## Math-1010

#### Lesson 4-1 Factoring Quadratics with Lead Coefficient Not = 1, Irrational and Complex Conjugates

Simplify Rational Expressions

Factor the quadratic expressions.

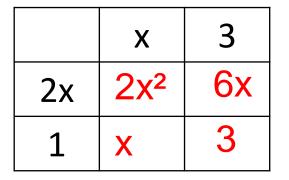
$$3x^3 + 15x^2 - 42x \qquad 5x^3 - 25x^2 - 20x$$

What if there is no common factor AND the lead coefficient is NOT equal to 1?  $a_{x}x^{2} + bx + c$ 

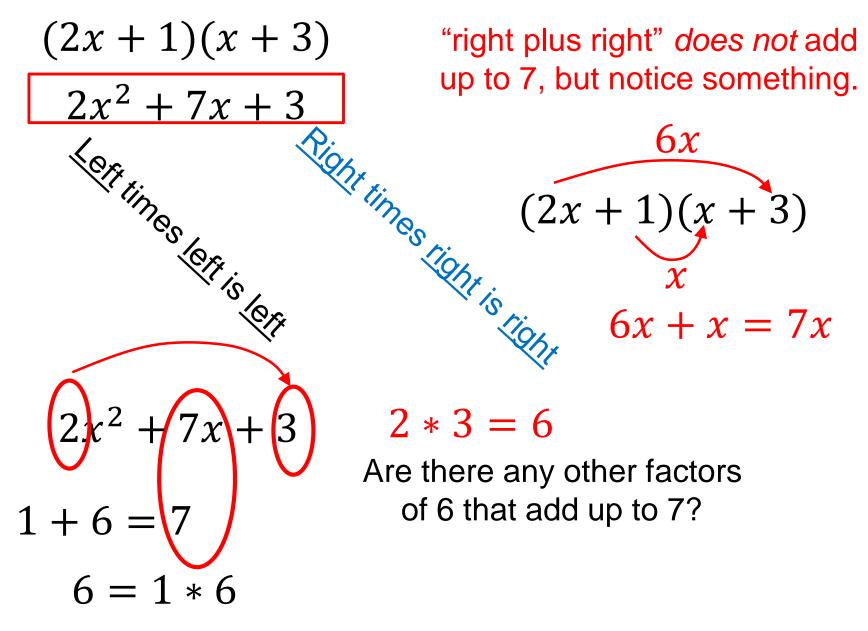
(These come from multiplying binomials that also do not have lead coefficients of 1.) (2x + 1)(x + 3)

Use the "box method" to multiply the binomials

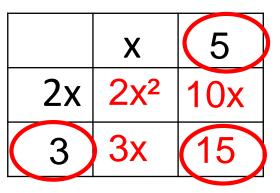
$$2x^2 + 7x + 3$$



Notice a nice pattern when you multiply this out ("simplify")



These are all of the terms in "the box"



30 = 10 \* 3

 $2x^2 + 13x$ 

10 + 3 = 13

2 \* 15 = 30

Are there any <u>other</u> factors of 30 that add up to 13?

15

This tells us to break  $\underline{13x}$  into  $\underline{10x + 3x}$ 

 $2x^2 + 13x + 15$ 

 $2x^2 + 10x + 3x + 15$ 

What is the bottom-left term in the box?

 $x^{*}(\underline{3}) = 3x$ 

What is the top-right term in the box?

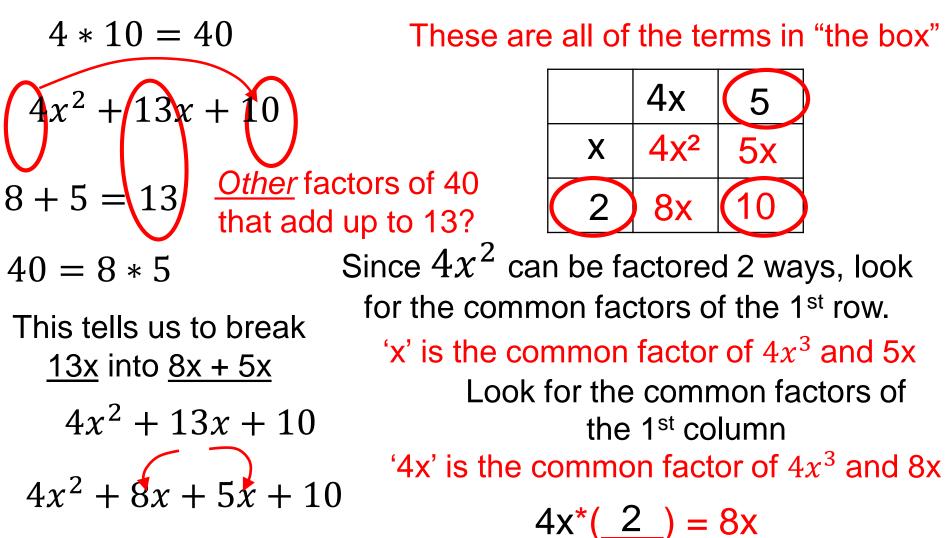
 $2x^{*}(5) = 10x$ 

Final check: 3\*5 = 15?

Factored form:

 $2x^2 + 13x + 15$ 

 $\rightarrow (2x+3)(x+5)$ 



Factored form:

 $4x^2 + 13x + 10$  $\rightarrow (x+2)(4x+5)$  Final check: 2\*5 = 10?

 $x^{*}(5) = 5x$ 

These are all of the terms in "the box"

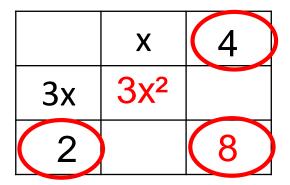
$$3x^{2} + 14x + 8$$

$$2 + 12 = 14$$

$$\frac{0 \text{ ther}}{14} \text{ factors of } 24$$

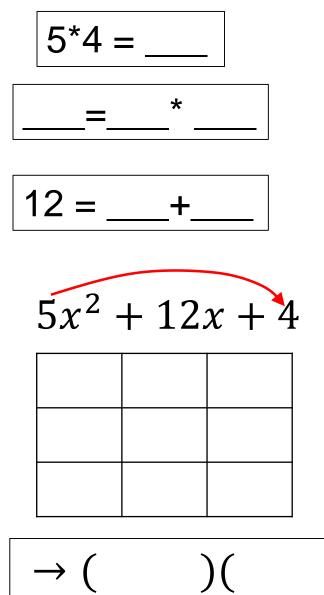
$$24 = 2 * 12$$

3 \* 8 - 24

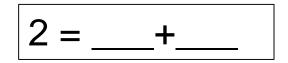


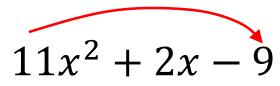
This tells us to break  $\underline{14x}$  into  $\underline{3x^2 + 14x + 8}$ 

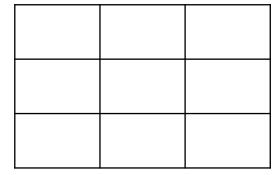
#### Factor





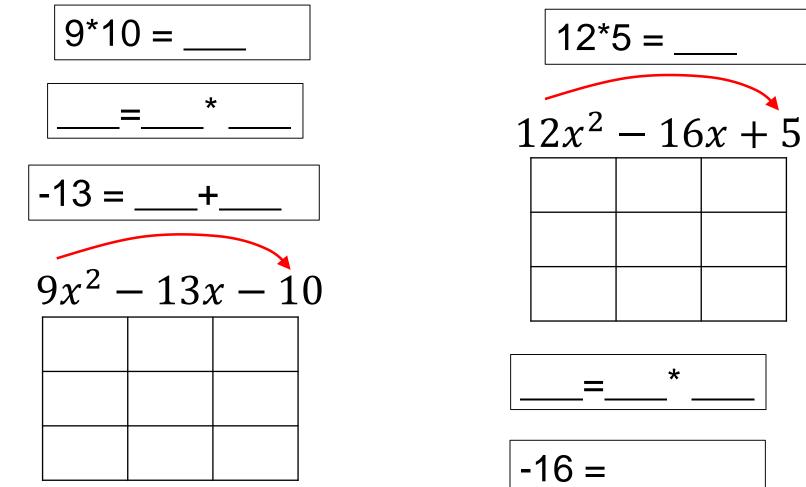








Factor



$$x^2 - 1$$

"the difference of two squares"

$$x^{2} + 0x - 1$$
 Two numbers multiplied = (-1)  
and added = 0  
 $(-1)(+1)$ 

(x-1)(x+1) "conjugate binomial pair"  $x^2 - 1 \rightarrow (x - 1)(x + 1)$  $x^2 - 4 \rightarrow (x - 2)(x + 2)$  $x^2 - 9 \rightarrow (x - 3)(x + 3)$  $x^2 - 16 \rightarrow (x - 4)(x + 4)$ 

$$x^{2} - 1 \rightarrow (x - 1)(x + 1)$$
  
"conjugate binomial pair"  

$$x^{2} - 4 \rightarrow (x - 2)(x + 2)$$
  
"conjugate binomial pair"

Follows this pattern

$$\rightarrow \left(\sqrt{x^2} - \sqrt{4}\right) \left(\sqrt{x^2} + \sqrt{4}\right)$$

What if the second term is not a "perfect square"?

$$x^{2} - 2 \rightarrow (x - \sqrt{2})(x + \sqrt{2})$$
  
"irrational conjugate binomial pair"  

$$x^{2} - 3 \rightarrow (x - \sqrt{3})(x + \sqrt{3})$$
  
"irrational conjugate binomial pair"

"Nice" 3rd Degree Polynomial (with no constant term)

$$ax^{3} + bx^{2} + cx + d$$
  
 $3x^{3} + 12x^{2} - 36x$ 

It has no <u>constant</u> term so it can easily be factored into '3x' times a quadratic factor.

$$3x(x^2 + 4x - 12)$$

**If** the quadratic factor is "nice" we can factor that into 2 binomials.

$$3x(x+6)(x-2)$$

#### Your turn:

$x^3 + 5x^2 + 4x$	Factor out the common factor.		
$x(x^2 + 5x + 4)$	Factor the trinomial		
x(x+4)(x+1)			
$2x^3 - 10x^2 - 28x$	$\chi$ Factor out the common factor.		
$2x(x^2 - 5x - 14)$	Factor the trinomial		
2x(x-7)(x+2)			

# Another "Nice" 3<sup>rd</sup> Degree Polynomial $ax^3 + bx^2 + cx + d$

This has the <u>constant</u> term, but it has a very useful feature:

$$1x^{3} + 2x^{2} + 2x + 4$$
$$1x^{3} + 2x^{2} + 2x + 4$$

What pattern do you see?

$$\frac{3rd}{1st} = \frac{2}{1}$$
  $\frac{4th}{2nd} = \frac{4}{2} = \frac{2}{1}$ 

If we divide the coefficients of the 1<sup>st</sup> and 3<sup>rd</sup> terms, and the coefficient of the 2<sup>nd</sup> term and the constant, we get the <u>same number</u>.

"factor by grouping" if it has this nice pattern.

$$1x^3 + 2x^2 + 2x + 4$$

Group the 1<sup>st</sup> and last pair of terms with parentheses.

$$(1x^3+2x^2) + (2x+4)$$

Factor out the common term from the first group.

$$x^2(x+2) + (2x+4)$$

Factor out the common term from the last group.

$$x^{2}(x+2) + 2(x+2)$$

Common factor of (x + 2)

 $x^{2}(x+2) + 2(x+2)$ 

These two binomials are now common factors of

$$x^2$$
 and 2

Factor out the common binomial term.

$$(x+2)(x^2+2)$$

Does if follow the "nice pattern"? If so, "factor by grouping"

$$2x^3 + 3x^2 + 4x + 6$$

An easier method is "box factoring" (if it has this nice pattern).

$$1x^3 + 2x^2 + 2x + 4$$

These 4 terms are the *numbers in the box*.

Find the *common factor* of the 1<sup>st</sup> row.

Fill in the rest of the box.

Rewrite as two binomials muliplied

 $(x+2)(x^2+2)$ 

<u> </u>		
	x	2
$x^2$	<i>x</i> <sup>3</sup>	$2x^2$
2	2x	4

Which of the following 3<sup>rd</sup> degree polynomials have the "nice pattern" we saw on the previous slide?

$$2x^{3} - 3x^{2} - 6x + 9 \quad \frac{3rd}{1st} = \frac{4th}{2nd} = -3$$

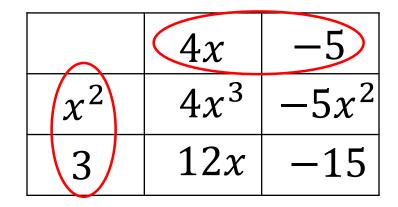
$$4x^{3} - 5x^{2} + 12x - 15 \quad \frac{3^{rd}}{1^{st}} = \frac{4^{th}}{2^{nd}} = 3$$

$$4x^{3} + 8x^{2} + 5x + 10 \quad \frac{3^{rd}}{1^{st}} = \frac{4^{th}}{2^{nd}} = \frac{5}{4}$$

### All of them!

Find the zeroes using "box factoring"

$$4x^3 - 5x^2 + 12x - 15$$
$$(4x - 5)(x^2 + 3)$$



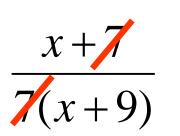
$$2x^3 - 3x^2 + 8x - 12$$
())

**Imaginary Numbers** 

$$x = \sqrt{-1}$$
$$x^{2} = (\sqrt{-1})^{2}$$
$$x^{2} = -1$$

Factor the following:

 $x^{2} + 3$  $x^{2} + 4$  $x^{2} - 5$  $x^{2} - 17$ 



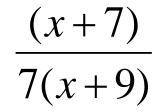
No !!  $\frac{x+7}{7(x+9)}$  Cannot use the Inverse Property of Multiplication on Addends.

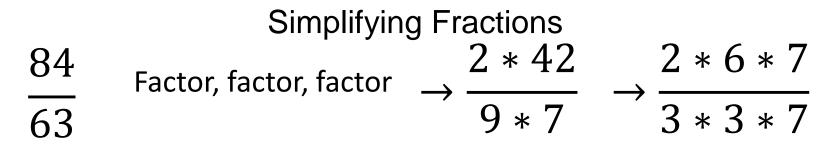
### Addition and Subtraction mean:

Combine the terms into <u>one term</u> (if you can)

If you can't combine them (unlike terms) they still are connected to each other.

Put binomials into a parentheses.

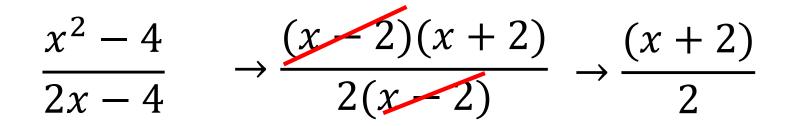




$$\rightarrow \frac{2 * 2 * 3 * 7}{3 * 3 * 7} \rightarrow \frac{4}{3}$$

$$\frac{(x+3)(x-4)(x+5)}{(x+5)(x+4)(x+3)}$$

Simplifying Rational Expressions <u>Factor, factor, factor!</u>



$$\frac{2x^2-8x-24}{x^2-x-6}$$

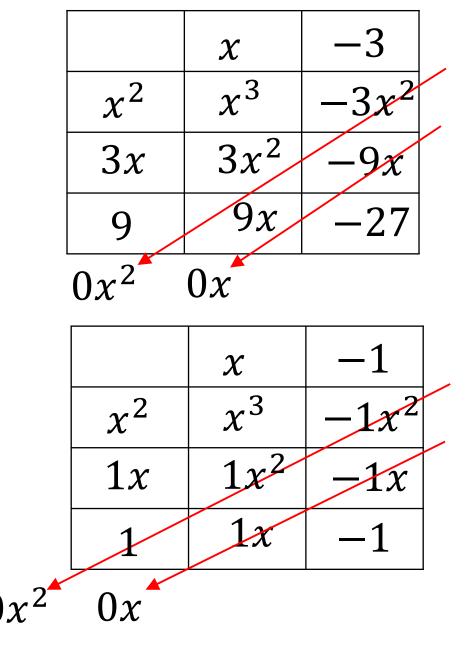
**Multiply** 

$$(x-3)(x^2+3x+9)$$
  
 $x^3-27$ 

There are NO  $\chi^2$  terms and NO 'x' terms

<u>The Difference of cubes</u>: factors as the cubed root of each term multiplied by a  $2^{nd}$  degree polynomial.  $x^3 - 1$ 

$$(x-1)(ax^{2} + bx + c)$$
$$(x-1)(x^{2} + x + 1)$$



The Sum of cubes: factors as the cubed root of each term 5 2xmultiplied by a 2<sup>nd</sup> degree  $4x^{2}$  $8x^3$ 20xpolynomial.  $8x^3 + 125$ -20x-10xY  $(2x+5)(ax^2+bx+c)$ 50x12525 **Ο**γ

$$(2x+5)(4x^2-10x+25)$$