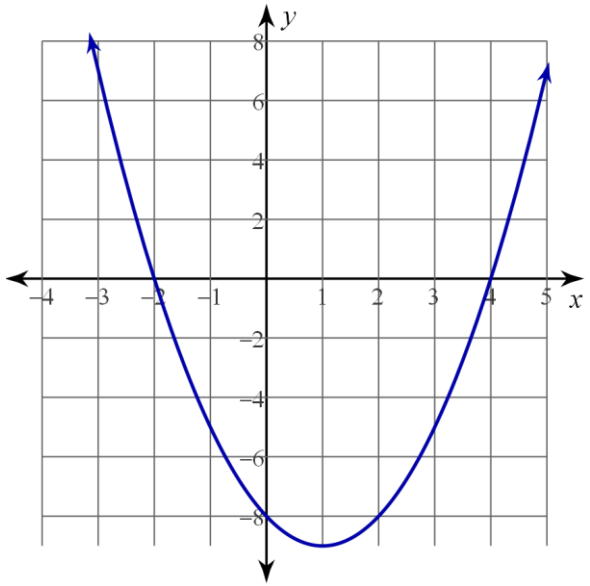
Math 1010 Lesson 4-7

Intercept Form quadratic equation



Vertex Form: gives you…? $y = -2(x+3)^2 + 4$

1. Vertex (-3, 4)

2. Transformations of the parent function (shape of the graph)

Reflected (x-axis), VSF=2, left 3, up 4

Standard Form: gives you…? $y = -2x^2 - 12x - 15$

1. Vertical Stretch Factor VSF = 2
2. Whether it has been reflected across x-axis or not. Yes it has.
3. Y-intercept: (0, -15)
4. A way to calculate the x-coordinate of the vertex: $x = \frac{-b}{2a}$

$$x = \frac{-(-12)}{2(-2)} = \frac{12}{-4} = -3$$

5. An equation to plug the x-coordinate of the vertex into, so that you can find the y-coordinate of the vertex.

$y = -2x^2 - 12x - 14$ $\quad y = -2(-3)^2 - 12(-3) - 14 = 4$

Factor the following quadratic expressions:

$$x^2+10x+21 \rightarrow (x+7)(x+3)$$

$$x^2-6x-16 \rightarrow (x-8)(x+2)$$

$$x^2-9x+18 \rightarrow (x-6)(x-3)$$

Standard Form Quadratic Equation

$$y=x^2+10x+21 \rightarrow y=(x+7)(x+3)$$

$$y=x^2-6x-16 \rightarrow y=(x-8)(x+2)$$

$$y=x^2-9x+18 \rightarrow y=(x-6)(x-3)$$

Intercept Form Quadratic Equation

Vocabulary

X-intercept: the x-y pair where the graph crosses the x-axis.

The y-value of an x-intercept always equals Zero

The Zero Product Property: If two numbers are multiplied together and the product equals zero, then one or both of the factors must equal zero.

$A * B = 0 \rightarrow$ either $A = 0$ or $B = 0$ or both A and B equal zero.

Intercept form Quadratic Equation

$$y = (x+4)(x-2)$$

The y-value of an x-intercept always equals Zero

$$0 = (x+4)(x-2)$$

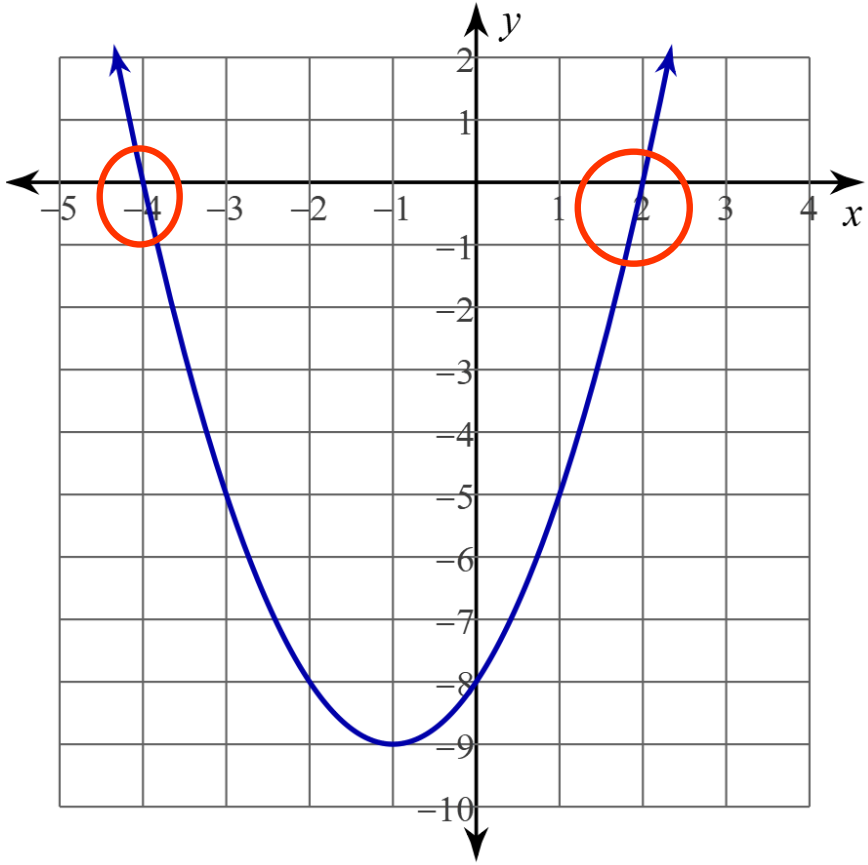
$$O = A * B$$

Zero Product Property: either

(x + 4) = 0 or (x – 2) = 0

x + 4 = 0 x – 2 = 0

x = -4 x = +2



Intercept form Quadratic Equation

$$y = (x-1)(x-3)$$

The y-value of an x-intercept always equals Zero

$$0 = (x-1)(x-3)$$

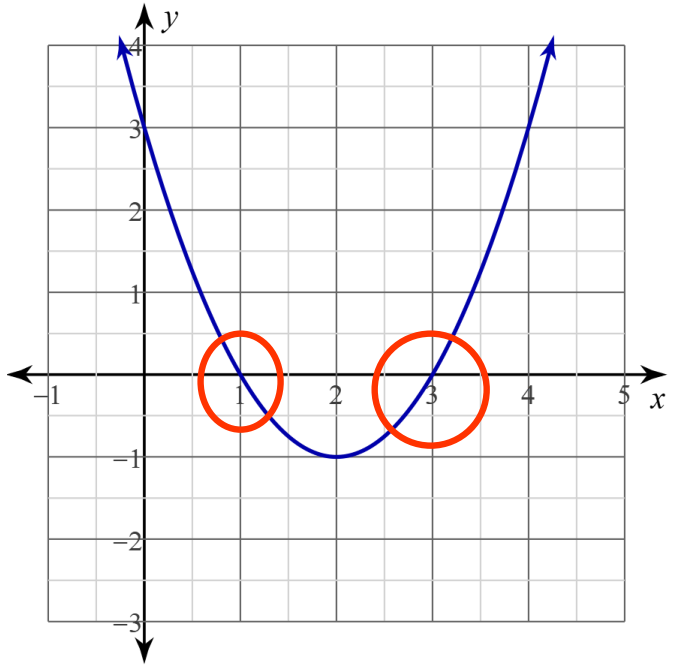
$$\mathbf{O} = \boldsymbol{A} * \boldsymbol{B}$$

Zero Product Property: either

(x – 1) = 0 or (x – 3) = 0

x – 1 = 0 x – 3 = 0

x = 1 x = 3



Standard Form Quadratic Equation is converted to an

Intercept Form Quadratic Equation by factoring

Convert to intercept form and list the zeroes.

$y = x^2 + 10x + 21 \rightarrow y = (x+7)(x+3)$

x = -7 x = -3

$y = x^2 - 6x - 16 \rightarrow y = (x-8)(x+2)$

x = 8 x = -2

$y = x^2 - 9x + 18 \rightarrow y = (x-6)(x-3)$

x = 6 x = 3

Convert the following Standard Form Quadratic Equations to Intercept Form (by factoring)

$y = x^2 + 3x - 10 \rightarrow y = (x+5)(x-2)$

x = -5 x = 2

$y = x^2 - 8x - 20 \rightarrow y = (x-10)(x+2)$

x = 10 x = -2

$y = x^2 - 10x + 24 \rightarrow y = (x-6)(x-4)$

x = 6 x = 4

What are the x-intercepts for each of these equations?

Intercept Form Quadratic Equation:

Vertical Stretch Factor!

'x-intercepts are 'p' and 'q'

$$y = (-1)a(x - p)(x - q)$$

If negative: reflected across x-axis.

'x-intercepts are: '1' and '3'

$$y = -3(x + 2)(x + 4)$$

$$y = (x - 1)(x - 3)$$

Opens down

'x-intercepts are: '-2' and '-4'

Each set of parentheses is called a "factor". Why?

Convert to Intercept Form

$$y = 2x^2 + 6x + 4$$

Always factor out the common factor first.

$$y = 2(x^2 + 3x + 2)$$

Now factor the trinomial.

$$y = 2(x + 2)(x + 1)$$

What are the x-intercepts?

'x-intercepts are: '-2' and '-1'

Which way (up/down) does the parabola open?

Up (not reflected across x-axis)

What is the vertical stretch factor?

VSF = 2

Convert to Intercept Form

$$y = 3x^2 - 15x - 18$$

Always factor out the common factor first.

$$y = 3(x^2 - 5x - 6)$$

Now factor the trinomial.

$$y = 3(x - 6)(x + 1)$$

What are the x-intercepts?

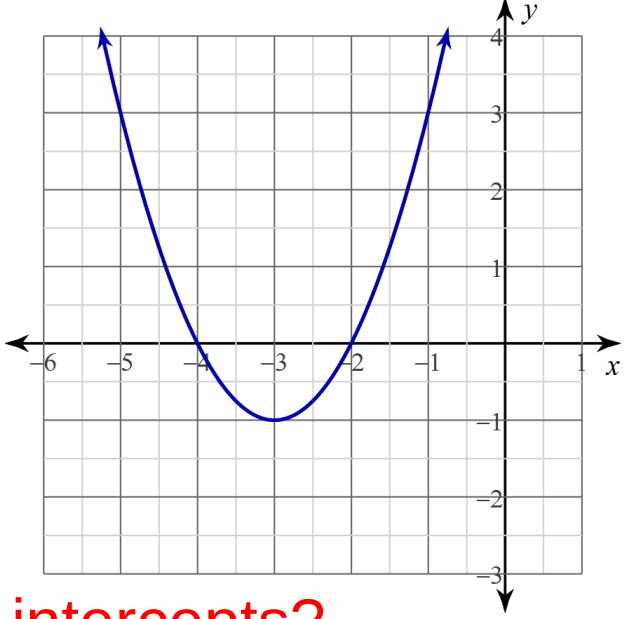
'x-intercepts are: '6' and '-1'

Which way (up/down) does the parabola open?

Up (not reflected across x-axis)

What is the vertical stretch factor?

VSF = 3

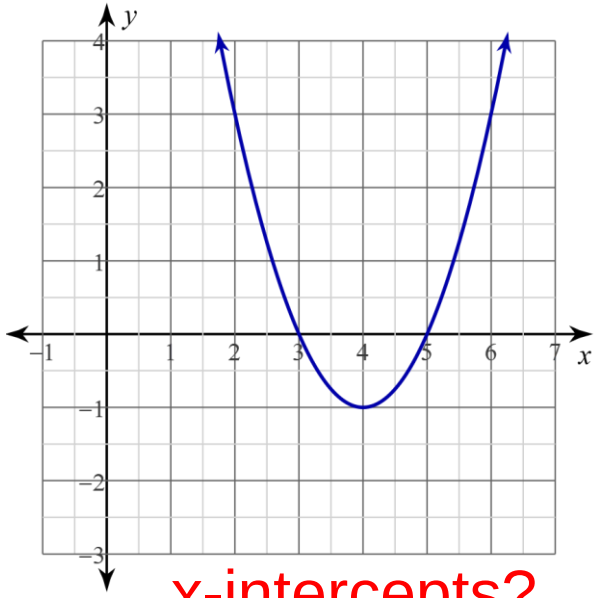


x-intercepts?

'-4' and '-2'

axis of symmetry?

x = -3



x-intercepts?

'3' and '5'

axis of symmetry?

x = 4

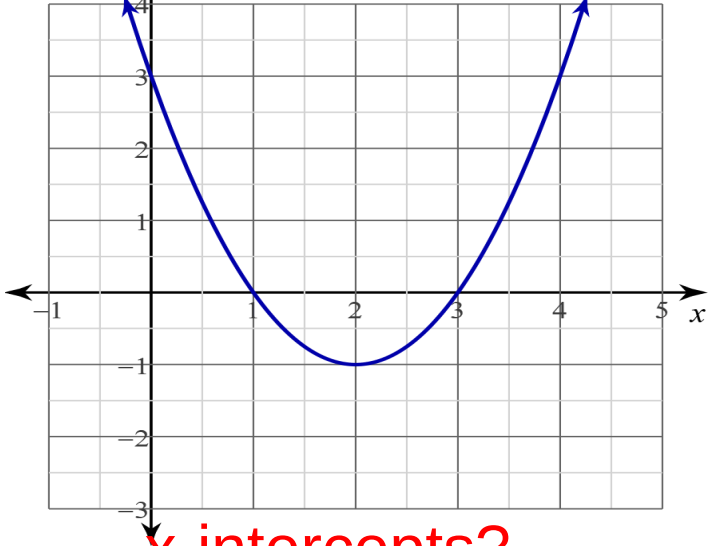
How can you use the x-intercepts to determine the x-coordinate of the vertex?

x-coordinate of the vertex?

(-3, ___)

x-coordinate of the vertex?

(4, ___)



x-intercepts?

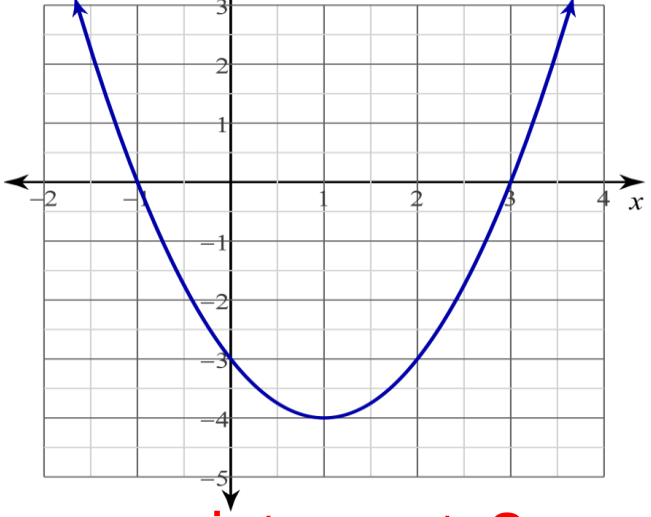
‘x-intercepts are ‘1’ and ‘3’

axis of symmetry?

x = 2

x-coordinate of the vertex?

(2, ___)



x-intercepts?

‘x-intercepts are ‘-1’ and ‘3’

axis of symmetry?

x = 1

x-coordinate of the vertex?

(1, ___)

How can you use the x-intercepts to determine the x-coordinate of the vertex?

Half-way between two numbers is the average of the two numbers. The axis of symmetry is exactly half-way between the two x-intercepts.

x-intercepts?

$y = (x+5)(x-1)$

x = -5 x = 1

y-coordinate of the vertex?

Vertex: (-2, -9)

x-coordinate of the vertex?

$x = \frac{-5+1}{2} = \frac{-4}{2}$ = -2 (-2, ___)

$y = (-2+5)(-2-1)$ = -9

x-intercepts?

$y = (x-6)(x-4)$

x = 6 x = 4

y-coordinate of the vertex?

Vertex: (5, -1)

x-coordinate of the vertex?

$x = \frac{6+4}{2} = \frac{10}{2}$ = 5 (5, ___)

$y = (5-6)(5-4)$ = -1

What is the vertex?

$y = (x+2)(x-4)$

x = -2 x = 4

$x = \frac{-2+4}{2} = \frac{2}{2}$ = 1

(1, ___)

$y = (1+2)(1-4)$ $y = (3)(-3)$ $y = -9$

(1, -9)

$y = (x-6)(x-4)$

x = 6 x = 4

$x = \frac{6+4}{2} = \frac{10}{2}$ = 5

(5, ___)

$y = (5-6)(5-4)$ $y = (-1)(1)$ $y = -1$

(5, -1)

Putting it all together!

What are the x-intercepts?
What is the vertex?
What is the graph?

$$y = -3x^2 + 6x + 72$$

$$y = -3(x^2 - 2x - 24)$$

$$y = -3(x - 6)(x + 4)$$

$$x = \frac{6-4}{2} \qquad = \frac{2}{2} \qquad = 1$$

x = 6 x = -4

(1, ___)

$$y = -3(1-6)(1+4) \quad y = -3(-5)(5) \quad y = 75$$

(1, 75)

Area of a Rectangle

Perimeter: the distance around the rectangle.

$P = 2L + 2W$

$Area = L * W$

You have 100 feet of fence.

$100 = 2L + 2W$

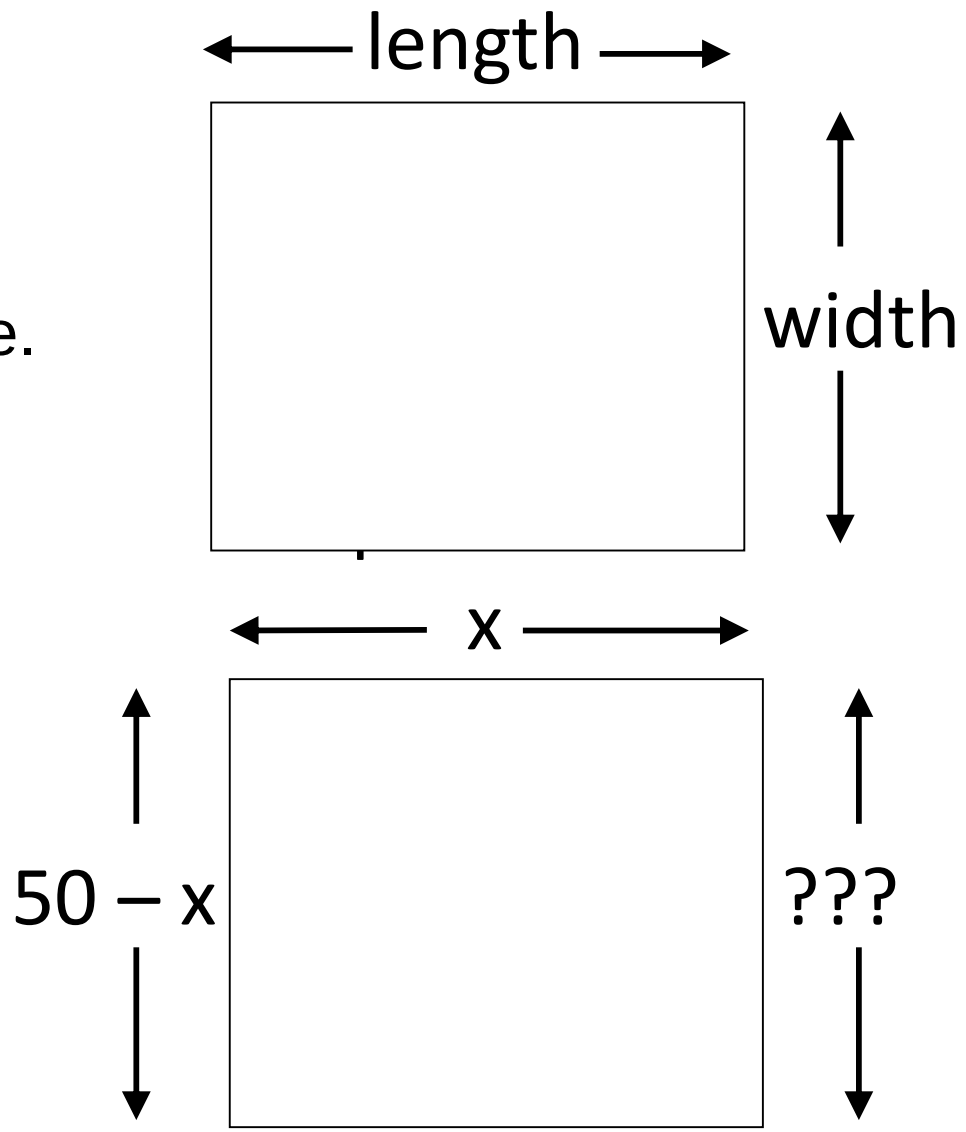
$50 = L + W$

$50 – L = W$

$50 – x = W$

$Area = L * W$

$Area = x(50 – x)$



Area of a Rectangle

Area = L * W

100 = x(50 – x)

A = x(50 – x)

What are the x-intercepts?

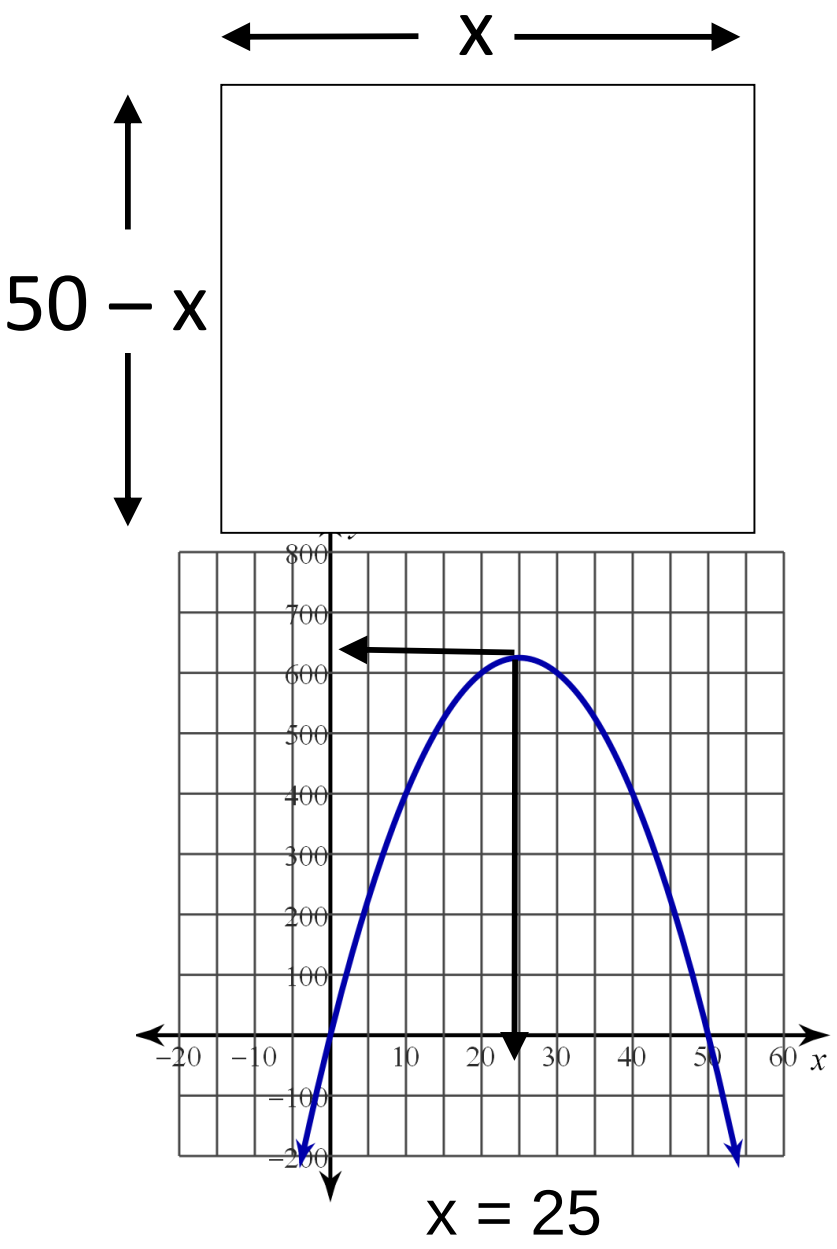
x = 0, 50

What is the vertex? x = 25

What is the maximum area?

A = 25(50 – 25)

A = $(25)^2$ = 625



Solve using the Quadratic formula.

$$y = ax^2 + bx + c$$

$$y = x^2 + 5x + 1$$

a = 1 b = 5 c = 1

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(\ \ )}{2(\ \ )} \pm \frac{\sqrt{(\ \ )^2 - [4(\ \ )(\ \ )]}}{2(\ \ )}$$

$$x = \frac{-(5)}{2(1)} \pm \frac{\sqrt{(5)^2 - [4(1)(1)]}}{2(1)}$$

$$x = \frac{-5}{2} \pm \frac{\sqrt{21}}{2}$$

If the quadratic CANNOT be factored, the solutions are “ugly.”

Remember solving by completing the square?

$$y = x^2 + 5x + 1$$

$$y = (x + 5/2)^2 - 21/4$$

Set y = 0

$$21/4 = (x + 5/2)^2$$

Isolate the square

$$\pm \frac{\sqrt{21}}{\sqrt{4}} = x + \frac{5}{2}$$

Take square root
(don't forget +/-)

Simplify and solve

$$-\frac{5}{2} \pm \frac{\sqrt{21}}{2} = x$$

Remember converting to vertex form?

$y = x^2 + 5x + 1$ $\quad x = \frac{-b}{2a}$ $\quad x = \frac{-5}{2}$

$y = \left(x + \frac{5}{2}\right)^2 + K$ $\quad K = f\left(-\frac{5}{2}\right) = \left(\frac{-5}{2}\right)^2 + 5\left(-\frac{5}{2}\right) + 1$

$y = \left(x + \frac{5}{2}\right)^2 - \frac{21}{4}$ $\quad K = f\left(-\frac{5}{2}\right) = -\frac{21}{4}$

$21/4 = (x + 5/2)^2$ Isolate the square Set y = 0, add 21/4

$\pm \frac{\sqrt{21}}{\sqrt{4}} = x + \frac{5}{2}$ Take square roots (don't forget +/-)

$-\frac{5}{2} \pm \frac{\sqrt{21}}{2} = x$ Simplify and solve