

$\sqrt{9} = \sqrt{(x-2)^2}$

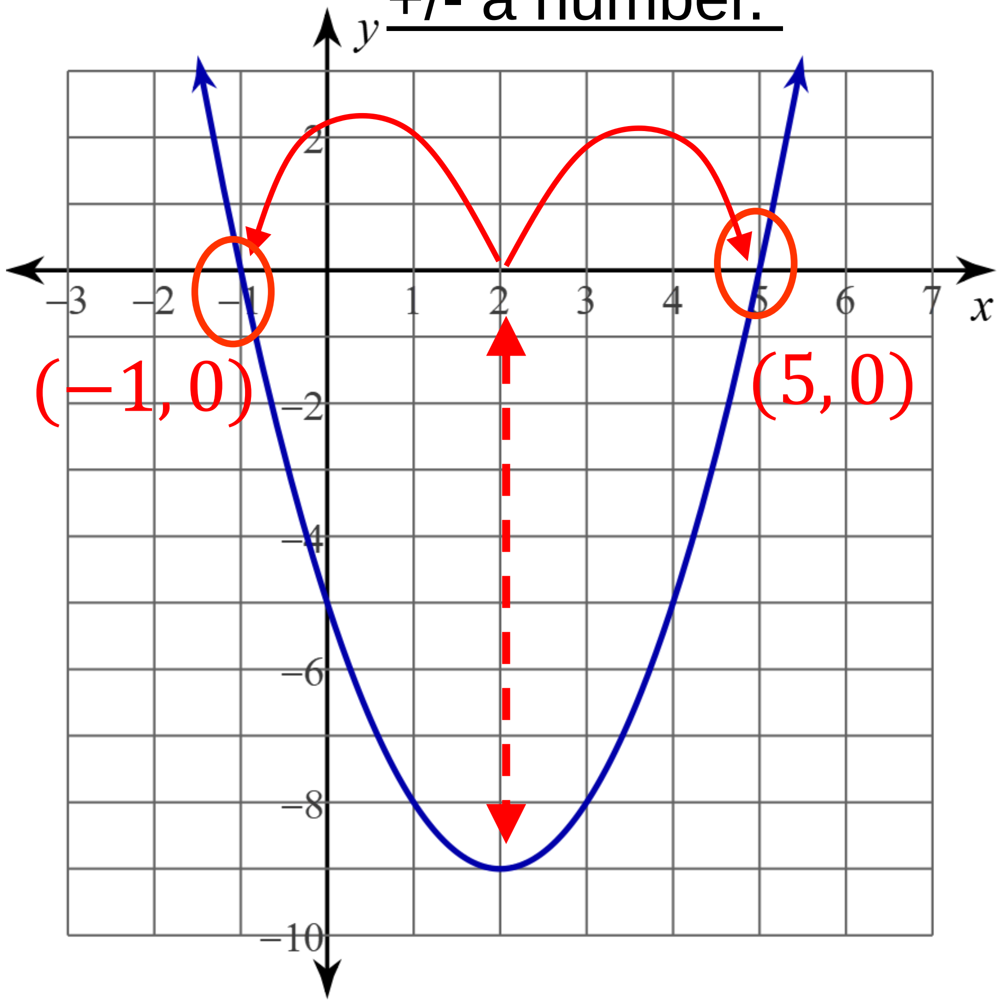
$3 = x - 2$

f(0)=5

Solve for 'x'

$(5, 0)$

$x = 2 + 3 \qquad x = 2 - 3$



You can find the zeroes of vertex form using "isolate the square, undo the square".

Vertex form → extract a square root.

$$y = a(x-h)^2 + k \qquad y = (x-1)^2 - 9$$

Let y = 0 $\quad 0 = (x-1)^2 - 9$

Isolate the squared term $\quad 9 = (x-1)^2$

"take square roots" $\quad \sqrt{9} = \sqrt{(x-1)^2} \qquad \pm 3 = x - 1$

Solve for 'x' $\quad 1 \pm 3 = x \quad$ simplify $\quad x = 4, -2$

Or, (harder) covert to standard form, then intercept form.

$$y = (x-1)^2 - 9 \qquad y = (x-4)(x+2)$$

$$y = x^2 - 2x + 1 - 9 \qquad 0 = (x-4)(x+2)$$

$$x = 4, -2$$

$$y = x^2 - 2x - 8$$

$$y = (x+3)^2 - 8$$

1. Which form of the quadratic is this? Vertex Form

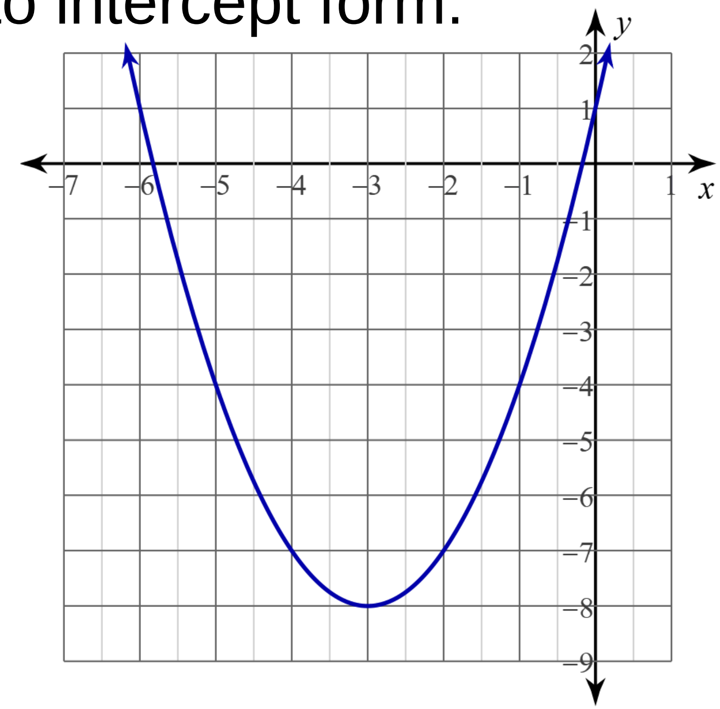
2. transformations of parent function? Left 3, down 8

3. vertex? (-3, -8)

4. Equivalent standard form equation. $y = x^2 + 6x + 1$

5. Convert standard form equation into intercept form:

It can't be factored



6. Draw a graph of the function.

7. Are the zeroes real or imaginary?

The graph crosses the x-axis
→ it has real number zeroes.

8. How can we find the zeroes?

If you can't factor the Standard Form version of the Vertex Form equation that must be a way to find the zeroes!

Vertex form → take square roots.

$$y = a(x-h)^2 + k \qquad y = (x+3)^2 - 8$$

Let y = 0 $\quad 0 = (x+3)^2 - 8$

Isolate the squared term $\quad 8 = (x+3)^2$

"take square roots" $\quad \sqrt{8} = \sqrt{(x+3)^2}$

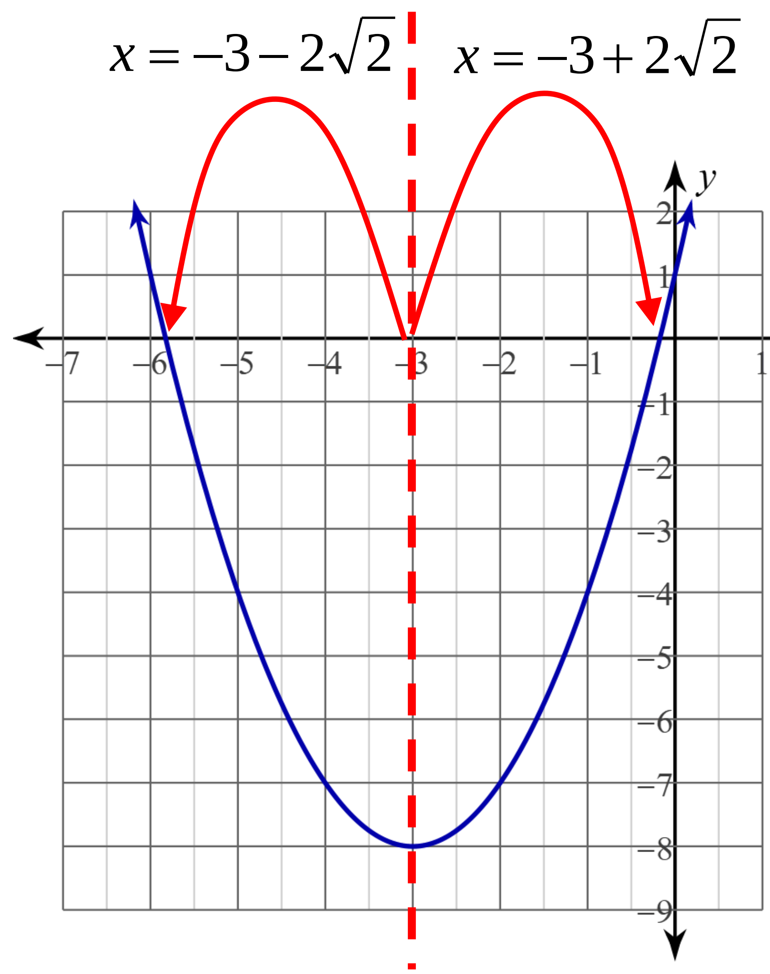
$\pm\sqrt{8} = x+3$ Simplify the radical

$\pm\sqrt{2*2*2} = x+3$

$\pm 2\sqrt{2} = x+3$ Solve for 'x'

$x = -3 \pm 2\sqrt{2}$

x-coord of vertex



$$y = (x-2)^2 - 4$$

Let y = 0

$$0 = (x-2)^2 - 4$$

Isolate the squared term $4 = (x-2)^2$

“Extract a square root” $\pm\sqrt{4} = \sqrt{(x-2)^2}$

$$\pm 2 = x - 2$$

Solve for ‘x’ $2 \pm 2 = x$

$$x = 2 + 2$$

simplify $x = 4,0$

$$x = 2 - 2$$

Or, covert to standard form, then intercept form.

$$y = (x-2)^2 - 4$$

$$y = x^2 - 4x + 4 - 4$$

$$y = x^2 - 4x$$

$$y = x(x-4)$$

$$0 = x(x-4)$$

$$x = 0, 4$$

If standard form can't be factored, you need another way to find the zeroes of the equation.

$$y = 2(x+7)^2 - 10$$

$$y = 2(x^2 + 14x + 49) - 10$$

$$y = 2x^2 + 28x + 96 - 10$$

$$y = 2x^2 + 28x + 86$$

$$y = x^2 + 14x + 43$$

43 is a prime number, it only has factors of 1 and 43

$$y = 2(x+7)^2 - 10$$

Let y = 0

$$0 = 2(x+7)^2 - 10$$

Isolate the Square term

$$10 = 2(x+7)^2$$

Divide by 2 (both sides)

$$5 = (x+7)^2$$

"take square roots"

$$\pm\sqrt{5} = \sqrt{(x+7)^2}$$

$$\pm\sqrt{5} = x + 7$$

subtract 7 from both sides

$$-7 \pm \sqrt{5} = x$$

$$x = -7 + \sqrt{5}$$

$$x = -7 - \sqrt{5}$$

Find the “zeroes” by “Extracting a square root”

$$y = (x - 2)^2 - 5$$

$$y = 3(x + 4)^2 - 12$$

$$y = -3(x - 4)^2 - 27$$