Math-1010
Lesson 4-8

Imaginary Numbers

Use the "Box Method" to multiply Polynomial $(2x+1)(x+3)$

	x	3
$2x$	$2x^2$	6x
1	x	3

$$2x^2 + x + 6x + 3$$

$$\Rightarrow 2x^2 + 7x + 3$$

$$(2x+1)(x+3) = 2x^2 + 7x + 3$$

Notice something

$$2*3=6$$

$$2x^2 + 7x + 3$$

$$1+6=7$$

$$6=1*6$$

$$3 * -10 = -30$$

What are the factors of -30 that add up to -1?

$$3x^2 - x - 10$$

$$-6 + 5 = -1$$

$$-30 = -6 * 5$$

$$3x^2 - x - 10$$

$$3x^2 - 6x + 5x - 10$$

This tells us to break -x into -6x + 5x

Group the first two and last two terms

$$(3x^2 - 6x) + (5x - 10)$$

Factor out the common factor

$$3x(x-2) + 5(x-2)$$

Factor out the common factor

$$(3x+5)(x-2)$$

Factor

2(-7) = -14 -14 = 1(-14) -13 = 1 – 14

$$2x^2 - 13x - 7 \rightarrow 2x^2 + 1x - 14x - 7 \rightarrow (2x^2 + 1x) + (-14x - 7)$$

$$\rightarrow x(2x+1) - 7(2x+1) \quad \rightarrow (2x+1)(x-7)$$

3(-4) = -12 -12 = 6(-2) 4 = 6 – 2

$$3x^2 + 4x - 4 \rightarrow 3x^2 + 6x - 2x - 4 \rightarrow (3x^2 + 6x) + (-2x - 4)$$

$$\rightarrow 3x(x+2) - 2(x+2) \quad \rightarrow (x+2)(3x-2)$$

4(-5) = -20 -20 = -10(2) -8 = -10 + 2

$$4x^2 - 8x - 5 \rightarrow 4x^2 - 10x + 2x - 5 \rightarrow (4x^2 - 10x) + (2x - 5)$$

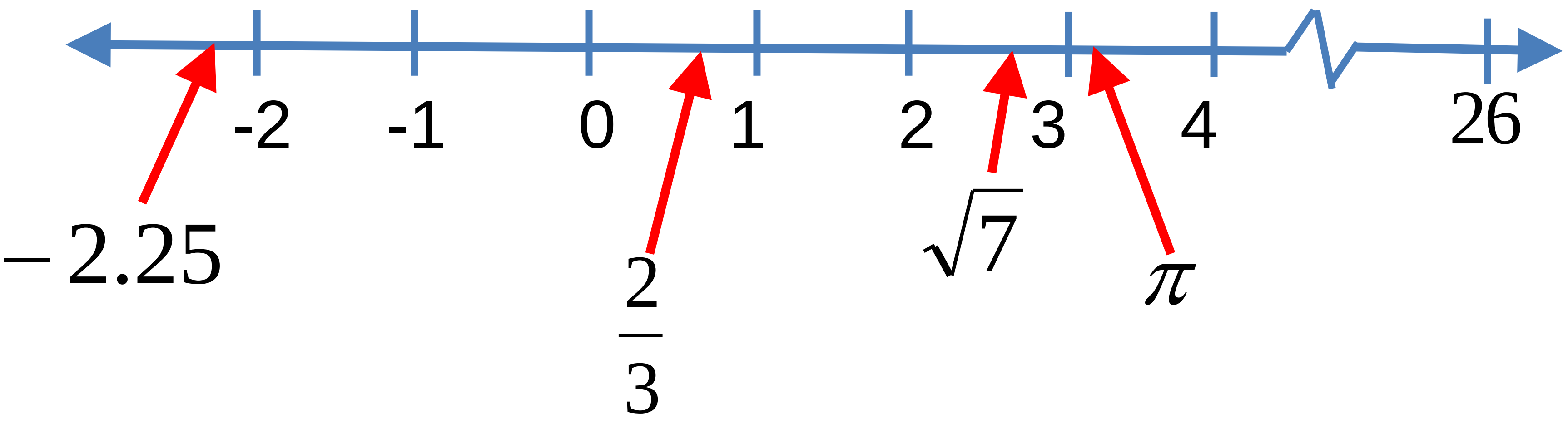
$$\rightarrow 2x(2x-5) + 1(2x-5) \quad \rightarrow (2x-5)(2x+1)$$

Vocabulary

<u>imaginary numbers</u>: a number that includes the square root of a negative number. They <u>are not</u> on the number line!

$\sqrt{-1}$ $\qquad$ $i\sqrt{3}$ $\qquad$ $\sqrt{-2}$ $\qquad$ $3\sqrt{-6}$

Are there any imaginary numbers on the number line?



$$i = \sqrt{-1}$$

If we apply the Property of Equality (square both sides)

$$i^2 = -1$$

ALWAYS replace i^2 with -1.

Rewrite the following so that there are NO negatives under the square root symbol and NO i^2 's .

$$\sqrt{-5} \rightarrow i\sqrt{5}$$

$$3\sqrt{-5} \rightarrow 3i\sqrt{5}$$

$$-4\sqrt{-5} \rightarrow -4i\sqrt{5}$$

$$5-2\sqrt{-3} \rightarrow 5-2i\sqrt{3}$$

$$-2i^2\sqrt{-3} \rightarrow -2(-1)\sqrt{3}$$

$$\rightarrow 2\sqrt{3}$$

New number systems are needed when a number system is not "closed" for a particular operation (the square root of -1)

Even the complex number system is not close for all operations!

$$\frac{5}{0} \Rightarrow ???$$

We always write Complex Numbers that contain both a real AND an imaginary part in the form: a + bi

2 + 3i

Real number

Imaginary number

Order of Operations (and those silly parentheses)

Write out what PEMDAS means.

The Correct Order of operations:

1st: operations inside parentheses $(2x+3x)^2 \quad \rightarrow (5x)^2$

You MUST remove parentheses BEFORE
you can perform any other operations!

2nd: operations caused by exponents $(2x^2)^3 \quad \rightarrow 2^3 x^6$

Exponent of a Product → removes parentheses

(repeated multiplication of the base) $(2x^2)^3 \rightarrow (2x^2)(2x^2)(2x^2)$

$$(2x^2)^3 \rightarrow 2*2*2*x^2*x^2*x^2 \quad \rightarrow 8x^6$$

Exponent of a Sum → removes parentheses $(2-3i)^2$

$$\rightarrow (2-3i)(2-3i) \quad \rightarrow (2-3i)(2-3i) \rightarrow 2(2-3i)-3i(2-3i)$$

(repeated multiplication of the base) $\rightarrow 4-6i-6i+9i^2 \quad \rightarrow -5-12i$

Order of Operations (and those silly parentheses)

The Correct Order of operations:

3rd Multiplication $2(3x) \rightarrow 2*3*x \rightarrow 6x$

multiplication→ removes parentheses

4th: Division $\frac{(4x)}{2}$

From the Inverse Property of Multiplication: $5*\frac{1}{5}=1$

Division by a number is the same thing as multiplication by the reciprocal of the number.

$5 \div 5 = 1$

→ We can DIVIDE before we MULTIPLY since the commutative property says “the order of multiplication doesn’t matter”.

$$\frac{(4x)}{2} \rightarrow \frac{2*2*x}{1}*\frac{1}{2} \rightarrow 2x$$

$$\frac{(4x)}{2} \rightarrow \frac{2^2*x}{2^1} \rightarrow \frac{2^2*x}{2^1} \rightarrow 2^2*2^{-1}*x \rightarrow 2x$$

Adding Complex Numbers

$(2 + 3i) + 4(5 - 6i)$

1st: operations inside <u>parentheses</u>

You MUST remove parentheses BEFORE you can perform any other operations!

Use multiplication to remove the parentheses. If no obvious multiplication exists, you may multiply by "1"

$1 * (2 + 3i) + 4(5 - 6i)$

$2 + 3i + 20 - 24i$

$2 + 20 + 3i - 24i$

$22 - 21i$

Simplify:

$(2 - 3i) - (-4 - 5i)$ $6 + 2i$

$7i - (2 - 3i)$ $-2 + 10i$

$(3 - 2i) - (2 - 4i)$ $1 + 2i$

$(-5 + 6i) + (5 - 2i)$ $4i$

Multiplying Complex Numbers

(3i)(-4i) Parentheses “touching” means multiply

Multiplication removes parentheses

→ 3 * i * (-4) * i → 3 *(-4) * i * i

$$-12i^2$$

$$i^2 = -1$$ Substitute (-1) for *i*^2

$$-12(-1)$$

$$\to 12$$

$(2i)(-3)(4 - 5i)$

$(\quad)(\quad)(\quad)$ Three expressions are being multiplied!!!!

You CANNOT multiply three number/expressions in one step!

Pick two expressions to multiply first.

Path-1

$(2i)(-3)(4 - 5i)$

$(2i)(-12 + 15i)$

$(2i)(-12) + (2i)(15i)$

$-24i + 30i^2$

$-30 - 24i$

Path-2

$(2i)(-3)(4 - 5i)$

$(-6i)(4 - 5i)$

$-24i + 30i^2$

$-30 - 24i$

Simplify:

$(2i)(-5i)(-4 - 3i)$ $6 + 2i$

$(2i)(4 - 3i)(-7i)$ $-2 + 10i$

$(3 - 2i)(2 - 4i)$ → $6 - 4i - 12i + 8i^2$ → $-2 - 16i$

$2(-5 + 6i)(5 - 2i)$ → $(-10 + 12i)(5 - 2i)$

→ $-50 + 20i + 60i - 24i^2$

→ $26 + 80i$

Dividing Complex Numbers

$$\frac{3i^2}{2i} \rightarrow \frac{3i}{2}$$

$$\frac{-3}{2i} * \frac{i}{i} \rightarrow \frac{-3i}{2i^2} \rightarrow \frac{-3i}{-2} \rightarrow \frac{(-1)*3i}{(-1)*2} \rightarrow \frac{3i}{2}$$

Identity Property Of Mult.

Inverse Property Of Mult.

Simplify

$$\frac{6i}{-4i^2} \rightarrow \frac{3i}{2}$$

$$\frac{-5}{3i} \rightarrow \frac{5i}{3}$$

$$\frac{4}{-8i} \rightarrow \frac{i}{2}$$

$$\frac{2-3i}{5i} * \frac{i}{i} \rightarrow \frac{2i-3i^2}{5i^2}$$

Identity Property Of Mult.

$$\rightarrow \frac{2i+3}{-5}$$

$$\rightarrow \frac{3+2i}{-5} * \frac{-1}{-1} \rightarrow \frac{-3-2i}{5}$$

Identity Property Of Mult.