Math-1010

Lesson 4-1
Factoring Quadratics with Lead Coefficient Not = 1,
Irrational and Complex Conjugates

Simplify Rational Expressions

Factor the quadratic expressions.

$3x^3 + 15x^2 - 42x$

$5x^3 - 25x^2 - 20x$

What if there is no common factor AND the lead coefficient is NOT equal to 1?

$ax^2 + bx + c$

(These come from multiplying binomials that also do not have lead coefficients of 1.)

$(2x + 1)(x + 3)$

Use the "box method" to multiply the binomials

$2x^2 + 7x + 3$

	x	3
2x	$2x^2$	6x
1	x	3

Notice a nice pattern when you multiply this out (“simplify”)

$$(2x+1)(x+3)$$

$$\boxed{2x^2+7x+3}$$

Left times left is left

Right times right is right

“right plus right” *does not* add up to 7, but notice something.

$6x$

$$(2x+1)(x+3)$$

x

$$6x+x=7x$$

$$2x^2+7x+3$$

$$1+6=7$$

$$6=1*6$$

$$2*3=6$$

Are there any other factors of 6 that add up to 7?

$$2 * 15 = 30$$

$$2x^2 + 13x + 15$$

$$10 + 3 = 13$$

$$30 = 10 * 3$$

Are there any *other* factors of 30 that add up to 13?

This tells us to break 13x into 10x + 3x

$$2x^2 + 13x + 15$$

$$2x^2 + 10x + 3x + 15$$

These are all of the terms in "the box"

	x	5
2x	$2x^2$	10x
3	3x	15

What is the bottom-left term in the box?

x*(3) = 3x

What is the top-right term in the box?

2x*(5) = 10x

Final check: 3*5 = 15?

Factored form:

$$2x^2 + 13x + 15$$

$$\rightarrow (2x + 3)(x + 5)$$

$4 * 10 = 40$

$4x^2 + 13x + 10$

$8 + 5 = 13$

Other factors of 40 that add up to 13?

$40 = 8 * 5$

This tells us to break 13x into 8x + 5x

$4x^2 + 13x + 10$

$4x^2 + 8x + 5x + 10$

Factored form:

$$4x^2 + 13x + 10$$
$$\rightarrow (x + 2)(4x + 5)$$

These are all of the terms in "the box"

	4x	5
x	$4x^2$	5x
2	8x	10

Since $4x^2$ can be factored 2 ways, look for the common factors of the 1st row.

'x' is the common factor of $4x^3$ and 5x

Look for the common factors of the 1st column

'4x' is the common factor of $4x^3$ and 8x

4x*(2) = 8x

x*(5) = 5x

Final check: 2*5 = 10?

$$3 * 8 = 24$$

These are all of the terms in “the box”

$$3x^2 + 14x + 8$$

	x	4
3x	$3x^2$	
2		8

$$2 + 12 = 14$$

Other factors of 24 that add up to 14?

$$24 = 2 * 12$$

This tells us to break 14x into ___________

$$3x^2 + 14x + 8$$

Factor

5*4 = ____

____=____* ____

12 = ____+____

$$5x^2 + 12x + 4$$

$\rightarrow$ (   )(   )

11*(-9) = ____

____=____* ____

2 = ____+____

$$11x^2 + 2x - 9$$

$\rightarrow$ (   )(   )

Factor

9*10 = ____

____=____* ____

-13 = ____+____

$$9x^2 - 13x - 10$$

12*5 = ____

$$12x^2 - 16x + 5$$

____=____* ____

-16 =

____+____= -16

$x^2 - 1$

"the difference of two squares"

$x^2 + 0x - 1$

Two numbers multiplied = (-1)
and added = 0

$(-1)(+1)$

$(x - 1)(x + 1)$

"***conjugate binomial pair***"

$$x^2 - 1 \quad \rightarrow (x - 1)(x + 1)$$

$$x^2 - 4 \quad \rightarrow (x - 2)(x + 2)$$

$$x^2 - 9 \quad \rightarrow (x - 3)(x + 3)$$

$$x^2 - 16 \quad \rightarrow (x - 4)(x + 4)$$

$$x^2 - 1 \quad \rightarrow (x-1)(x+1)$$

"***<u>conjugate binomial pair</u>***"

$$x^2 - 4 \quad \rightarrow (x-2)(x+2)$$

"***<u>conjugate binomial pair</u>***"

Follows this pattern

$$\rightarrow \left(\sqrt{x^2} - \sqrt{4}\right)\left(\sqrt{x^2} + \sqrt{4}\right)$$

What if the second term is not a "perfect square"?

$$x^2 - 2 \quad \rightarrow \left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)$$

"***<u>irrational conjugate binomial pair</u>***"

$$x^2 - 3 \quad \rightarrow \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$$

"***<u>irrational conjugate binomial pair</u>***"

"Nice" 3rd Degree Polynomial (with no constant term)

$$ax^3 + bx^2 + cx + d$$

$$3x^3 + 12x^2 - 36x$$

It has no constant term so it can easily be factored into '3x' times a quadratic factor.

$$3x(x^2 + 4x - 12)$$

If the quadratic factor is "nice" we can factor that into 2 binomials.

$$3x(x + 6)(x - 2)$$

Your turn:

$x^3 + 5x^2 + 4x$ Factor out the common factor.

$x(x^2 + 5x + 4)$ Factor the trinomial

$x(x + 4)(x + 1)$

$2x^3 - 10x^2 - 28x$ Factor out the common factor.

$2x(x^2 - 5x - 14)$ Factor the trinomial

$2x(x - 7)(x + 2)$

Another "Nice" 3rd Degree Polynomial

$$ax^3 + bx^2 + cx + d$$

This has the constant term, but it has a very useful feature:

$$1x^3 + 2x^2 + 2x + 4$$

$$1x^3 + 2x^2 + 2x + 4$$

What pattern do you see?

$$3rd/1st = 2/1 \qquad 4th/2nd = 4/2 = 2/1$$

If we divide the coefficients of the 1st and 3rd terms, and the coefficient of the 2nd term and the constant, we get the same number.

“factor by grouping” if it has this nice pattern.

$$1x^3 + 2x^2 + 2x + 4$$

Group the 1st and last pair of terms with parentheses.

$$(1x^3+2x^2) + (2x + 4)$$

Factor out the common term from the first group.

$$x^2(x + 2) + (2x + 4)$$

Factor out the common term from the last group.

$$x^2(x + 2) + 2(x + 2)$$

Common factor of (x + 2)

$$x^2(x+2)+2(x+2)$$

These two binomials are now common factors of

$$x^2 \text{ and } 2$$

Factor out the common binomial term.

$$(x+2)(x^2+2)$$

Does if follow the “nice pattern”? If so, “factor by grouping”

$$2x^3+3x^2+4x+6$$

An easier method is "box factoring" (if it has this nice pattern).

$$1x^3 + 2x^2 + 2x + 4$$

These 4 terms are the *numbers in the box*.

Find the *common factor* of the 1st row.

Fill in the rest of the box.

Rewrite as two binomials muliplied

$$(x + 2)(x^2 + 2)$$

	x	2
x^2	x^3	$2x^2$
2	$2x$	4

Which of the following 3rd degree polynomials have the "nice pattern" we saw on the previous slide?

$2x^3 - 3x^2 - 6x + 9$ $\qquad 3rd/_{1st} = 4th/_{2nd} = -3$

$4x^3 - 5x^2 + 12x - 15$ $\qquad 3^{rd}/1^{st} = 4^{th}/2^{nd} = 3$

$4x^3 + 8x^2 + 5x + 10$ $\qquad 3^{rd}/1^{st} = 4^{th}/2^{nd} = 5/4$

All of them!

Find the zeroes using "box factoring"

$$4x^3 - 5x^2 + 12x - 15$$

	$4x$	-5
x^2	$4x^3$	$-5x^2$
3	$12x$	-15

$$(4x - 5)(x^2 + 3)$$

$$2x^3 - 3x^2 + 8x - 12$$

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Imaginary Numbers

$$x = \sqrt{-1}$$

$$x^2 = \left(\sqrt{-1}\right)^2$$

$$x^2 = -1$$

Factor the following:

$$x^2 + 3$$

$$x^2 + 4$$

$$x^2 - 5$$

$$x^2 - 17$$

$$\frac{x+7}{7(x+9)}$$

No !!
Cannot use the Inverse Property of Multiplication on Addends.

Addition and Subtraction mean:

Combine the terms into one term (if you can)

If you can't combine them (unlike terms) they still are connected to each other.

Put binomials into a parentheses.

$$\frac{(x+7)}{7(x+9)}$$

Simplifying Fractions

$$\frac{84}{63}$$

Factor, factor, factor

$$\rightarrow \frac{2 * 42}{9 * 7} \rightarrow \frac{2 * 6 * 7}{3 * 3 * 7}$$

$$\rightarrow \frac{2 * 2 * \cancel{3} * \cancel{7}}{\cancel{3} * 3 * \cancel{7}} \rightarrow \frac{4}{3}$$

$$\frac{\cancel{(x+3)}(x-4)\cancel{(x+5)}}{\cancel{(x+5)}(x+4)\cancel{(x+3)}}$$

Simplifying Rational Expressions

<u>Factor, factor, factor!</u>

$$\frac{x^2 - 4}{2x - 4} \rightarrow \frac{(x-2)(x+2)}{2(x-2)} \rightarrow \frac{(x+2)}{2}$$

$$\frac{2x^2 - 8x - 24}{x^2 - x - 6}$$

Multiply

$(x-3)(x^2+3x+9$

x^3-27

	x	-3
x^2	x^3	$-3x^2$
$3x$	$3x^2$	$-9x$
9	$9x$	-27

$0x^2$ $0x$

There are NO x^2 terms and NO 'x' terms

The Difference of cubes: factors as the cubed root of each term multiplied by a 2nd degree polynomial.

x^3-1

$(x-1)(ax^2+bx+c)$

$(x-1)(x^2+x+1)$

	x	-1
x^2	x^3	$-1x^2$
$1x$	$1x^2$	$-1x$
1	$1x$	-1

$0x^2$ $0x$

The Sum of cubes: factors as the cubed root of each term multiplied by a 2nd degree polynomial. $8x^3 + 125$

$$(2x+5)(ax^2+bx+c)$$

	$2x$	5
$4x^2$	$8x^3$	$20x^2$
$-10x$	$-20x^2$	$-50x$
25	$50x$	125

$0x^2$ $0x$

$$(2x+5)(4x^2-10x+25)$$