Math-1010

Lesson 4-4

Solve Rational Equations

And

Applications of rational functions

(MathXL 5.5 and 5.6)

Solving Rational Equations

Method #1: Obtain common denominators for each term (then multiply by the common denominator).

Method #2: Multiply by (what will be) the “brute force” common denominator

Unit of Measure: the unit that is used to measure quantities.

Examples:

Quantity	**Unit of Measure**
Height	Inches, feet, miles, etc.
Weight	Pounds, ounces, kilograms, grams
Temperature	Degrees F, Degrees C, Degrees K

Sometimes ratios of quantities become new quantities

When you see the word "per" it is a ratio of quantities.

Quantity	**Ratio of:**	**Unit of Measure**
Speed	Distance/time **Job/time**	Mile/hr (mile "**per**" hr) (rooms painted/hr)

Relative Speed: when measuring speed you must "stand somewhere" then measure the speed of an object relative to your position. If stand at the side of the road, a car that is approaching you at 25 mph has a relative speed (to you) of 25 mph.

If you are driving at the speed limit of 60 mph and a car passes you going 70 mph, his relative speed is 10 mph faster than you. 70 – 60 = 10

If you are sitting on the Front Runner and it is going 60 mph:

a) You are going 0 mph relative to the train
b) You are going 60 mph relative to the ground

If you are walking to the front of the Front Runner at a walking speed of 3 mph and it is going 60 mph:

a) You are going 3 mph relative to the train
b) You are going 63 mph relative to the ground

Speed is a rate. We can add/subtract speeds to determine total speed.

We can add/subtract rates to determine total rates.

Jose takes 3 hours to clean a house
(time rate of cleaning: one house per 3 hours → Rate Jose = $\frac{1\text{ house}}{3\text{ hours}}$

George takes 4 hours to clean a house
(time rate of cleaning: one Job per 4 hours → Rate George $= \frac{1\text{ house}}{4\text{ hours}}$

How Long for both to clean one house by working together?

Rate George + Rate Jose = Combined Rate (George & Jose)

$$Rate_G + Rate_J = Rate_{G+J}$$

$$\frac{1\text{ house}}{4\text{ hrs}} + \frac{1\text{ house}}{3\text{ hours}} = \frac{1\text{ house}}{t\text{ hours}} \qquad \frac{1}{4} + \frac{1}{3} = \frac{1}{t}$$

Multiply by the common denominator

$$\frac{12t}{4} + \frac{12t}{3} = \frac{12t}{t}$$

simplify

$$3t + 4t = 12 \qquad 7t = 12 \qquad t = 12/7$$

$$t = 1.7\text{ hrs}$$

Rational equations involving rates are usually quantities related to time.

You are an intern at an architecture firm. The company is designing an auditorium that will be annexed to the local high school building. The rate of traffic flow through the exits is an important consideration. The auditorium will have three exit doors. Two exits are single doors of slightly different sizes. The first exit, by itself, can be used to empty the auditorium in 10 minutes. The second exit can be used to empty the room in 8 minutes. The third exit is a double-wide door that, by itself, can be used to empty the auditorium in 5 minutes.

Quantity: Exit #1 Rate: "empty the room / 10 min"
Exit #2 Rate: "empty the room / 8 min"
Exit #3 Rate: "empty the room / 5 min"

We can add/subtract rates to determine total rates.

$$Rate_{\#1} + Rate_{\#2} + Rate_{\#3} = Rate_{total}$$

$$\frac{room}{10\text{ min}} + \frac{room}{8\text{ min}} + \frac{room}{5\text{ min}} = \frac{room}{t\text{ min}}$$

$$\frac{1}{10} + \frac{1}{8} + \frac{1}{5} = \frac{1}{t}$$

Solve for 't'.

$$\frac{1}{10}+\frac{1}{8}+\frac{1}{5}=\frac{1}{t}$$

Solve for 't'.

Method 2: Multiply by (what will be) the "brute force" common denominator.

$$5*8*10*t\left(\frac{1}{10}+\frac{1}{8}+\frac{1}{5}\right)=\left(\frac{1}{t}\right)\ 5*8*10*t$$

As you multiply each term, the denominator will be eliminated

$$\frac{5*8*t}{1}+\frac{5*10*t}{1}=\frac{5*8*10}{1}$$

$$40t+50t=400$$

$$40t+50t=400$$

$$90t=400$$

$$\div 90 \qquad \div 90$$

$$\boxed{t=4.4\ min}$$

Jamie, Paul and Stan can paint a room together in 2 hours. If Paul does the job alone, he can paint the room in 5 hours. If Stan works alone, he can paint the room in 6 hours. If Jamie works alone, how long would it take her to paint the room?

$$Rate_{J+P+S} = Rate_J + Rate_P + Rate_S$$

$$\frac{room}{2\text{ hrs}} = \frac{room}{\text{"t" hrs}} + \frac{room}{5\text{ hrs}} + \frac{room}{6\text{ hrs}}$$

$$\frac{1}{2} = \frac{1}{t} + \frac{1}{5} + \frac{1}{6}$$

Multiply by the common denominator (2*5*6*t)

$$\frac{5*6*t}{1} = \frac{2*5*6}{1} + \frac{2*6*t}{1} + \frac{2*5*t}{1}$$

$$30t = 60 + 12t + 10t$$

simplify

$$30t = 60 + 22t$$

$$8t = 60$$

$$\div 8 \qquad \div 8$$

$$t = 7.5\text{ hrs}$$

Tanya and Cam can each wash a car and vacuum its interior in 2 hours. Pat needs 3 hours to do this same job by himself. If Pat, Cam and Tanya work together, how long will it take them to clean a car?

$$Rate_{T+C+P} = Rate_T + Rate_C + Rate_P$$

$$\frac{car}{t} = \frac{car}{2\text{ hrs}} + \frac{car}{2\text{ hrs}} + \frac{car}{3\text{ hrs}}$$

$$\frac{1}{t} = \frac{1}{2} + \frac{1}{2} + \frac{1}{3}$$

Multiply by the common denominator (6 t)

$$\frac{6t}{t} = \frac{6t}{2} + \frac{6t}{2} + \frac{6t}{3}$$

simplify

$$6 = 3t + 3t + 2t$$

$$6 = 8t$$

$$\frac{6}{8} = t$$

$$t = 0.75\text{ hrs}$$

Rational equations with 2 solutions.

$$1+\frac{8}{x-5}=-\frac{9}{x}$$ Obtain common denominators.

$$\frac{x(x-5)}{x(x-5)}+\frac{8x}{x(x-5)}=\frac{-9(x-5)}{x(x-5)}$$ Multiply left/right by x(x – 5)

(all denominators are eliminated in 1 step)!

$$\boxed{x(x-5)+8x=-9(x-5)}$$

Solve for x

$$x^2-5x+8x=-9x+45$$ Distributive Property

$$x^2+3x=-9x+45$$ Simplify left side

$$x^2+12x-45=0$$ Add 9x, subtract 45 (both sides)

$$(x+15)(x-3)=0$$ factor

(continued)

$$1+\frac{8}{x-5}=-\frac{9}{x}$$

$$x^2+12x-45=0$$ factor

$$(x+15)(x-3)=0$$

$$x=-5,3$$

Check for extraneous solutions.

Neither solution is an excluded value!

Extraneous Solution: a solution obtained algebraically that is not in the domain of the original equation.

$$\frac{2}{x-3}+\frac{1}{x}=\frac{x-1}{x-3}$$

What are the excluded values

$$x \neq 0,3$$

Obtain a common denominator

Multiply by the common denominator

$$\frac{2x}{x(x-3)}+\frac{(x-3)}{x(x-3)}=\frac{x(x-1)}{x(x-3)}$$

$$\boxed{2x+x-3=x(x-1)}$$

Solve for ‘x’

$$3x-3=x^2-x$$

Get in “standard form”

$$0=x^2-4x+3$$

Factor

$$0=(x-3)(x-1)$$

(continued)

$$\frac{2}{x-3}+\frac{1}{x}=\frac{x-1}{x-3}$$

$0 = x^2 - 4x + 3$ Factor

excluded values $x \neq 0,3$

$0 = (x-3)(x-1)$

$x = 3, 1$

Check for extraneous solutions.

$x = 1$ Only one solution, the other is extraneous