Math-1010
Lesson 4-6
Converting Standard Form Quadratic Equations to Vertex Form.

1) Finding zeroes of un-factorable quadratic equations by:
   a) Converting to Vertex Form (using x = -b/2a)
   b) Converting to Vertex form (using “completing the square)
2) Simplifying “Complex Fractions”

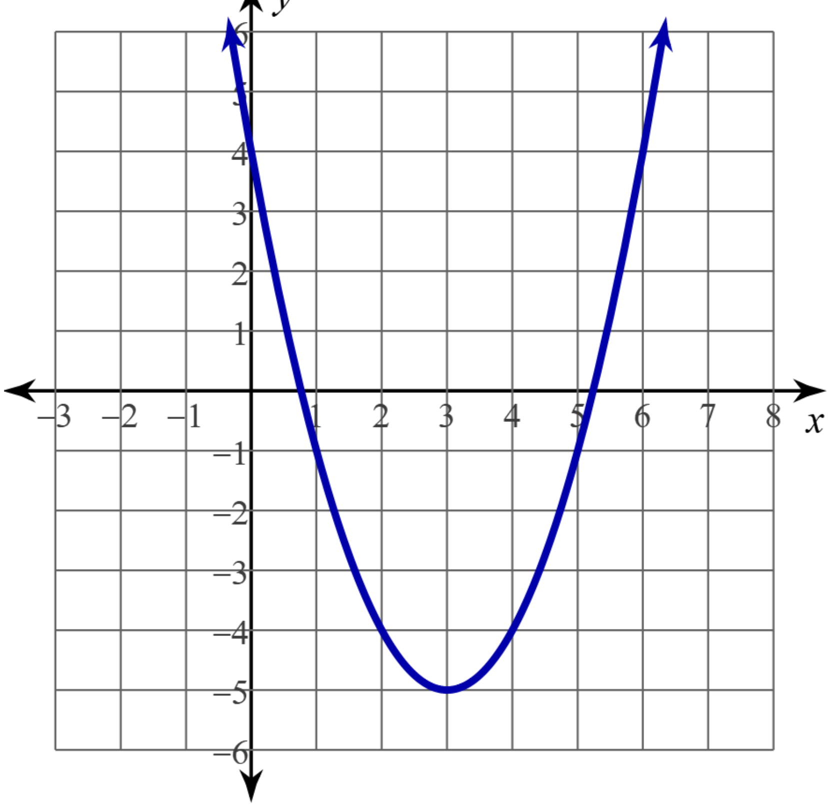
$$y = x^2 - 6x + 4$$

Can this be factored?

The x-intercepts are *<u>"ugly"</u>*

What is the vertex form equation?

$$y = (x-3)^2 - 5$$



Has anyone been taught the Quadratic Formula?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

I like this version more.

What is the purpose of the formula?

The ‘x’ in the formula are the x-intercepts of the *standard form equation.* $y = ax^2 + bx + c$

$$y = ax^2 + bx + c$$

$$y = x^2 - 6x + 4$$

a = 1 b = -6 c = 4

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(\quad)}{2(\quad)} \pm \frac{\sqrt{(\quad)^2 - [4(\quad)(\quad)]}}{2(\quad)}$$

$$x = \frac{-(-6)}{2(1)} \pm \frac{\sqrt{(-6)^2 - [4(1)(4)]}}{2(1)}$$

$$x = \frac{6}{2} \pm \frac{\sqrt{36 - 16}}{2}$$

$$x = 3 \pm \frac{\sqrt{20}}{2}$$

$$x = 3 \pm \frac{\sqrt{4}\sqrt{5}}{2}$$

$$x = 3 \pm \frac{2\sqrt{5}}{2}$$

$$x = 3 \pm \sqrt{5}$$

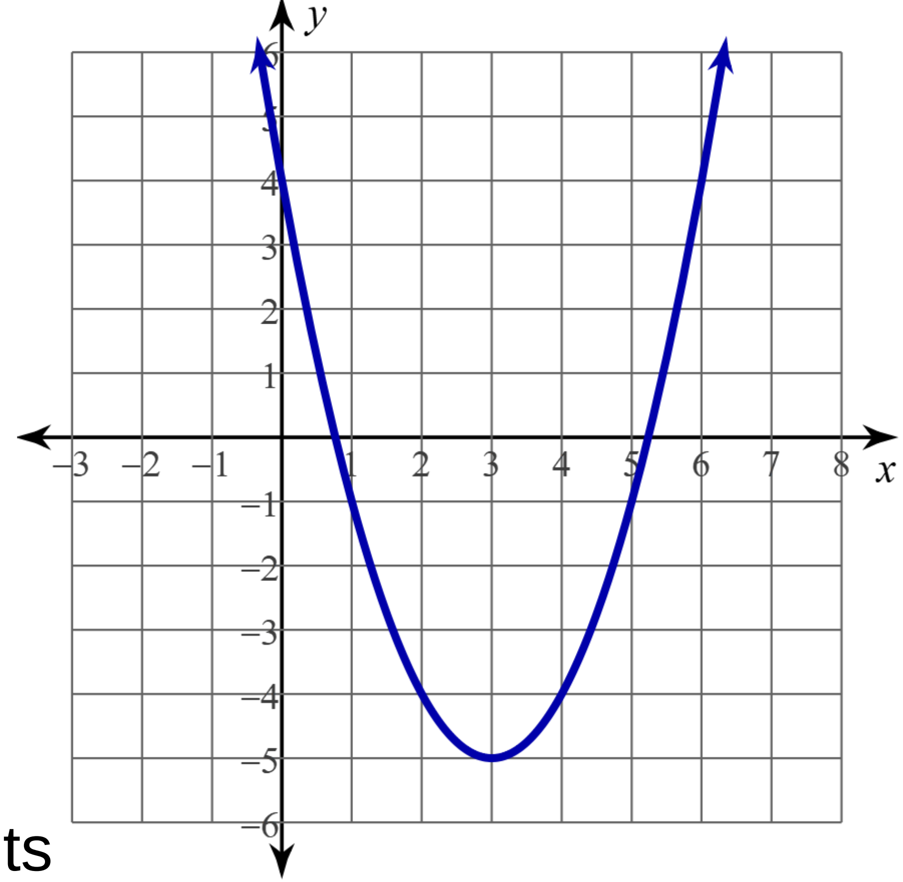
You need to be able to do this. Any questions?

Standard Form Equation

$$y = x^2 - 6x + 4$$

Vertex Form Equation

$$y = (x-3)^2 - 5$$



The x-intercepts that came from the quadratic formula were:

$$x = 3 \pm \sqrt{5}$$

How could you get the x-intercepts from the *vertex form equation*?

Set ‘y’ to zero. Isolate the square, “undo” the square.

$$0 = (x-3)^2 - 5$$

$$5 = (x-3)^2$$

$$\pm\sqrt{5} = x - 3$$

$$x = 3 \pm \sqrt{5}$$

How can we convert *Standard Form Quadratic Equations* directly into *Vertex form*? (without converting to *Intercept Form* first?)

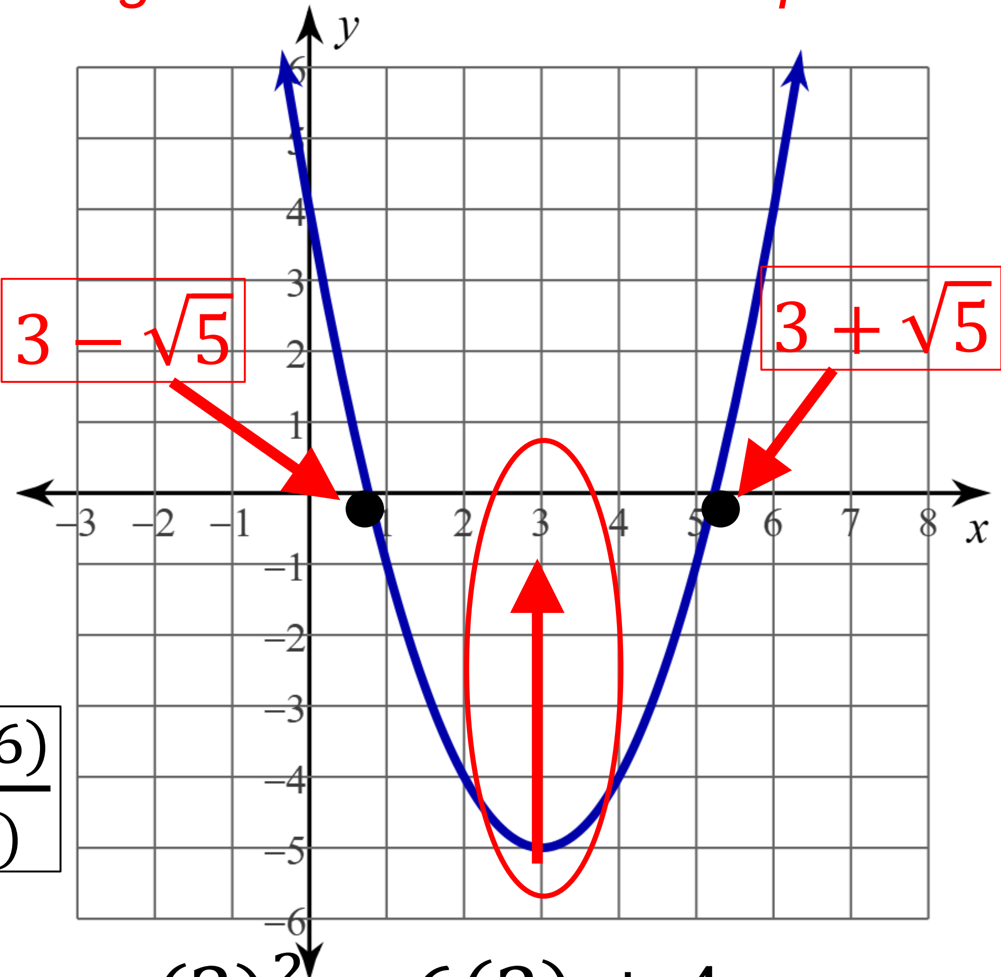
Remember the quadratic formula gave us these x-intercepts.

$$y = x^2 - 6x + 4$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = 3 \pm \sqrt{5}$$

The x-coordinate of the vertex is 3.



x-coord. of vertex $= \frac{-b}{2a} = \frac{-(-6)}{2(1)}$

y-coord. of vertex $= f(\frac{-b}{2a})$

$$y = (3)^2 - 6(3) + 4$$

$$y = -5$$

What is the x-coordinate of the vertex?

$$y = 2x^2 + 16x + 24$$

a = 2 b = 16

$$\text{x-coord. of vertex} = \frac{-b}{2a}$$

$$\frac{-b}{2a} = \frac{-16}{2(2)} = -4$$

Vertex: $(-4, f(-4))$

What is the y-coordinate of the vertex?

$$f(-4) = 2(-4)^2 + 16(-4) + 24$$

$$f(-4) = -8$$

Vertex: $(-4, -8)$

What is the Vertex form equation?

$\text{VSF} = 2, \text{vertex} = (-4, -8)$

$$y = 2(x + 4)^2 - 8$$

What is the x-coordinate of the vertex?

$$y = x^2 - 6x + 13$$

a = 1 b = -6

$$\text{x-coord. of vertex} = \frac{-b}{2a}$$

$$\frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$$

Vertex: $(3, f(3))$

What is the y-coordinate of the vertex?

$$f(3) = (3)^2 - 6(3) + 13$$

$$f(3) = 4$$

Vertex: $(3, 4)$

What is the Vertex form equation?

$\text{VSF} = 1, \text{vertex} = (3, 4)$

$$y = (x - 3)^2 + 4$$

Convert the following *non-factorable* standard form equations into vertex form. Find the x-intercepts.

$$y = x^2 - 2x - 12$$

$$y = x^2 + 20x + 99$$

$$y = x^2 - 14x + 50$$

Write the <u>names</u> of the 3 forms of quadratic equation in the circles.

What process would you use to ….?

$y = x^2 - 3x - 18$

Standard form

factor

$y = (x - 6)(x + 3)$

$y = 2x^2 - 4x - 6$

multiply

Intercept Form

$y = 2(x - 3)(x + 1)$

Vertex form

What process would you use to ….?

Standard form

$$y = x^2 + 6x + 11$$

multiply

$$y = (x-3)(x-3) + 2$$

Intercept Form

$$y = (x-3)^2 + 2$$

Vertex form

What process would you use to ….?

Standard form

$$y = ax^2 + bx + c$$

$$h = {}^{-b}\!/_{2a}$$

$$k = f\left({}^{-b}\!/_{2a}\right)$$

$$f(x) = a(x-h)^2 + k$$

Vertex form

$$y = x^2 - 8x - 10$$

$$x = \frac{-(-8)}{2(1)}$$

$$x = 4$$

$$y = f(4) \quad = (4)^2 - 8(4) - 10$$

$$y = f(4) = -26$$

$$y = (x-4)^2 - 26$$

Intercept Form

What process would you use to ….?

Standard form

$$y = ax^2 + bx + c$$

"complete the square"

$$f(x) = a(x-h)^2 + k$$

Vertex form

Intercept Form

Convert the vertex form quadratic into a standard form quadratic.

$$y = (x+1)^2 = (x+1)(x+1) = x^2 + 2x + 1$$

$$y = (x+2)^2 = (x+2)(x+2) = x^2 + 4x + 4$$

$$y = (x+3)^2 = (x+3)(x+3) = x^2 + 6x + 9$$

What pattern do you notice with the coefficients of ‘x’ in the standard form quadratic equation?

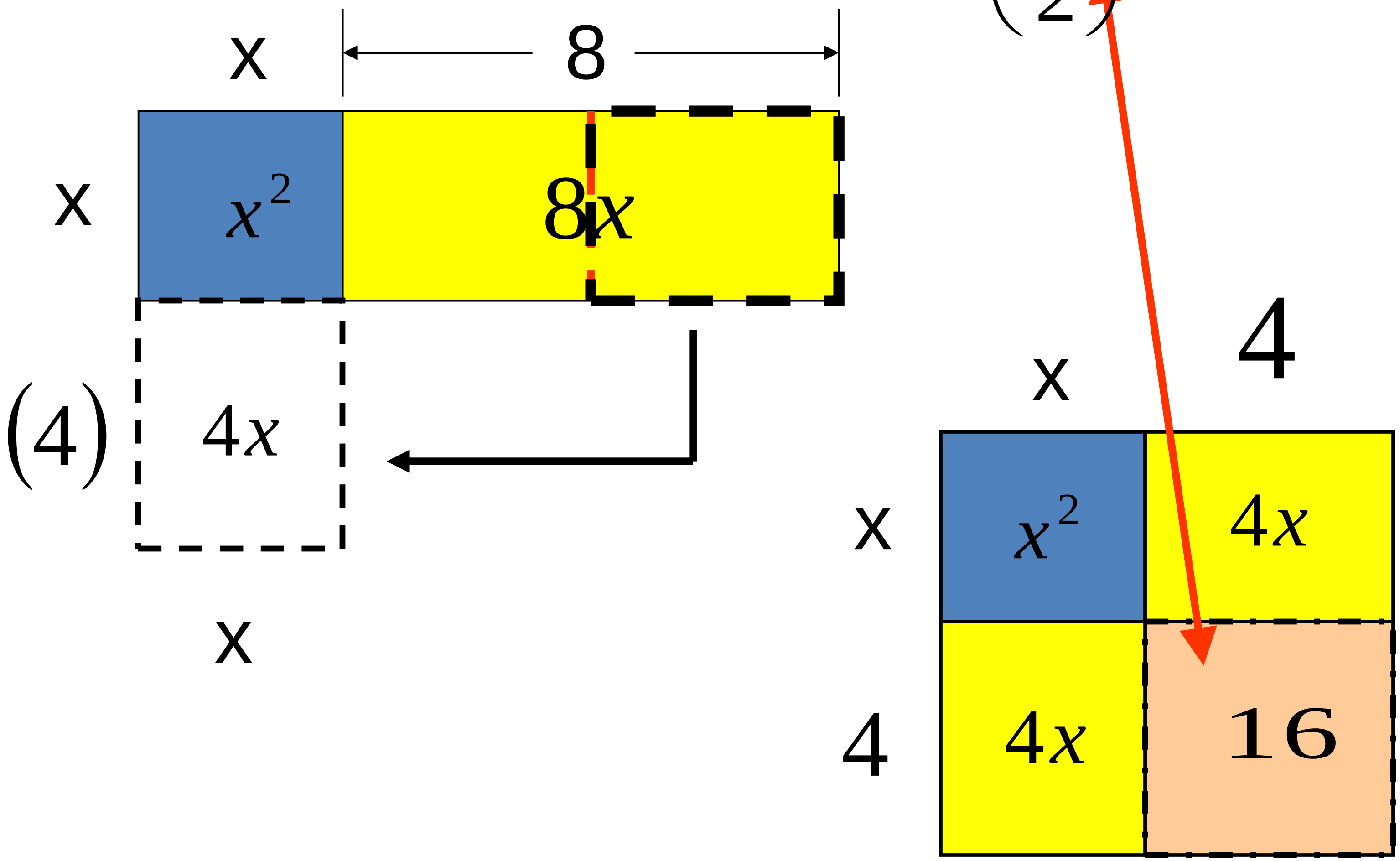
What pattern do you notice with the constant term of the standard form quadratic equation?

Generalize these patterns for the following vertex form quadratic equation.

$$y = (x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$$

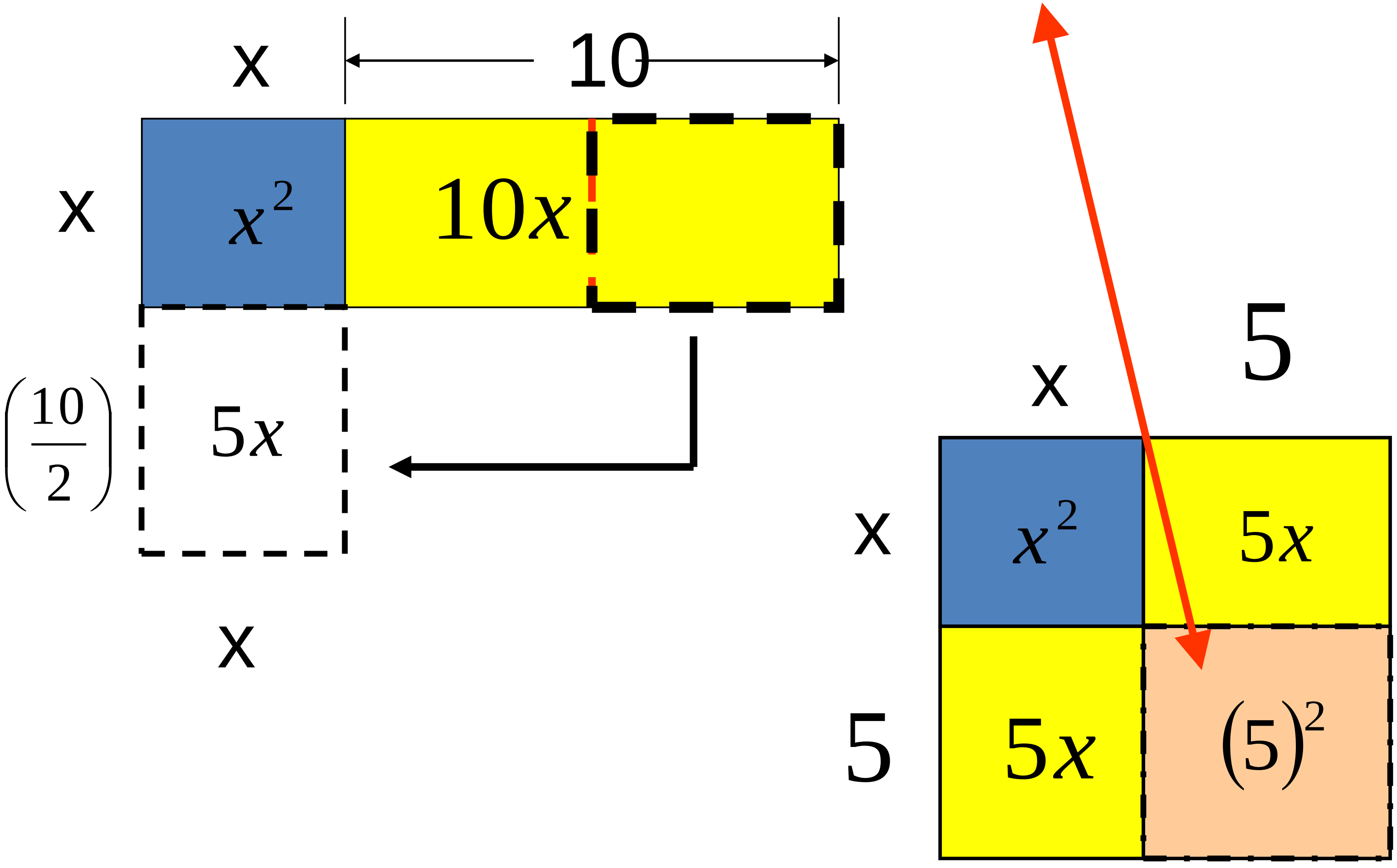
1st we need to figure out what number to add to "Complete the Square"

$$x^2 + 8x + \left(\frac{8}{2}\right)^2$$



Completing the Square

$$x^2 + 10x + 25$$



Which of the following can be written as a binomial squared?

$f(x) = x^2 + 2x + 1 \quad = (x+1)^2$ **Yes**

$f(x) = x^2 + 4x + 3 \quad = (x+1)(x+3)$ **No**

$f(x) = x^2 + 4x + 4 = (x+2)(x+2) \quad = (x+2)^2$ **Yes**

$f(x) = x^2 + 5x + 6 \quad = (x+2)(x+3)$ **No**

$f(x) = x^2 + 6x + 8 \quad = (x+2)(x+4)$ **No**

$f(x) = x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2$ **Yes**

What patterns do you recognize that will help you predict if it can be written as the square of a binomial?

What must the number 'c' be equal to for it to be a "perfect square trinomial"?

C = ? $x^2 + 2x + c$ $x^2 + 2x + 1$

$$c = \left(\frac{2}{2}\right)^2 = 1$$

C = ? $x^2 - 14x + c$ $x^2 - 14x + 49$

$$c = \left(\frac{-14}{2}\right)^2 = 49$$

C = ? $x^2 - 16x + c$ $x^2 - 16x + 64$

$$c = \left(\frac{-16}{2}\right)^2 = 64$$

Rewrite the equation as the square of a binomial:

$$y = x^2 + 16x + 64$$

$$y = (x+8)^2$$

$$y = x^2 - 4x + 4$$

$$y = (x-2)^2$$

$$y = x^2 + 12x + 36$$

$$y = (x+6)^2$$

$$y = x^2 - 14x + 49$$

$$y = (x-7)^2$$

What form (<u>standard form</u>, <u>vertex form</u>, or <u>intercept form</u>) would you call the binomial squared?

Converting a standard form into a vertex form.

$$y = x^2 + 6x + 12$$

What number is needed to complete the square? $\left(\frac{6}{2}\right)^2 = 9$

$$y = x^2 + 6x + 9 - 9 + 12$$

add 9 and subtract 9 (from right side).

$$y = \left(x^2 + 6x + 9\right) + (-9 + 12)$$

Notice the perfect square trinomial!!!

$$y = (x+3)^2 + 3$$

Convert the perfect square trinomial to the square of a binomial

$$y = a(x-h)^2 + k$$

Vertex form!!!!!!

Converting a <u>standard form</u> into a <u>vertex form</u>.

$$y = x^2 - 4x - 7$$

What number is needed to complete the square? $\left(\frac{-4}{2}\right)^2 = 4$

$$y = x^2 - 4x + 4 - 4 - 7$$

Add 4, subtract 4

$$y = \left(x^2 - 4x + 4\right) + (-4 - 7)$$

Notice the perfect square trinomial!!!

$$y = (x-2)^2 - 11$$

Convert to the square of a binomial

$$y = a(x-h)^2 + k$$

Vertex form!!!!!!

Finding "zeroes" of <u>un-factorable standard form quadratic equations.</u>

<u>Complete the square</u>

$$y = ax^2 + bx + c$$

↓ (by completing the square)

$$y = a(x-h)^2 + k$$

↓ (by extracting a square root)

x intercepts

Turn standard form into vertex form, then solve directly.

Simplifying complex fractions:

Complex Fraction is a fraction in the numerator and a fraction in the denominator.

$$\frac{2/3}{4/5} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} * \frac{5}{4} = \frac{2}{2} * \frac{5}{3*2} = \frac{5}{6}$$

How do you divide fractions?

Multiply by reciprocal.

Simplify the complex fraction.

$$\frac{\frac{5}{x+4}}{\frac{3}{x+4}} \rightarrow \frac{5}{x+4} \div \frac{3}{x+4} \rightarrow \frac{5}{x+4} * \frac{x+4}{3}$$

$$\rightarrow \frac{5}{3}$$

$$\frac{\frac{x}{x+2}}{\frac{3}{x+3}}$$

Combine the numerator and denominator fractions into <u>one</u> fraction

$$\frac{\frac{x}{3}-6}{2+\frac{3}{x}} \rightarrow \frac{\frac{x}{3}-6}{\frac{2}{1}+\frac{3}{x}} \rightarrow \frac{\frac{x}{3}-6}{\frac{2x}{x}+\frac{3}{x}} \rightarrow \frac{\frac{x}{3}-6}{\frac{2x+3}{x}}$$

$$\rightarrow \left(\frac{x}{3}-6\right) \div \frac{2x+3}{x} \rightarrow \left(\frac{x}{3}-6\right) * \frac{x}{2x+3}$$

$$\rightarrow \left(\frac{x}{3}-\frac{18}{3}\right) * \frac{x}{2x+3} \rightarrow \left(\frac{x-18}{3}\right) * \frac{x}{2x+3}$$

$$\boxed{\rightarrow \frac{x(x-18)}{3(2x+3)}}$$