Multiplying Binomials

The "Box Method"

$(x-3)(x+4)$

	x	4
x	x^2	4x
-3	-3x	-12

$x^2 + x - 12$

Standard Form
Quadratic Expression

$(x-1)(x+5)$

	x	5
x	x^2	5x
-1	-x	-5

$x^2 + 4x - 5$

$(x+2)(x+6)$

	x	6
x	x^2	6x
2	2x	12

$x^2 + 8x + 12$

$(x-4)(x+4)$

	x	4
x	x^2	4x
-4	-4x	-16

$x^2 + 0x - 16$

$x^2 - 16$

$(x+2)(x+3)$ multiply

x^2 "left times left is the left term"

$(x+2)(x+3)$ "right times right is the right term"

$x^2 \quad +6$

$(x+2)(x+3)$ "inner"

$x^2 \quad +2x \quad +6$

$(x+2)(x+3)$ "outer"

$x^2 + 2x + 3x + 6 = x^2 + (2+3)x + (2*3)$

$$(x+2)(x+3)$$

$$= x^2 + (2+3)x + (2*3)$$

$$= x^2 + 5x + 6$$

Left times left is left

Right plus right is middle

Right times right is right

$$(x-6)(x+1)$$

$$= x^2 + (-6+1)x + (-6*1)$$

Left times left is left

Right plus right is middle

Right times right is right

$$= x^2 - 5x - 6$$

x^2+5x+6

Left times left is left

$(\_\_+\_\_)(\_\_+\_\_)$

$(x+\_\_)(x+\_\_)$ Right times right is right

$(x+\_\_)(x+\_\_)$ Right plus right is middle

$(x+2)(x+3)$

What are the factors of 6 that add up to 5?

$(x+4)(x+5) \quad = x^2 + (\_\_\_\_\_)x + (\_\_\_\_)$

$= \_\_\_\_\_$

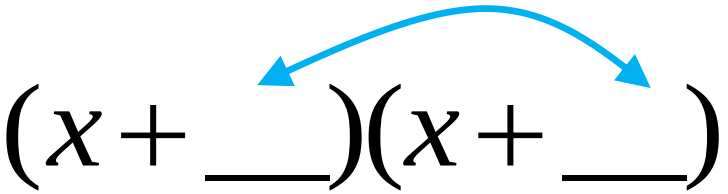
$(x-6)(x+1) \; = x^2 + (\_\_\_\_\_)x + (\_\_\_\_\_)$

$= \_\_\_\_\_$

Try the following:

$$x^2 - 3x - 4 \quad = (x-4)(x+1)$$

$$(x+\_\_)(x+\_\_)$$ Right times right is right



$$(x+\_\_)(x+\_\_)$$ Right plus right is middle

$$(-4)(1) = -4$$

$$(-4)+(1) = -3$$

What are the factors of -4 that add up to -3?

Try the following:

$$x^2+8x+15 = (x+3)(x+5)$$

$(x+\_\_)(x+\_\_)$ Right times right is right

$(x+\_\_)(x+\_\_)$ Right plus right is middle

$(3)(5)=15$

$3+5=8$

What are the factors of 15 that add up to 8?

Factor the following:

$x^2 + 10x + 21 \rightarrow (x+3)(x+7)$

$x^2 - 6x - 16 \rightarrow (x-8)(x+2)$

$x^2 - 9x + 18 \rightarrow (x-6)(x-3)$

$x^2 + 3x + 2 \rightarrow (x+2)(x+1)$

$2x^2 + 4x + 2$

Always factor out the common factor first.

$2(x^2 + 2x + 1)$

Now factor the trinomial.

$2(x + 1)(x + 1)$

Your turn:

$6x^2 + 24x + 18$

Always factor out the common factor 1st.

$6(x^2 + 4x + 3)$

Now factor the trinomial.

$6(x+1)(x+3)$

$3x^2 + 9x + 6 \rightarrow 3(x+2)(x+1)$

$4x^2 - 20x - 48 \rightarrow 4(x-7)(x+2)$

$x^2 - 1$ "the difference of two squares"

$x^2 + 0x - 1$ Two numbers multiplied = (-1) and added = 0

$(-1)(+1)$

$(x-1)(x+1)$

Conjugate pair (of binomials)

two binomials whose terms are exactly the same except +/- for one pair of terms

$(x-1)(x+1)$ $(-x+1)(x+1)$

Which of the following are conjugate pairs?

$(x-4)(x+4)$ Yes, they are.

$(5-x)(5+x)$ Yes, they are.

$(3x+2)(3x+2)$ NO, they are NOT.

$(x-\sqrt{5})(x+\sqrt{5})$ Yes, they are.

$(3x+2)(3x+2)$ NO, they are NOT.

$(2x-7)(2x+7)$ Yes, they are.

$$x^2 - 2$$

We can call this the “difference of two squares”

$$x^2 + 0x - 2$$

Two numbers multiplied = (-2)
and added = 0

$$(-\sqrt{2})(+\sqrt{2})$$

$$(x - \sqrt{2})(x + \sqrt{2})$$

Your turn: Multiply the conjugate pairs.

$(x-1)(x+1) = x^2-1$

$(x-\sqrt{2})(x+\sqrt{2}) = x^2-2$

$(x-\sqrt{3})(x+\sqrt{3}) = x^2-3$

$(x-\sqrt{4})(x+\sqrt{4}) = x^2-4 = (x-2)(x+2)$

$(x-\sqrt{5})(x+\sqrt{5}) = x^2-5$

$(x-\sqrt{a})(x+\sqrt{a}) = x^2-a$

$(x-\sqrt{17})(x+\sqrt{17}) = x^2-17$

Can we use this as a pattern in order to factor the difference of two squares?

A "nice" one.

Your turn: factor the following binomials

$$x^2 - 9 = (x - \sqrt{9})(x + \sqrt{9})$$
$$= (x - 3)(x + 3)$$

$$x^2 - 6 = (x - \sqrt{6})(x + \sqrt{6})$$

Multiply this out: $(x + i)(x - i)$

$$x^2 - xi + xi - i^2$$

Inverse property of addition!

i squared = -1

$$x^2 - (-1)$$

$$x^2 + 1$$

Multiply: $(x+i\sqrt{2})(x-i\sqrt{2})$ x^2+2

What about the sum of two squares?

$x^2+1=(x-i)(x+i)$

$x^2+2=(x-i\sqrt{2})(x+i\sqrt{2})$

$x^2+3=?\quad=(x-i\sqrt{3})(x+i\sqrt{3})$ Can you see the pattern?

$x^2+4=?\quad=(x-i\sqrt{4})(x+i\sqrt{4})\quad=(x-2i)(x+2i)$

$x^2+a=(x-i\sqrt{a})(x+i\sqrt{a})$ General form.

$x^2+7=?\quad=(x-i\sqrt{7})(x+i\sqrt{7})$