

SM2 HANDOUT 5-4 (Zeroes of Vertex Form Quadratic Equations.

$$y = ax^2 + bx + c$$

In General, if the standard form quadratic has no x-term ($b = 0$),

then we have a “nice” 2-term quadratic equation
($a \neq 1$, and $b = 0$) that is easily factored. $y = ax^2 + c$

Always factors into: $y = (\sqrt{ax^2} + \sqrt{c})(\sqrt{ax^2} - \sqrt{c})$
Check it with the box.

$$y = (9x + 5)(9x - 5)$$

	9x	-5
9x		
5		

$$y = (7x + i)(7x - i)$$

Check it with the box.

	7x	i
7x		
-i		

Irrational Conjugates Theorem:

Complex Conjugates Theorem:

Review: simplify the following expressions

$$\frac{\sqrt{12}}{2}$$

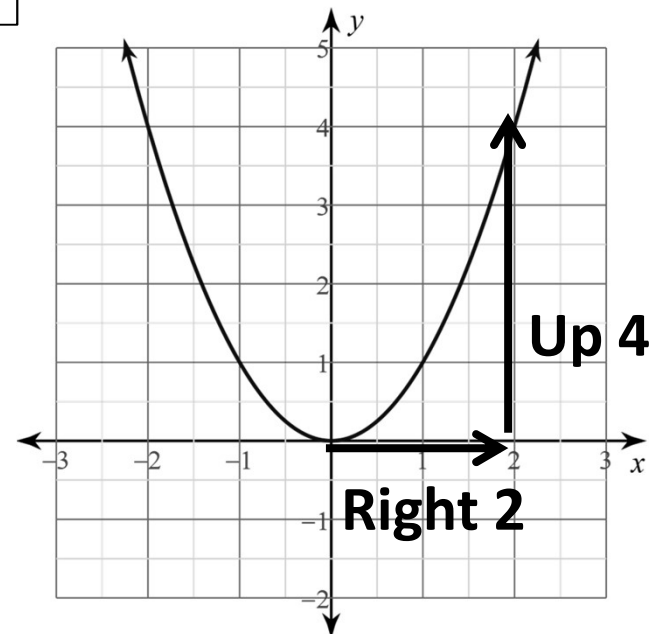
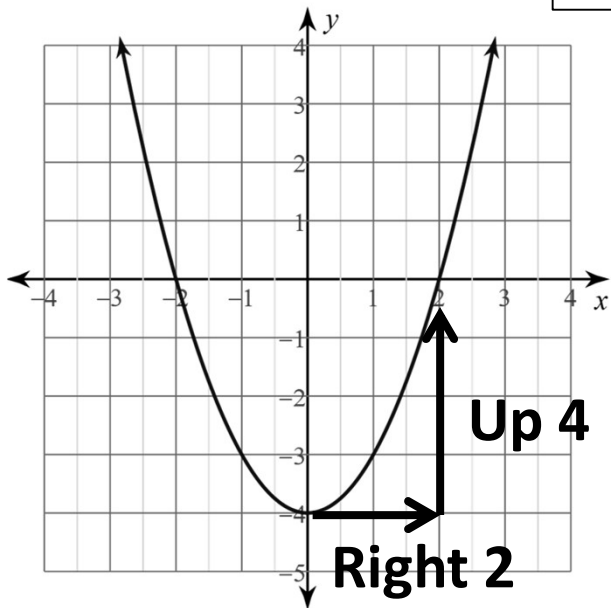
$$\frac{1}{\sqrt{2}}$$

Describe the transformation to the parent function:

$$g(x) = x^2 - 4$$



$$f(x) = x^2$$



Notice the x-intercepts are $(-2, 0)$ and $(2, 0)$

“Zero” of a 2-variable equation: the input value of an equation that causes the output to equal zero.

“Zeroes” often (but not always) are the x-intercepts of the graph.

“Zero” of a 2-variable equation: the input value that causes the output to equal zero.

1) Find Zeroes by taking square roots

$$y = x^2 - 4 \quad \text{Set 'y = 0'}$$

$$0 = x^2 - 4 \quad \text{Get the 'x' squared term by itself:}$$

$$x^2 = 4 \quad \text{How do you "undo" a square function?}$$

$$\sqrt{x^2} = \sqrt{4}$$

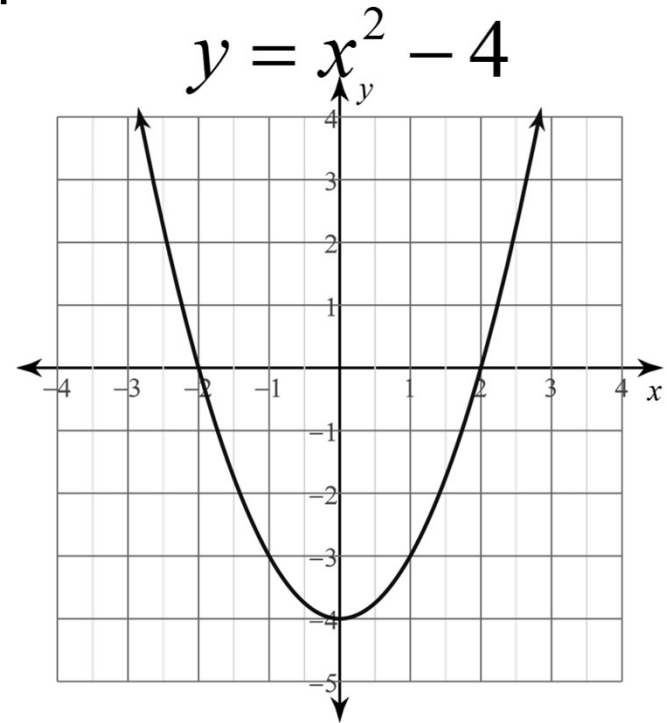
$$x^2 = 4$$

$$x = 2, -2$$

$$(2)^2 = 4 \quad \text{"+" number squared is positive}$$

Why is it both "+" and "-" 2?

$$(-2)^2 = 4 \quad \text{"-" number squared is positive}$$

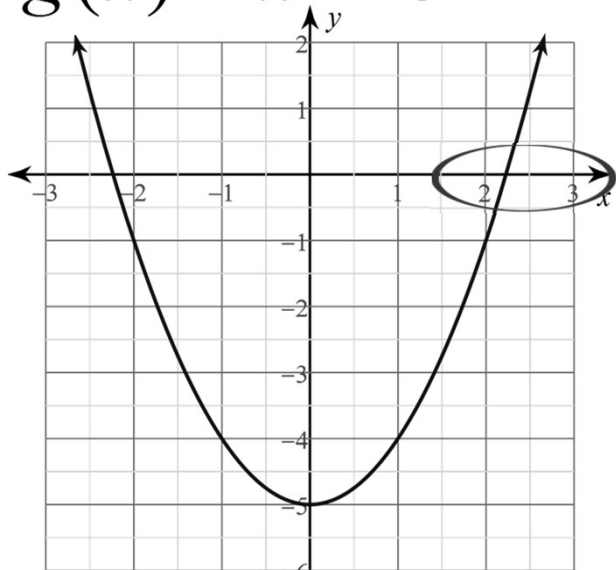


Describe the transformation: down 5

$$g(x) = x^2 - 5$$

Would you consider this equation “vertex form”?

yes



Notice the positive x-intercept is between 2 and 3.

What are the ‘x’ intercepts (?, 0) and (?, 0)

Another method to find the zeroes:
“Isolate the square, “undo” the square. $0 = x^2 - 5$

$$x^2 = 5$$
$$\sqrt{x^2} = \sqrt{5}$$

The “zeroes” are:

(1) Convert to intercept form:

$$y = x^2 - 5$$

$$0 = (x - \sqrt{5})(x + \sqrt{5})$$

$$(-\sqrt{5}, 0) \text{ and } (\sqrt{5}, 0)$$

The “zeroes” are:

Find the zeroes of the following equations by taking square roots.

$$y = x^2 - 1 \quad \text{Set 'y = 0''}$$

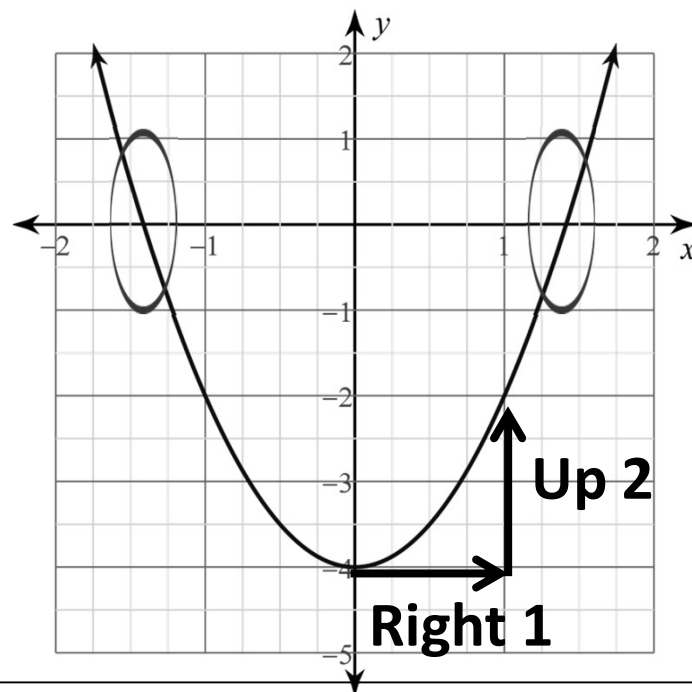
$$0 = x^2 - 1 \quad \text{Isolate the 'x' squared term}$$

$$x^2 = 1 \quad \text{Take square roots}$$

$$\sqrt{x^2} = \sqrt{1} \quad x = \pm 1$$

$$y = 2x^2 - 4 \quad \underline{\text{Describe the transformations}}$$

Find the zeroes



Find the zeroes of the following equations by taking square roots.

$$y = x^2 + 1$$

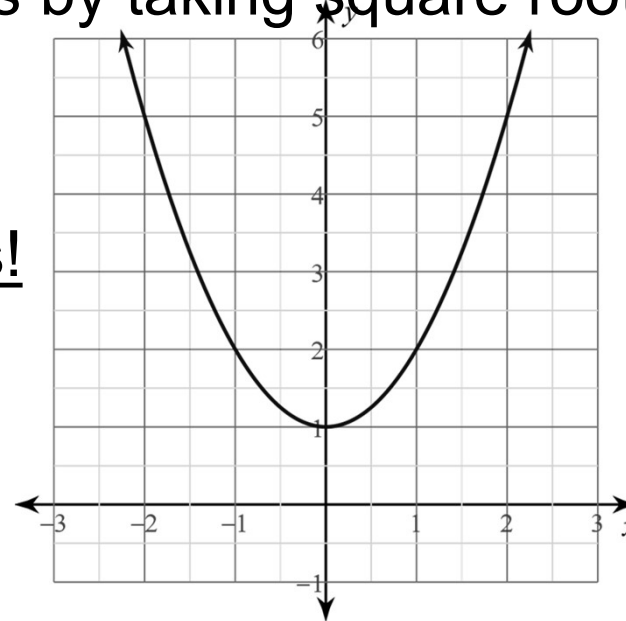
Up 1

Describe the transformations

There are no x-intercepts!

$$y = x^2 + 1$$

Find the zeroes by taking square roots.



Set 'y = 0'

Isolate the 'x' squared term

Take square roots

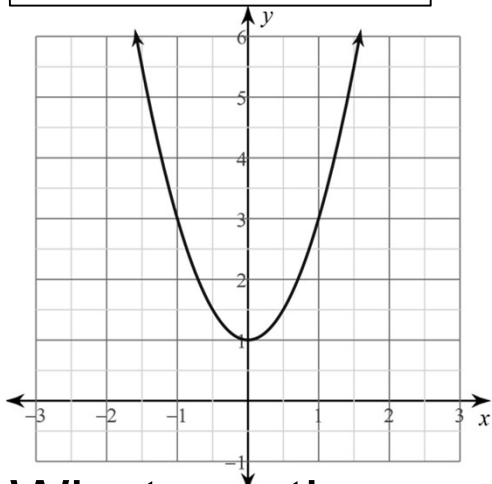
The zeroes are imaginary numbers!

We cannot graph imaginary numbers on the "real plane"

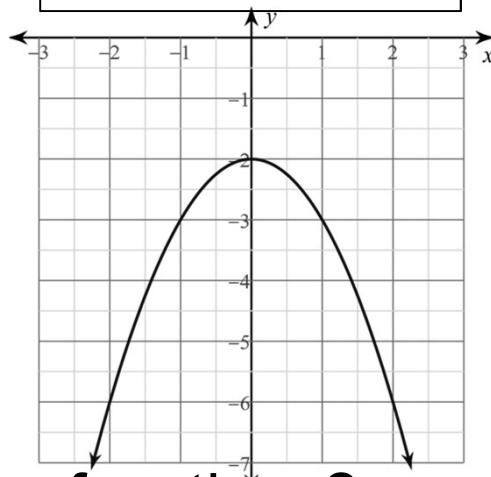
There are _____, but there are two zeroes of the equation!

Which of the following have x-intercepts?

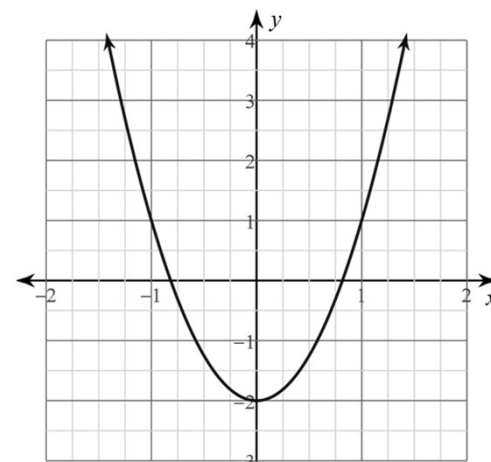
$$y = 2x^2 + 1$$



$$y = -x^2 - 2$$



$$y = 3x^2 - 2$$



What are the zeroes of the functions?

$$0 = 2x^2 + 1$$

$$-1 = 2x^2$$

$$\frac{-1}{2} = x^2 \quad \pm \frac{i}{\sqrt{2}} = x$$

$$\pm \frac{\sqrt{-1}}{\sqrt{2}} = x \quad \pm \frac{i\sqrt{2}}{2} = x$$

$$0 = -x^2 - 2$$

$$x^2 = -2$$

$$x = \pm\sqrt{-2}$$

$$x = \pm i\sqrt{2}$$

$$0 = 3x^2 - 2$$

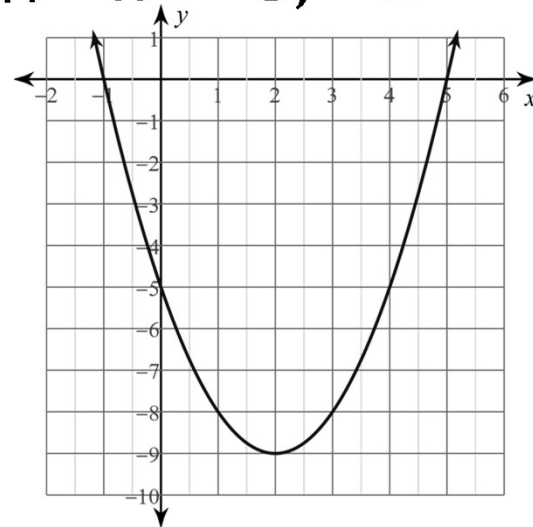
$$2 = 3x^2$$

$$\frac{2}{3} = x^2$$

$$\pm \frac{\sqrt{2}}{\sqrt{3}} = x = \pm \frac{\sqrt{6}}{3}$$

$$y = (x - 2)^2 - 9$$

1. Transformations of the parent function? Right 2, down 9
2. Which form of the quadratic is this? Vertex Form
3. What is the vertex? (2, -9)
4. Convert the equation to standard form. $y = x^2 - 4x - 5$
5. Convert standard form equation into intercept form:
$$y = (x - 5)(x + 1)$$
6. What are the zeroes of the equation? $x = 5, -1$
7. Draw a graph of the function.
8. Are the zeroes real or imaginary?
The graph crosses the x-axis
→ it has real number zeroes.



Can we use “isolate the square, “undo” the square to find the zeroes of the equation?

$$y = (x - 2)^2 - 9$$

x-coord of vertex

Zeroes: x-coord. of vertex

$$y = a(x - h)^2 + k$$

→ Let $y = 0$ $0 = (x - 2)^2 - 9$

Isolate the squared term

$$0 = (x - 2)^2 - 9$$

$$+9 \qquad +9$$

$$9 = (x - 2)^2$$

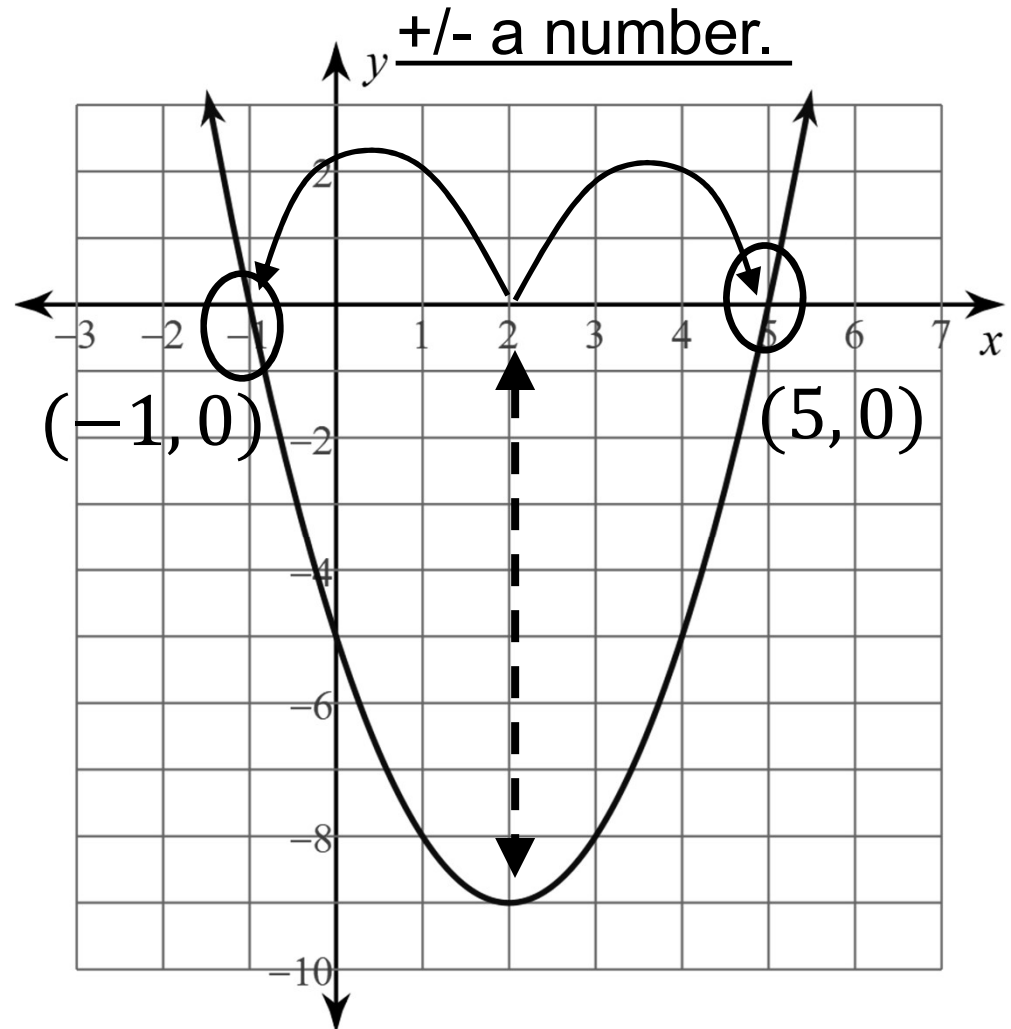
“take square roots”

$$\sqrt{9} = \sqrt{(x - 2)^2}$$

$$3 = x - 2 \qquad f(0)=5$$

$$\text{Solve for 'x'} \qquad (5, 0)$$

$$x = 2 + 3 \qquad x = 2 - 3$$



You can find the zeroes of vertex form using “isolate the square, undo the square”.

Vertex form \rightarrow extract a square root.

$$y = a(x - h)^2 + k \quad y = (x - 1)^2 - 9$$

\longrightarrow Let $y = 0$ $0 = (x - 1)^2 - 9$

Isolate the squared term $9 = (x - 1)^2$

“take square roots” $\sqrt{9} = \sqrt{(x - 1)^2} \quad \pm 3 = x - 1$

Solve for ‘x’ $1 \pm 3 = x$ simplify $x = 4, -2$

Or, (harder) covert to standard form, then intercept form.

$$y = (x - 1)^2 - 9 \quad y = (x - 4)(x + 2)$$

$$y = x^2 - 2x + 1 - 9 \quad 0 = (x - 4)(x + 2)$$

$$y = x^2 - 2x - 8 \quad x = 4, -2$$

$$y = (x + 3)^2 - 8$$

1. Which form of the quadratic is this?

2. transformations of parent function?

3. vertex?

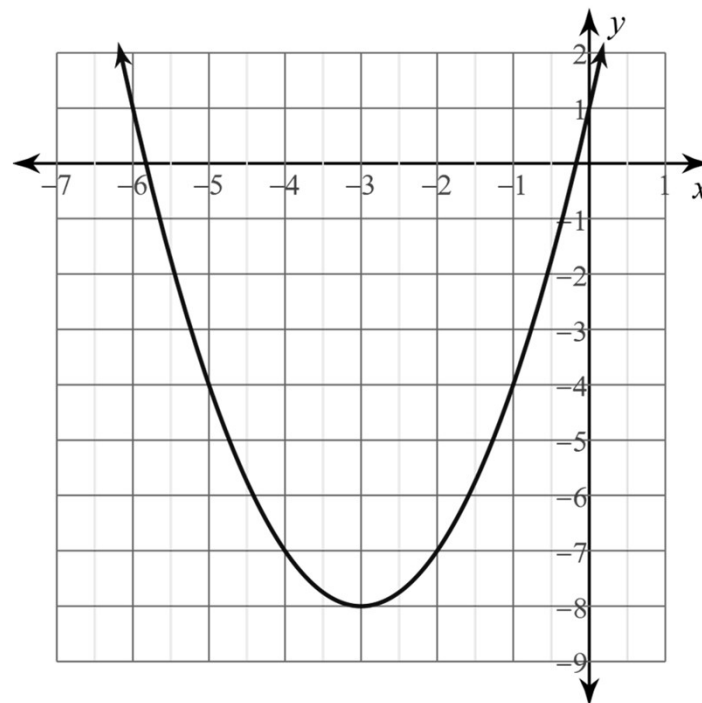
4. Equivalent standard form equation. $y = x^2 + 6x + 1$

5. Convert standard form equation into intercept form:

6. Draw a graph of the function.

7. Are the zeroes real or imaginary?

8. How can we find the zeroes?



If you can't factor the Standard Form version of the Vertex Form equation that must be a way to find the zeroes!

Vertex form → take square roots.

$$y = a(x - h)^2 + k \quad y = (x + 3)^2 - 8$$

→ Let $y = 0$ $0 = (x + 3)^2 - 8$

Isolate the squared term $8 = (x + 3)^2$

“take square roots” $\sqrt{8} = \sqrt{(x + 3)^2}$

$\pm \sqrt{8} = x + 3$ Simplify the radical

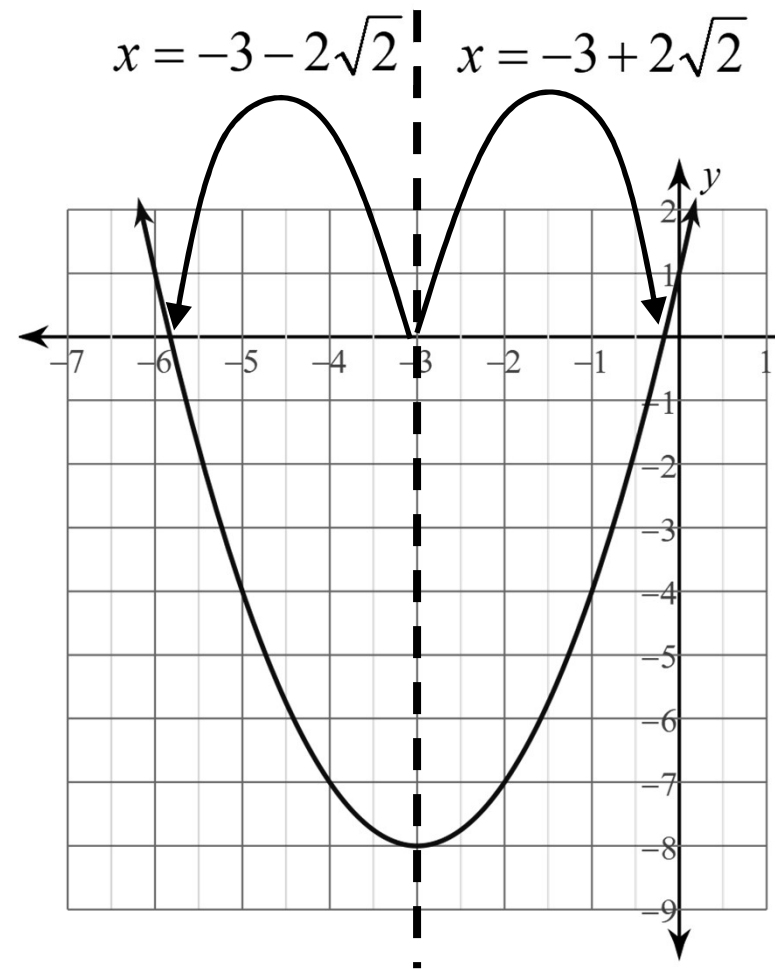
$\pm \sqrt{2 * 2 * 2} = x + 3$

$\pm 2\sqrt{2} = x + 3$ Solve for 'x'

$x = -3 \pm 2\sqrt{2}$



x-coord of vertex



$$y = (x - 2)^2 - 4$$

Let $y = 0$

$$0 = (x - 2)^2 - 4$$

Isolate the squared term

“Extract a square root”

Solve for ‘x’

simplify

Or, convert to standard form, then intercept form.

$$y = (x - 2)^2 - 4$$

$$y = x^2 - 4x + 4 - 4$$

$$y = x^2 - 4x$$

$$y = x(x - 4)$$

$$0 = x(x - 4)$$

$$x = 0, 4$$

If standard form can't be factored, you need another way to find the zeroes of the equation.

$$y = 2(x + 7)^2 - 10$$

Let $y = 0$

Isolate the Square term

Divide by 2 (both sides)

“take square roots”

subtract 7 from both sides

Find the “zeroes” by “Extracting a square root”

$$y = (x - 2)^2 - 5$$

$$y = 3(x + 4)^2 - 12$$

$$y = -3(x - 4)^2 - 27$$