Math-1010

Lesson 4-2

Add and Subtract Rational Expressions

What are "like terms" ?

Like variables: 2x + 3y + 4x

Multiples of the same variable

Like powers:
$$x^2 + y^2 + x^3 + x^2$$

same 'base' and same exponent.

Like radicals:
$$\sqrt{2} \neq \sqrt{3} \quad \pm 3\sqrt{2}$$

same 'radicand' and same index number.

Like fractions:

$$\left(\frac{2}{3}, \left(\frac{4}{3}, \frac{3}{4}\right)\right)$$

same denominator.

Adding Fractions

We can add "like fractions"

$$\frac{2}{3} + \frac{1}{3} + \frac{4}{3}$$

Combine the numerator over a common denominator.

$$=\frac{2+1+4}{3} = \frac{7}{3}$$

For fractions that involve integer numbers only, you can obtain a "<u>brute-force</u>" common denominator by multiplying the other fraction by:

"One in the form of" the other dominator divided by itself.

$$\frac{42}{42} * \frac{1}{30} + \frac{1}{42} * \frac{30}{30} \to \frac{42}{1260} + \frac{30}{1260} \to \frac{72}{1260}$$

This often results in fractions that have huge numbers in the numerator and denominator that must then be simplified.

It is much better to obtain the "least common denominator"

$$\frac{1}{30} + \frac{1}{42} = \frac{1}{6*5} + \frac{1}{6*7} = \frac{1}{6*5} + \frac{1}{6*7} + \frac{1}{6*5} + \frac{1}{6*7} + \frac{1}{6$$

(2) Multiply the 1st fraction by "<u>one in the form of</u>" the *missing factor* that the 2nd denominator has but the 1st one does not have.

$$\frac{1}{30} + \frac{1}{42} = \frac{1}{6*5} * \frac{7}{7} + \frac{1}{6*7} * \frac{5}{5} \longrightarrow \frac{7}{210} + \frac{5}{210}$$

This results in much smaller numbers in the numerator and denominator compared to the "brute-force" method.

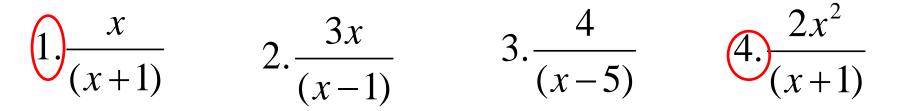
$$\rightarrow \frac{7+5}{210} \rightarrow \frac{12}{210} \rightarrow \frac{2*6}{2*105} \rightarrow \frac{2*2*3}{2*5*21} \rightarrow \frac{\cancel{2}*2*\cancel{3}}{\cancel{2}*5*\cancel{3}*7}$$
Factor the resulting fraction in order to simplify it.
$$\frac{1}{30} + \frac{1}{42} \rightarrow \frac{2}{35}$$

For rational expression, a "brute-force" common denominator is possible but much harder to simplify.

What type of <u>rational expressions</u> can you combine together using addition or subtraction?

"like rational expressions"

Which of these expressions are "like expressions"?



What is the <u>excluded value</u> for each expression?

$$1.\frac{x}{(x+1)} \qquad 2.\frac{3x}{(x-1)} \qquad 3.\frac{4}{(x-5)}$$
$$x \neq -1 \qquad x \neq 1 \qquad x \neq 5$$

Adding/Subtraction Rational Expressions

The easy problem:
$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

Combine the numerator over a <u>common</u> denominator.

The easy problem:
$$\frac{4}{(x-5)} + \frac{3x}{(x-5)} = \frac{4+3x}{(x-5)}$$

Combine the numerator over a common denominator.

Your turn: add/subtract

$$\frac{x+2}{2x^2} + \frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$=\frac{2(x-1)}{2^{*}x^{2}} = \frac{(x-1)}{x^{2}}$$

What property are we using? Inverse Property of Multiplication.

Can you do it this way?

$$\frac{x+2}{2x^2} + \frac{x-4}{2x^2} = \frac{x+2}{2^*x^*x} + \frac{x-(2^*2)}{2^*x^*x} = \frac{1}{x} + \frac{-2}{x}$$

NO! You CANNOT use the Inverse Property of Multiplication on <u>addends.</u> Your turn: add/subtract

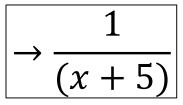
$$\frac{3x+2}{2x^2} - \frac{x-4}{2x^2} \rightarrow \frac{3x+2-(x-4)}{2x^2} \rightarrow \frac{3x+2-x+4}{2x^2}$$
Subtract the entire numerator
of the 2nd faction!

$$\rightarrow \frac{2x+6}{2x^2} \rightarrow \frac{2(x+3)}{2x^2} \rightarrow \frac{(x+3)}{x^2}$$

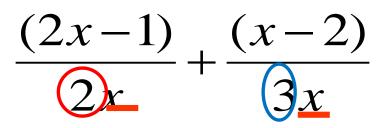
Simplify your fractions!

$$\frac{2x-7}{x^2-25} - \frac{x-2}{x^2-25} \longrightarrow \frac{2x-7-x+2}{x^2-25} \longrightarrow \frac{x-5}{x^2-25}$$

$$\rightarrow \frac{(x-5)}{(x-5)(x+5)} \quad \rightarrow \quad$$



No common denominator.



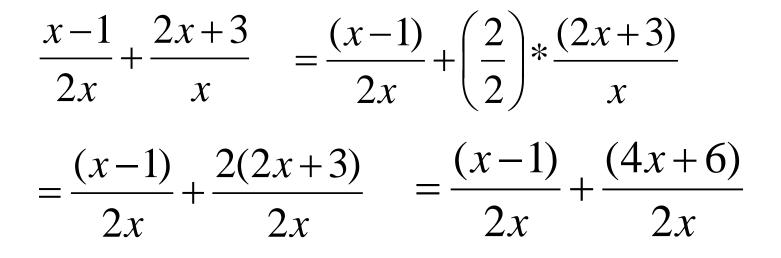
Find what <u>missing</u> from each denominator.

Multiply the left side fraction by one in the form of 3/3

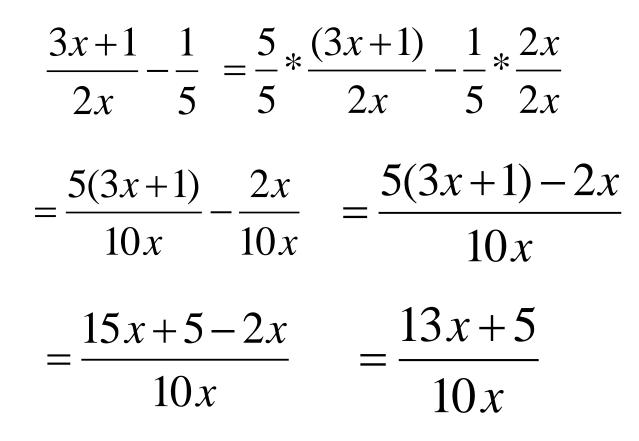
$$\frac{3}{3} * \frac{(2x-1)}{2x} + \frac{(x-2)}{3x} * \frac{2}{2} = \frac{3(2x-1)}{2*3*x} + \frac{2(x-2)}{2*3*x}$$

Multiply the right side fraction by one in the form of 2/2

$$=\frac{3(2x-1)+2(x-2)}{6x} = \frac{6x-3+2x-4}{6x} = \frac{8x-7}{6x}$$



$$=\frac{(x-1)+(4x+6)}{2x} = \frac{5x+5}{2x}$$

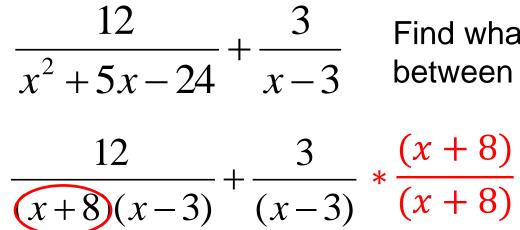


LCD: Least Common Denominator

$\frac{x-7}{x^2-5x}$	$\rightarrow \frac{x-7}{x(x-5)}$
and	
x - 2	× − 2
$x^2 - 25$	$\rightarrow \overline{(x-5)(x+5)}$

Rewrite each expression in terms of their LCD

$$\frac{x-7}{x^2-5x} = \frac{(x-7)(x+5)}{x(x-5)(x+5)}$$
$$\frac{x-2}{x^2-25} = \frac{x(x-2)}{x(x-5)(x+5)}$$



Multiply by "<u>one in the form of</u>" the missing factor of the denominator divided by the missing factor.

$$=\frac{12}{(x+8)(x-3)} + \frac{3}{(x-3)} * \frac{(x+8)}{(x+8)} = \frac{12+3(x+8)}{(x+8)(x-3)}$$
$$=\frac{12+3x+24}{(x+8)(x-3)} = \frac{3x+36}{(x+8)(x-3)} = \frac{3(x+12)}{(x+8)(x-3)}$$

Find what is *already common* between the two denominators.

$$\frac{(x+1)}{x^2 - 2x - 3} + \frac{2}{x - 3} = \frac{(x+1)}{(x+1)(x - 3)} + \frac{2}{x - 3}$$

Simplify before you multiply
(by "one in the form of..."
$$= \frac{1}{x - 3} + \frac{2}{x - 3} \begin{bmatrix} = \frac{3}{x - 3} \end{bmatrix}$$

But, it's not wrong to go a different (but more complicated) route.

 $\frac{(x+1)}{(x+1)(x-3)} + \frac{2}{x-3} * \frac{(x+1)}{(x+1)} \to \frac{(x+1)+2(x+1)}{(x+1)(x-3)}$ $\to \frac{3x+3}{(x+1)(x-3)} \to \frac{3(x+1)}{(x+1)(x-3)} = \frac{3}{x-3}$