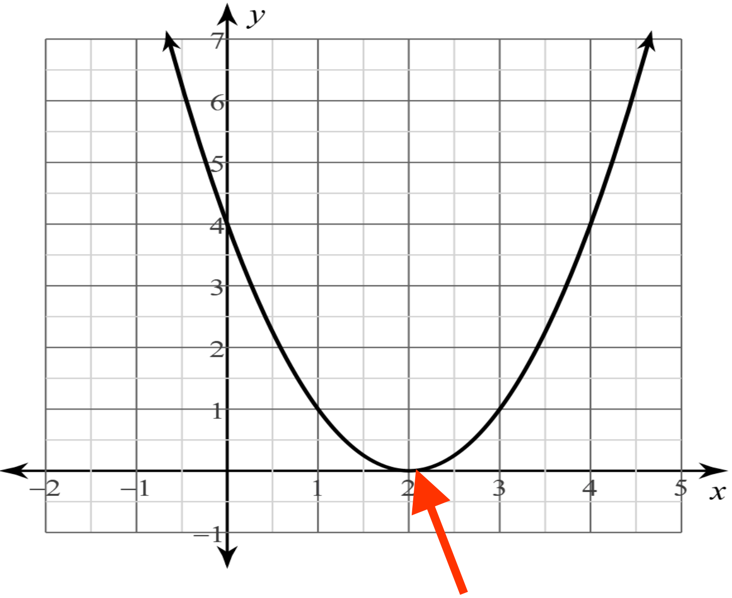
Math-1010
Lesson 4-6
The Standard Form Quadratic Function

We will learn about:

1) Symmetry
2) Axis of symmetry
3) Direction of opening
4) Vertical (y-int) intercept
5) Application Problems

Quadratic function: when graphed, is a U-shaped curve called a ___parabola___?



"Turning Point" (for an upward opening parabola) the location on the graph where it stops going down and "turns" back up

What other name do we have for this special point?

Vertex

Standard Form Quadratic Equation: a polynomial of the form $y = ax^2 + bx + c$ where the sign of the coefficient ‘a’ (either + or -) tells the direction of opening and the “constant” ‘c’ is the y-intercept.

Example: $y = x^2 - 6x + 4$

a = 1 b = -6 c = 4

“a” is positive → the parabola opens upward

The output value (y-value) corresponding to an input value (x-value) of 0 is always the y-intercept.

f(0) = y-intercept

f(0) = 4 “c” = 4

‘c’ is the y-intercept

a) What is the direction of opening?

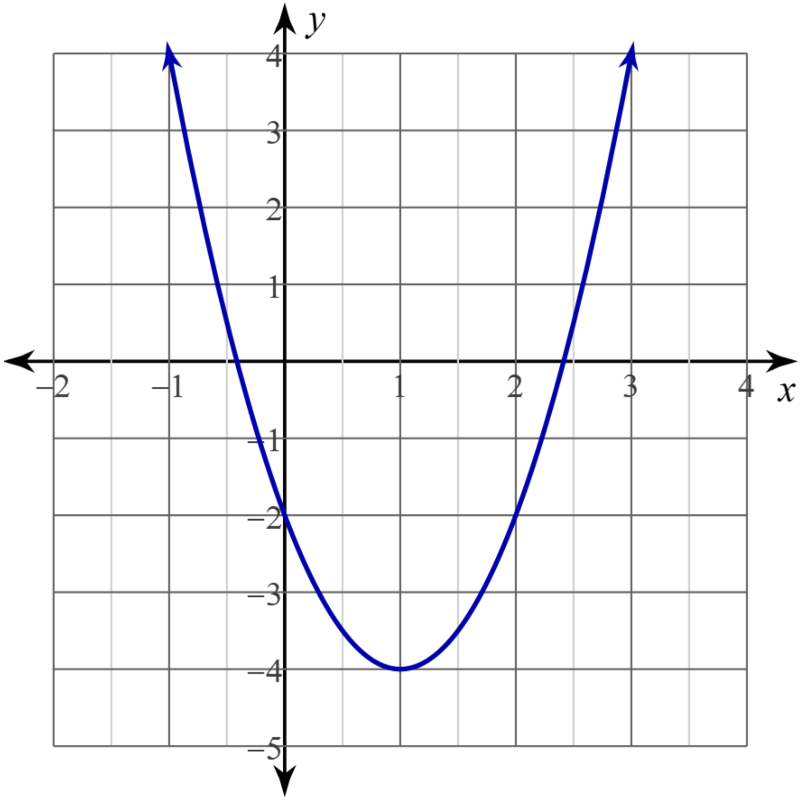
b) What is the y-intercept? (give an x-y pair)

$y = x^2 + 11x + 30$	upward	(0, 30)
$y = -3x^2 - 10x - 24$	downward	(0, -24)
$y = 0.5x^2 - 4$	upward	(0, -4)
$y = -4x^2$	downward	(0, 0)

a) Direction of Opening?

b) Vertical intercept?

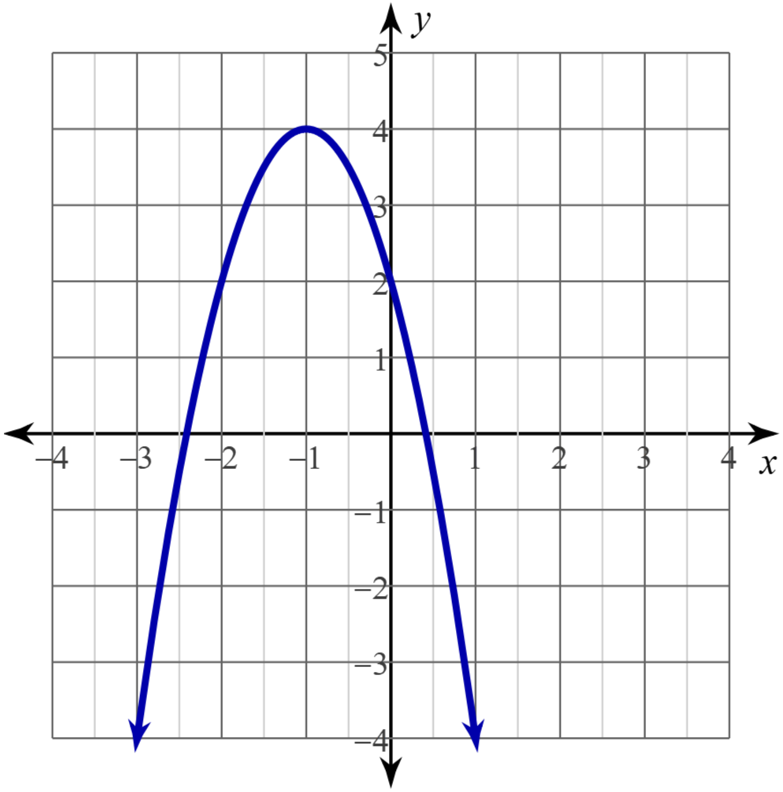
c) Turning point?



a) upward

c) (1, -4)

b) (0, -2)

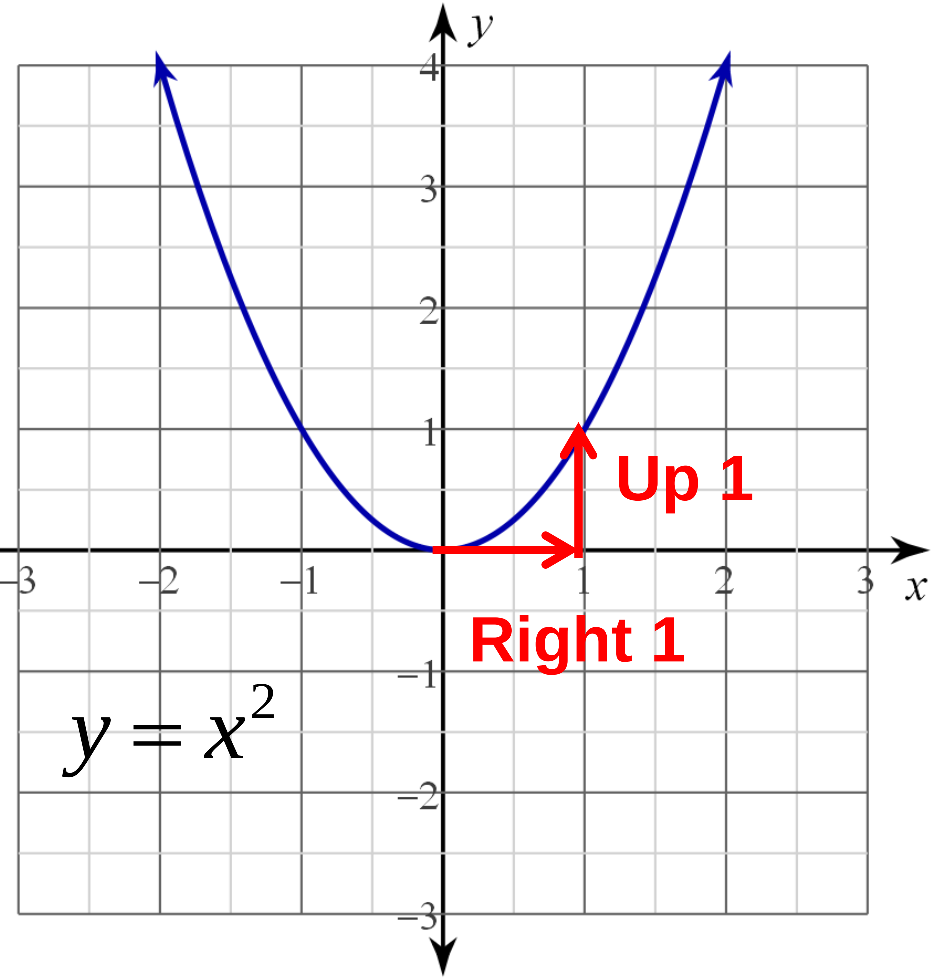


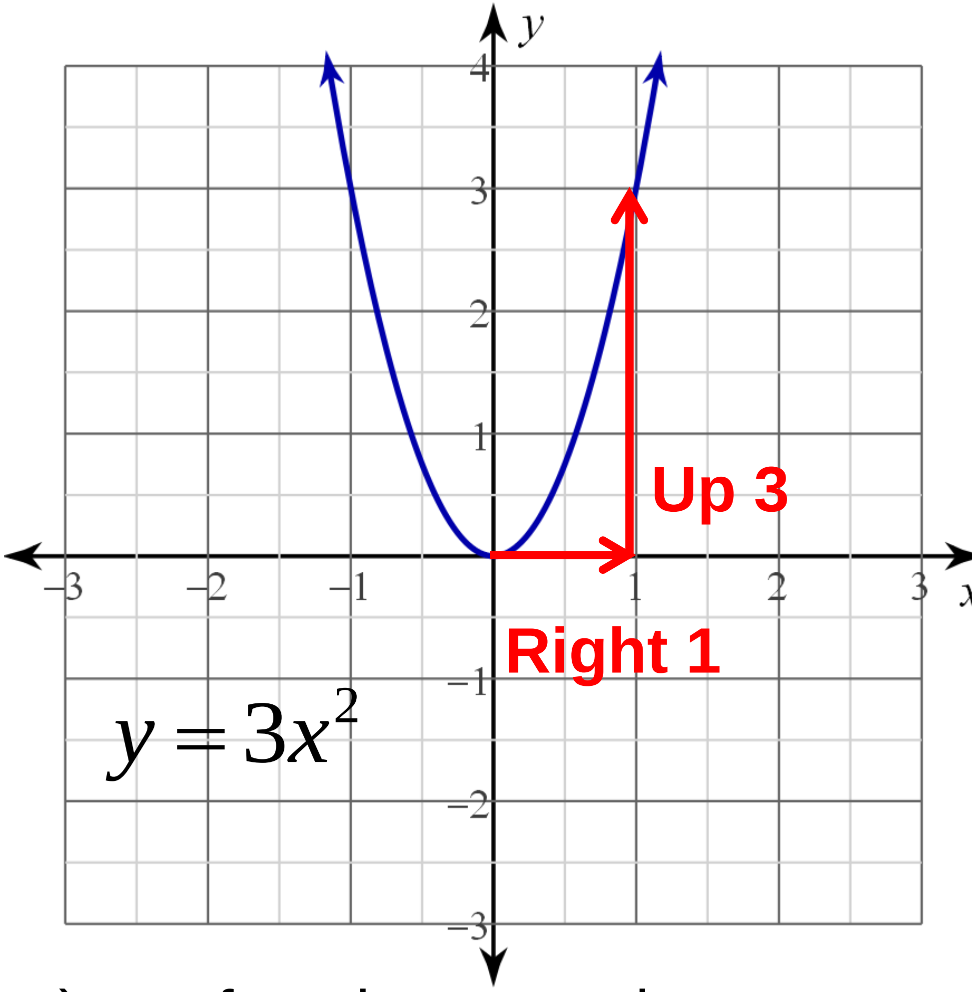
a) downward

c) (-1, 4)

b) (0, 2)

Coefficient ‘a’, not only tells the direction of opening, it also predicts how steep (and narrow) the graph is. $y = ax^2 + bx + c$





To compare how steep (narrow) two functions are: the equation with the larger value of ‘a’ will be narrower.

Arrange the equations from narrowest to widest.

1) $y = 2.5x^2 + 11x + 30$ Next widest

2) $y = -3x^2 - 10x - 24$ Widest

3) $y = 0.5x^2 - 4$ narrowest

4) $y = -1.5x^2$ Next narrowest

Why? $0.5 < 1.5 < 2.5 < 3$

How can we find the actual turning point of a *Standard Form Quadratic Equation*? (without actually graphing it?)

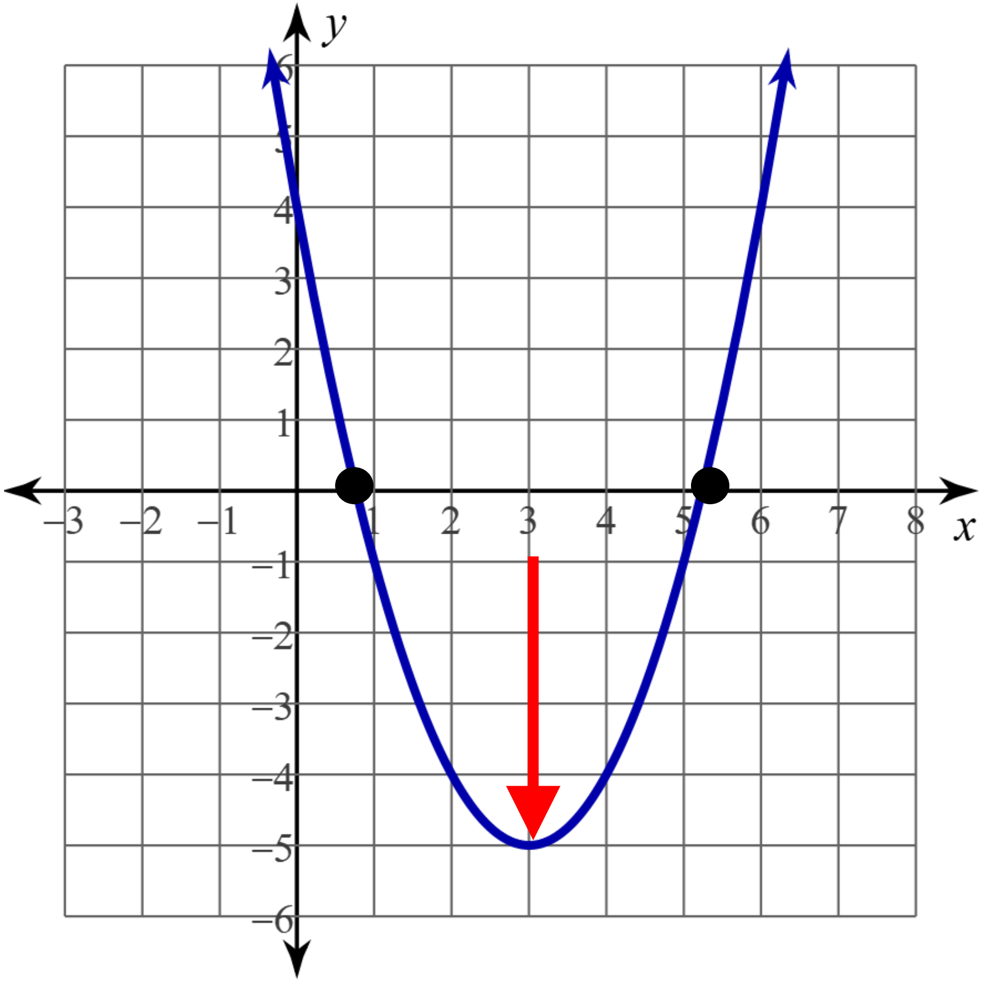
Remember the quadratic formula gave us the x-intercepts.

$$y = x^2 - 6x + 4$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = 3 \pm \sqrt{5}$$

The x-coordinate of the vertex is 3.



x-coord. of vertex = $\frac{-b}{2a}$

What is the x-coordinate of the vertex?

$$y = 2x^2 + 16x + 24$$

x-coord. of vertex = $\frac{-b}{2a}$

a = 2

b = 16

$$\frac{-b}{2a} = \frac{-16}{2(2)} = -4$$

Vertex: $(-4, f(-4))$

What is the y-coordinate of the vertex?

$$f(-4) = 2(-4)^2 + 16(-4) + 24$$

$$f(-4) = -8$$

Vertex: $(-4, -8)$

$$\text{VSF} = 2, \text{vertex} = (-4, -8)$$

What is the x-coordinate of the vertex?

$$y = x^2 - 6x + 13$$

a = 1 b = -6

$$\text{x-coord. of vertex} = \frac{-b}{2a}$$

$$\frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$$

Vertex: $(3, f(3))$

What is the y-coordinate of the vertex?

$$f(3) = (3)^2 - 6(3) + 13$$

$$f(3) = 4$$

Vertex: $(3, 4)$

What is the Vertex form equation?

$$\text{VSF} = 1, \text{vertex} = (3, 4)$$

Arrange the equations from narrowest to widest.

1) $y = 2.5x^2 + 11x + 30$ Next widest

2) $y = -3x^2 - 10x - 24$ Widest

3) $y = 0.5x^2 - 4$ narrowest

4) $y = -1.5x^2$ Next narrowest

Why? $0.5 < 1.5 < 2.5 < 3$

At the 2012 London Olympics, Poland's Tomasz Majewski became the 1^{st} repeat shot put champion since 1956. His winning throw was 71.8 feet. The path of the iron sphere can be modeled by the following equation where the output value is the height above the ground the input value is the horizontal distance from the point where it left his hand.

$$h(x) = -0.01509x^2 + x + 6$$

Does the parabola open upward or downward?

What was the shot's maximum height?

What is the y-intercept?

What is the practical significance of the y-intercept?

What is the practical significance of the positive x-intercept?

What is the x-intercept?

What is the practical domain?

What is the practical range?

$$y = -3x^2 + 12x + 5$$

What is the vertex?

$$x = \frac{-b}{2a} \qquad x = \frac{-(12)}{2(-3)} = \frac{-12}{-6} = 2$$

$$y = ax^2 + bx + c$$

a = -3   b = 12

Substitute x = 2 into the equation then "solve for y"

$$y = -3(2)^2 + 12(2) + 5$$

$$y = -12 + 24 + 5$$

$$y = 17$$

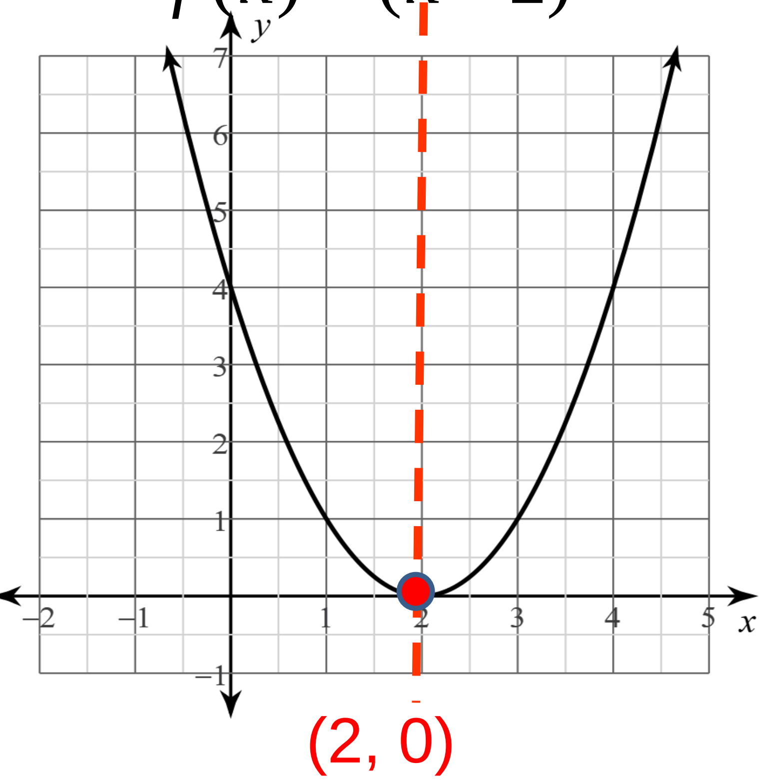
Where is the vertex? (2, 17)

What is the coefficient? $a = -3$

Does the parabola open upward or downward? *downward*

Axis of symmetry: the line that divides the graph into “mirror images” of each other.

$$f(x) = (x-2)^2$$



What point does the axis of symmetry pass through?

Vertex: (2, 0)

What is the equation of the vertical line passing through the vertex?

Axis of symmetry: x = 2

What is axis of symmetry for the following equations?

$$y = -2(x+4)^2 - 5$$

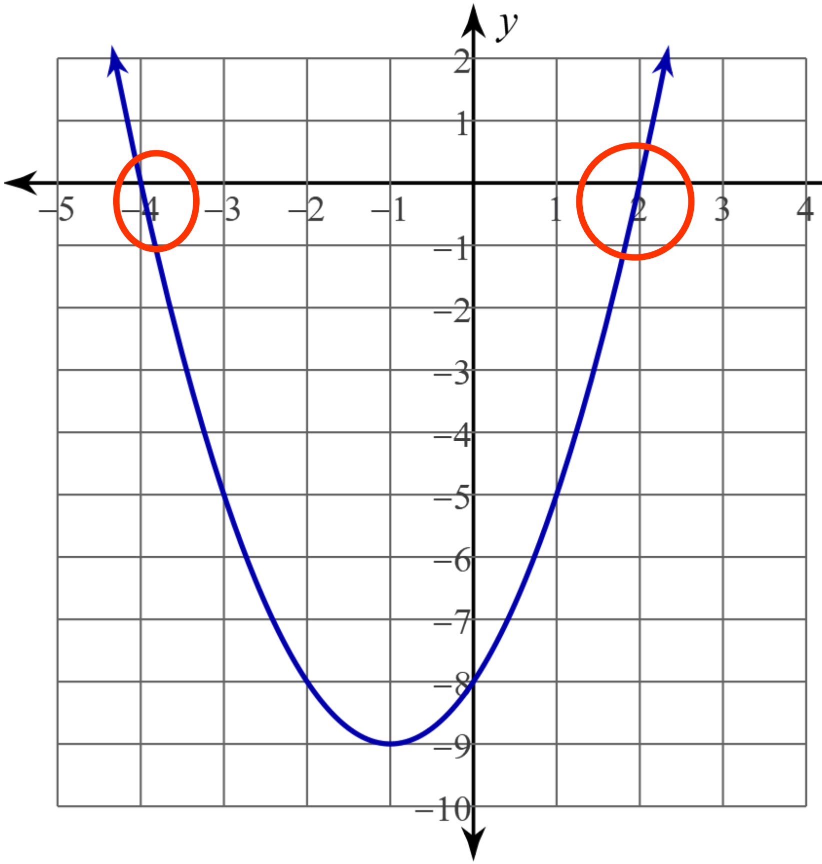
$$y = -6x^2 + 12x - 1$$

$$y = 3(x+2)(x-4)$$

X-Intercepts of the Quadratic Equation: the ***real number*** inputs that have a corresponding output of zero.

The y-value of an x-intercept always equals Zero

x = -4 x = 2



How can you tell from the equation if the graph has x-intercepts?

Which of the following has an x-intercept?

$$f(x) = -3(x-2)^2 + 4$$

$$g(x) = 2(x+3)^2 - 5$$

$$h(x) = -6(x-1)^2 - 7$$

$$k(x) = 4(x+6)^2 + 2$$

Which of the following has an x-intercept?

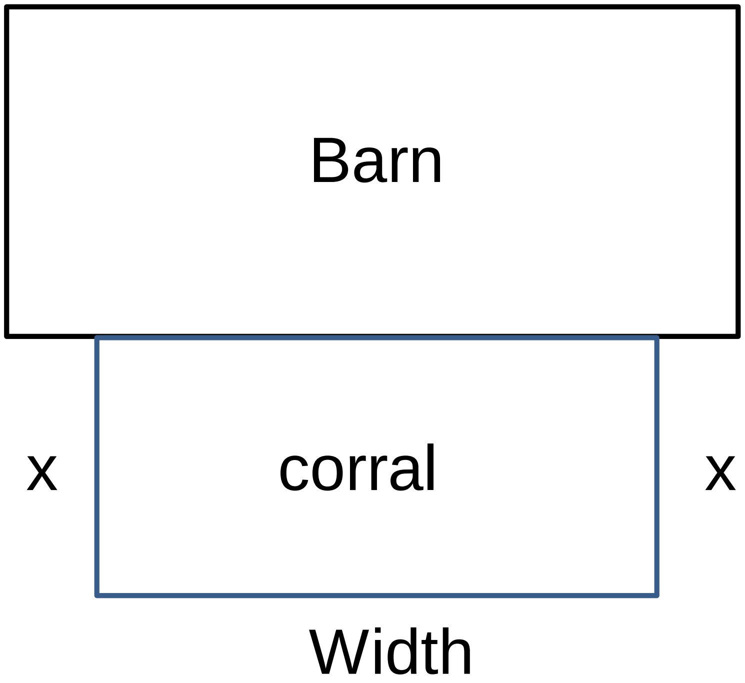
$$f(x) = -3x^2 + 18x - 5$$

$$g(x) = 2x^2 - 12x + 15$$

$$h(x) = -x^2 - 6x - 12$$

Quadratic Equation Modeling

A horse owner wants to build an economical, rectangular horse corral. She decides to use the barn as one side of the corral. If she has 82 feet of fence, what dimensions will give the largest area for the animal to “horse around” in?



x + x + width = 82

width = 82 – 2x

A = L*W

A = x(82 – 2x)

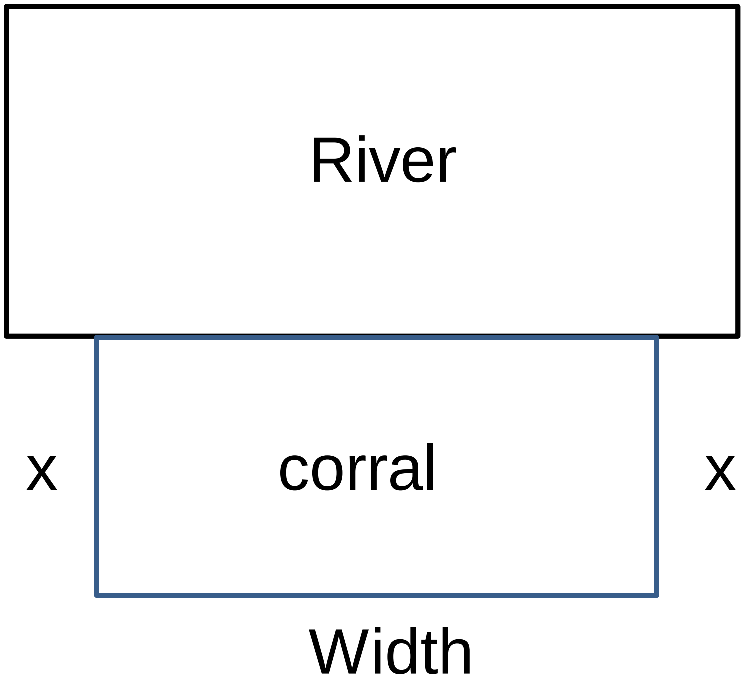
What type of equation is this?

$$y = x(82 - 2x)$$

How do you find the max y-value?

Quadratic Equation Modeling

A rancher wants use the side of a river as once side of his pasture. He has 1500 feet of fencing to build the other 3 sides of his rectangular fence. What dimensions will give the largest area?



x + x + width = 1500

width = 1500 – 2x

A = L*W

A = L(1500 – 2L)

What type of equation is this?

$$y = x(1500 - 2x)$$

How do you find the max y-value?

Transformation: an adjustment made to the parent function that results in a change to the graph of the parent function.

Changes could include:

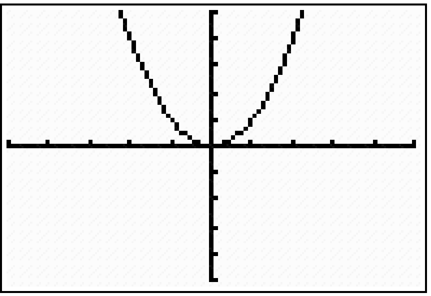
shifting the graph up or down,

Shifting the graph left or right

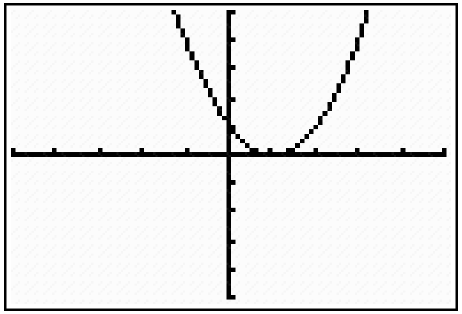
vertical stretching or horizontal stretching

Reflecting across x-axis or y-axis

$$f(x) = x^2$$



$$g(x) = (x-1)^2$$



Replacing 'x' in the parent function with 'x – 1' causes the graph to translate *right '1'*

Build a table of values for each equation for domain elements: -2, -1, 0, 1, 2.

x	y
-2	4
-1	1
0	0
1	1
2	4

x	y
-2	9
-1	4
0	1
1	0
2	1

Let's generalize the transformations

$f(x) = x^2$

$$y = (-1)a(x-h)^2 + k$$

Reflection across x-axis → (-1)

VSF → a

left/right → h

up/down → k

$y = -2(x-3)^2 + 4$ Reflected across x-axis, VSF = 2, right 3, up 4

$y = 3(x+5)^2 - 6$ VSF = 3, left 5, down 6

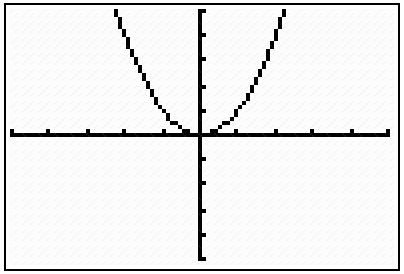
In order to graph the equation:

1) Move the vertex left/right and up/down

2) From the e vertex move right 1, then up/down by the VSF.

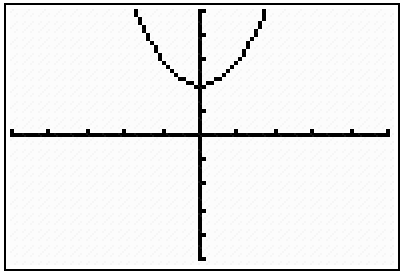
Describe how each transforms f(x) graphically.

$$f(x) = x^2$$



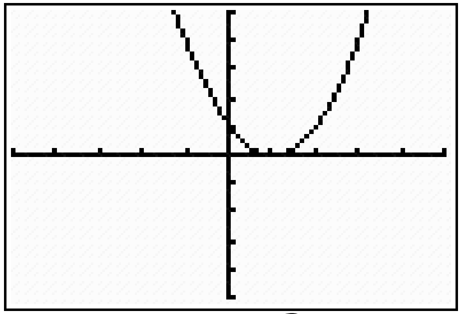
$$g(x) = x^2 + 2$$

f(x) up 2



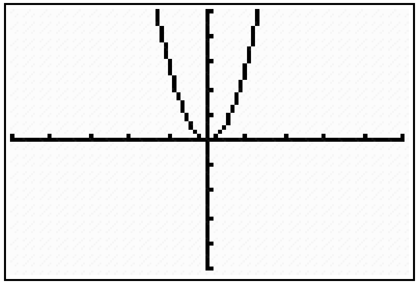
$$h(x) = (x-1)^2$$

f(x) right 1



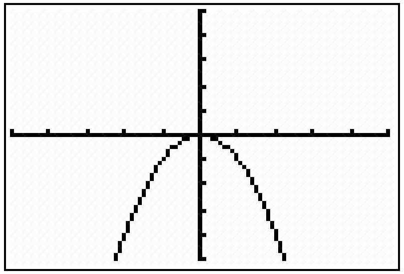
$$j(x) = 3x^2$$

f(x) vertically stretched by a factor of 3



$$k(x) = -x^2$$

f(x) reflected across the x-axis



vertical shift and horizontal Asymptote

$$f(x) = (-1)ab^{(x-c)} + d$$

If negative:
Reflect across x-axis

Horizontal shift

Initial value:
Crosses y-axis here

Growth factor:

$$f(x) = 4(3)^{x-2}$$