Math-1010

Lesson 4-2

Add and Subtract Rational Expressions

What are "like terms" ?

Like variables: $2x + 3y + 4x$

Multiples of the same variable

Like powers: $x^2 + y^2 + x^3 + x^2$

same 'base' and same exponent.

Like radicals: $\sqrt{2} + \sqrt{3} + 3\sqrt{2}$

same 'radicand' and same index number.

Like fractions: $\frac{2}{3}, \frac{4}{3}, \frac{3}{4}$

same denominator.

Adding Fractions

We can add “like fractions”

$$\frac{2}{3}+\frac{1}{3}+\frac{4}{3}$$

Combine the numerator over a <u>common</u> denominator.

$$=\frac{2+1+4}{3} \qquad =\frac{7}{3}$$

For fractions that involve integer numbers only, you can obtain a "brute-force" common denominator by multiplying the other fraction by:

"One in the form of" the other dominator divided by itself.

$$\frac{42}{42} * \frac{1}{30} + \frac{1}{42} * \frac{30}{30} \rightarrow \frac{42}{1260} + \frac{30}{1260} \rightarrow \frac{72}{1260}$$

This often results in fractions that have huge numbers in the numerator and denominator that must then be simplified.

It is much better to obtain the "least common denominator"

$$\frac{1}{30} + \frac{1}{42} = \frac{1}{6*5} + \frac{1}{6*7} = \frac{1}{6*5} * \frac{7}{7} + \frac{1}{6*7} * \frac{5}{5}$$

(1) Factor each denominator.

(2) Multiply the 1st fraction by "one in the form of" the *missing factor* that the 2nd denominator has but the 1st one does not have.

$$\frac{1}{30} + \frac{1}{42} = \frac{1}{6*5} * \frac{7}{7} + \frac{1}{6*7} * \frac{5}{5} \rightarrow \frac{7}{210} + \frac{5}{210}$$

This results in much smaller numbers in the numerator and denominator compared to the “brute-force” method.

$$\rightarrow \frac{7+5}{210} \rightarrow \frac{12}{210} \rightarrow \frac{2*6}{2*105} \rightarrow \frac{2*2*3}{2*5*21} \rightarrow \frac{\cancel{2}*2*\cancel{3}}{\cancel{2}*5*\cancel{3}*7}$$

Factor the resulting fraction in order to simplify it.

$$\boxed{\frac{1}{30} + \frac{1}{42} \rightarrow \frac{2}{35}}$$

For rational expression, a “<u>brute-force” common denominator</u> is possible but much harder to simplify.

What type of rational expressions can you combine together using addition or subtraction?

“like rational expressions”

Which of these expressions are “like expressions”?

1. $\frac{x}{(x+1)}$ 2. $\frac{3x}{(x-1)}$ 3. $\frac{4}{(x-5)}$ 4. $\frac{2x^2}{(x+1)}$

What is the excluded value for each expression?

1. $\frac{x}{(x+1)}$ 2. $\frac{3x}{(x-1)}$ 3. $\frac{4}{(x-5)}$

$x \neq -1$ $x \neq 1$ $x \neq 5$

Adding/Subtraction Rational Expressions

The easy problem: $\dfrac{2}{7}+\dfrac{3}{7}=\dfrac{2+3}{7}=\dfrac{5}{7}$

Combine the numerator over a common denominator.

The easy problem: $\dfrac{4}{(x-5)}+\dfrac{3x}{(x-5)}=\dfrac{4+3x}{(x-5)}$

Combine the numerator over a common denominator.

Your turn: add/subtract

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{\cancel{2}(x-1)}{\cancel{2}*x^2} = \frac{(x-1)}{x^2}$$

What property are we using?

Inverse Property of Multiplication.

Can you do it this way?

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{\cancel{x}+\cancel{2}}{\cancel{2}*\cancel{x}*x}+\frac{\cancel{x}-(\cancel{2}*2)}{\cancel{2}*\cancel{x}*x} = \frac{1}{x}+\frac{-2}{x}$$

NO!

You CANNOT use the Inverse Property of Multiplication on <u>addends.</u>

Your turn: add/subtract

$$\frac{3x+2}{2x^2} - \frac{x-4}{2x^2} \rightarrow \frac{3x+2-(x-4)}{2x^2} \rightarrow \frac{3x+2-x+4}{2x^2}$$

Subtract *the entire numerator*
of the 2nd faction!

$$\rightarrow \frac{2x+6}{2x^2} \quad \rightarrow \frac{\cancel{2}(x+3)}{\cancel{2}x^2} \quad \rightarrow \frac{(x+3)}{x^2}$$

Simplify your fractions!

$$\frac{2x-7}{x^2-25} - \frac{x-2}{x^2-25} \rightarrow \frac{2x-7-x+2}{x^2-25} \rightarrow \frac{x-5}{x^2-25}$$

$$\rightarrow \frac{(x-5)}{(x-5)(x+5)} \quad \boxed{\rightarrow \frac{1}{(x+5)}}$$

No common denominator.

$$\frac{(2x-1)}{2x} + \frac{(x-2)}{3x}$$

Find what missing from each denominator.

Multiply the left side fraction by one in the form of 3/3

$$\frac{3}{3} * \frac{(2x-1)}{2x} + \frac{(x-2)}{3x} * \frac{2}{2} = \frac{3(2x-1)}{2*3*x} + \frac{2(x-2)}{2*3*x}$$

Multiply the right side fraction by one in the form of 2/2

$$= \frac{3(2x-1)+2(x-2)}{6x} = \frac{6x-3+2x-4}{6x} = \frac{8x-7}{6x}$$

$$\frac{x-1}{2x}+\frac{2x+3}{x} \qquad =\frac{(x-1)}{2x}+\left(\frac{2}{2}\right)*\frac{(2x+3)}{x}$$

$$=\frac{(x-1)}{2x}+\frac{2(2x+3)}{2x} \qquad =\frac{(x-1)}{2x}+\frac{(4x+6)}{2x}$$

$$=\frac{(x-1)+(4x+6)}{2x} \qquad =\frac{5x+5}{2x}$$

$$\frac{3x+1}{2x}-\frac{1}{5}=\frac{5}{5}*\frac{(3x+1)}{2x}-\frac{1}{5}*\frac{2x}{2x}$$

$$=\frac{5(3x+1)}{10x}-\frac{2x}{10x}=\frac{5(3x+1)-2x}{10x}$$

$$=\frac{15x+5-2x}{10x}=\frac{13x+5}{10x}$$

LCD: Least Common Denominator

$$\frac{x-7}{x^2-5x} \quad \rightarrow \frac{x-7}{x(x-5)}$$

and

$$\frac{x-2}{x^2-25} \quad \rightarrow \frac{x-2}{(x-5)(x+5)}$$

Rewrite each expression in terms of their LCD

$$\frac{x-7}{x^2-5x} = \frac{(x-7)(x+5)}{x(x-5)(x+5)}$$

$$\frac{x-2}{x^2-25} = \frac{x(x-2)}{x(x-5)(x+5)}$$

$$\frac{12}{x^2+5x-24}+\frac{3}{x-3}$$

Find what is *already common* between the two denominators.

$$\frac{12}{(x+8)(x-3)}+\frac{3}{(x-3)} * \frac{(x+8)}{(x+8)}$$

Multiply by "one in the form of" the missing factor of the denominator divided by the missing factor.

$$=\frac{12}{(x+8)(x-3)}+\frac{3}{(x-3)} * \frac{(x+8)}{(x+8)} = \frac{12+3(x+8)}{(x+8)(x-3)}$$

$$=\frac{12+3x+24}{(x+8)(x-3)} = \frac{3x+36}{(x+8)(x-3)}$$

$$\boxed{=\frac{3(x+12)}{(x+8)(x-3)}}$$

$$\frac{(x+1)}{x^2-2x-3}+\frac{2}{x-3} = \frac{(x+1)}{(x+1)(x-3)}+\frac{2}{x-3}$$

Simplify *before* you multiply
(by "one in the form of…"

$$=\frac{1}{x-3}+\frac{2}{x-3} \quad \boxed{=\frac{3}{x-3}}$$

But, it's not wrong to go a different (but more complicated) route.

$$\frac{(x+1)}{(x+1)(x-3)}+\frac{2}{x-3}*\frac{(x+1)}{(x+1)} \rightarrow \frac{(x+1)+2(x+1)}{(x+1)(x-3)}$$

$$\rightarrow \frac{3x+3}{(x+1)(x-3)} \rightarrow \frac{3\cancel{(x+1)}}{\cancel{(x+1)}(x-3)} \quad \boxed{=\frac{3}{x-3}}$$