Math-1010 Lesson 3-3 (Textbook 3.3 and 3.4) (National Debt and Population Growth

Year	Total National Debt as of 30 Sept (in trillions of $)	"this year's debt" ÷ "last year's debt"
2005	8.50	n/a
2006	9.52	1.12
2007	10.66	1.12
2008	11.94	1.12
2009	13.37	1.12
2010	14.98	1.12
2011	16.78	1.12

Is the data linear or is it exponential?

How can you tell the difference between linear data and exponential data?

What is the growth factor?

$$\frac{\text{output}_{\text{next}}}{\text{output}_{\text{previous}}} = \text{growth factor}$$

Linear data: $\frac{\Delta \text{ output}}{\Delta \text{ input}} = \text{constant}$

Exponential data: $\frac{\text{output}_{\text{next}}}{\text{output}_{\text{previous}}} = \text{constant}$

How would you change the input values so that the relation is easier to graph?

Replace "year" with "years since 2005"

Graph the following data

Years since 2005	Total National Debt as of 30 Sept (in trillions of $)	"this year's debt" ÷ "last year's debt"
0	8.50	n/a
1	9.52	1.12
2	10.66	1.12
3	11.94	1.12
4	11.37	1.12
5	14.98	1.12
6	16.78	1.12

Exponential data:

$$\frac{\text{output}_{\text{next}}}{\text{output}_{\text{previous}}} = \text{constant}$$

$$= \text{growth factor}$$

$$= 1.12$$

Does the following equation pass through the data? $D(t) = 1.12^x$

Why not?

The y-intercept for the equation is (0, 1) and the y-intercept for the data is (0, 8.5).

Complete the following table.

t	CALCULATION OF THE NATIONAL DEBT	EXPONENTIAL FORM	NATIONAL DEBT (in trillions of dollars)
0	8.50	$8.50(1.12)^0$	8.50
1	$(8.50)1.12$	$8.50(1.12)^1$	9.52
2	$9.52(1.12)$	$8.50(1.12)^2$	10.66
3	$10.66(1.12)$	$8.50(1.12)^3$	11.94

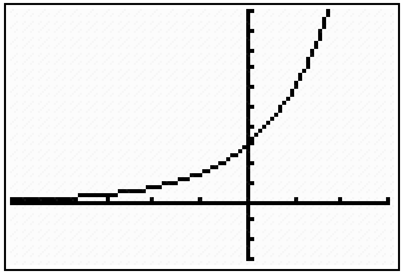
Use the pattern in the table to adjust the following equation so that it "fits" the data. $D(t) = 1.12^x$

$$D(t) = 8.50(1.12)^x$$

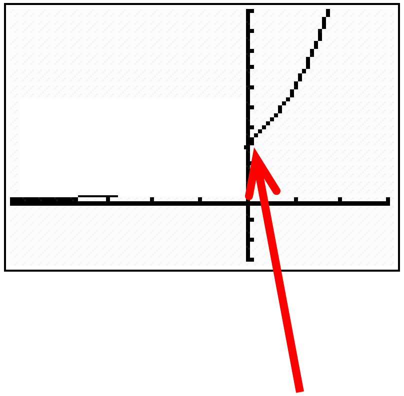
In general, what do we call the coefficient of the exponential function?

Vocabulary

Initial Value: (of the exponential) is the vertical stretch factor.



If in input is time ("stopwatch time") the initial value occurs when t = 0.



$$f(t) = 3(2)^t$$

Domain: x = [0, ∞)

$$f(0) = 3(2)^0 = ?$$

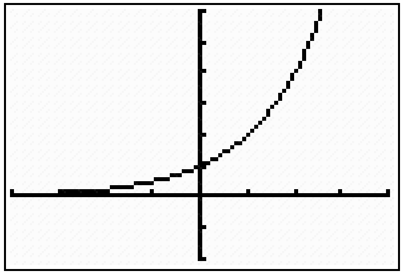
$$f(0) = 3$$

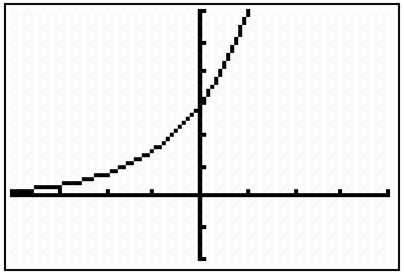
<u>General Form</u> of an "any base" exponential function:

$$f(x) = ab^x$$

Initial value of the function

Growth factor





f(0) = ? = 1

y-intercept

$$f(x) = 1(b)^x$$

f(0) = ? = 3

y-intercept

$$f(x) = 3(b)^x$$

Exponential Growth 'b' > 1

$$f(x) = ab^x$$

$$f(x) = 3(2)^x$$

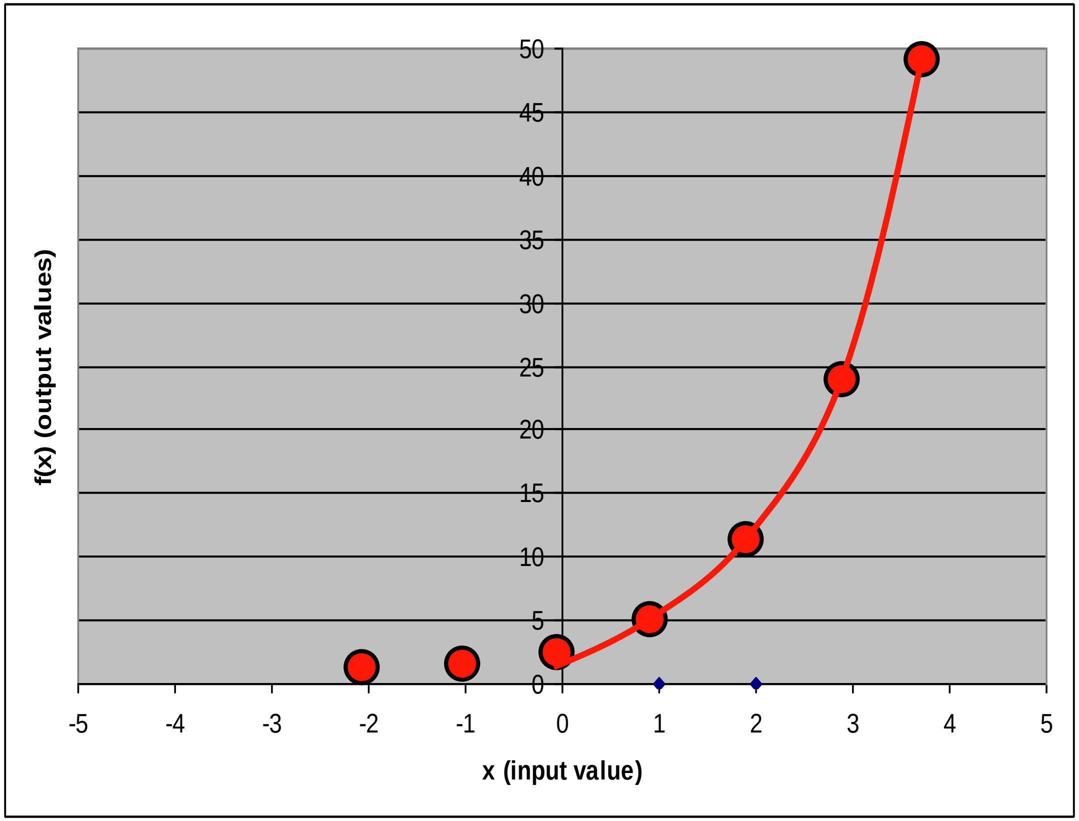


Table of values

x	$3(2)^x$	f(x)
-2	$3(2)^{-2}$	0.75
-1	$3(2)^{-1}$	1.5
0	$3(2)^0$	3
1	$3(2)^1$	6
2	$3(2)^2$	12
3	$3(2)^3$	24
4	$3(2)^4$	48

$\frac{1}{2}$, $\frac{1}{2}$, 2, 2, 2, 2

'a' is the initial value → f(0) = 'a'

'b' is called the growth factor

Exponential Decay

$$f(x) = ab^x$$

$$f(x) = 4(0.5)^x$$

0 < ‘b’ < 1

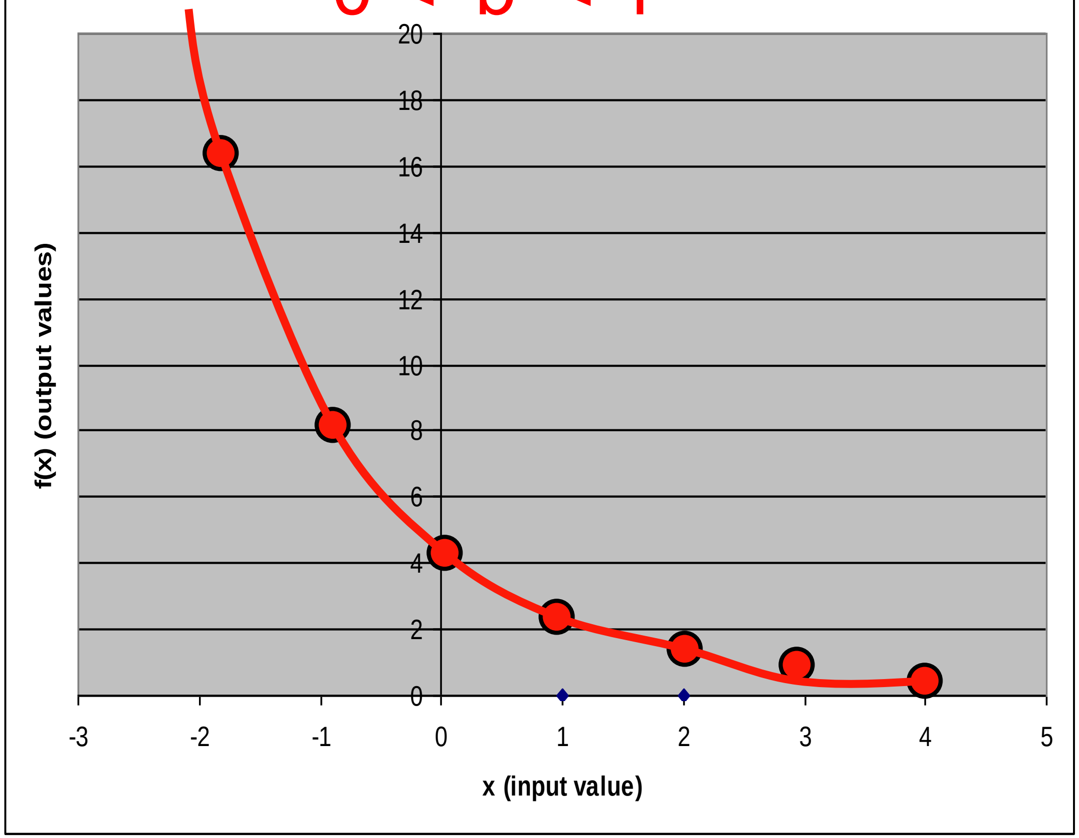


Table of values

x	$4(0.5)^x$	f(x)	
-2	$4(0.5)^{-2}$	16	2
-1	$4(0.5)^{-1}$	8	2
0	$4(0.5)^{0}$	4	½
1	$4(0.5)^{1}$	2	½
2	$4(0.5)^{2}$	1	½
3	$4(0.5)^{3}$	0.5	½
4	$4(0.5)^{4}$	0.25	

‘a’ is the initial value → f(0) = ‘a’

‘b’ is called the growth factor

Exponential Data: what is the equation? $f(x) = ab^x$

x	$g(x)$		$h(x)$	
-2	$4/9$		128	
		$\times 3$		$\times \frac{1}{4}$
-1	$4/3$		32	
		$\times 3$		$\times \frac{1}{4}$
0	4		8	
		$\times 3$		$\times \frac{1}{4}$
1	12		2	
		$\times 3$		$\times \frac{1}{4}$
2	36		$1/2$	

x-values increment by <u>one</u> each time.

y-values increment by the same <u>factor</u> each time.

This number is the "<u>growth factor</u>" (base of the exponential)

Find the function for this data.

x	$g(x)$		$h(x)$	
-2	$4/9$	$\times 3$	128	$\times \frac{1}{4}$
-1	$4/3$	$\times 3$	32	$\times \frac{1}{4}$
0	4	$\times 3$	8	$\times \frac{1}{4}$
1	12	$\times 3$	2	$\times \frac{1}{4}$
2	36		$1/2$	

"g(x)' is exponential → $g(x) = ab^x$

→growth factor = 3 $b = 3$

→initial value = 4 $a = 4$

$$g(x) = 4(3)^x$$

x	$g(x)$	$h(x)$
-2	4/9	128
	$\times 3$	$\times \frac{1}{4}$
-1	4/3	32
	$\times 3$	$\times \frac{1}{4}$
0	4	8
	$\times 3$	$\times \frac{1}{4}$
1	12	2
	$\times 3$	$\times \frac{1}{4}$
2	36	1/2

Find h(x)

h(x)' is exponential → $h(x) = ab^x$

→growth factor = 1/4 $b = 1/4$

→initial value = 8 $a = 8$

$$h(x) = 8\left(\frac{1}{4}\right)^x \qquad h(x) \neq \left(\frac{8}{4}\right)^x$$ → Why?

You buy a new car for $22,000 (ouch). Unfortunately your car will <u>depreciate</u> by 30% each year. What will the car be worth in 4 years?

t (year)	0	1	2	3	4
V(*t*) (thousands of dollars)	22	15.4	10.8	7.5	5.3

$22(0.70)^1$ $15.4(0.70)^1$ $10.8(0.70)^1$ $7.5(0.70)^1$

$22(0.70)^2$

$22(0.70)^3$

$22(0.70)^4$

If it is depreciates 30% in 1 year, what percentage of the original amount is it worth?

70% of the original amount after one year

You deposit $100 money into an account that pays 3.5% interest per year. How much money will be in the account at the end of the 1st year?

$$A(1) = \$100 + \$100(0.035)$$

Original amount ($100)will still be in the account.

There will be a small amount of growth(3% of $100)

Factor out the <u>common factor $100</u>

$$A(1) = \$100(1 + 0.035) = \$100(1.035)$$

$$A(2) = \$100(1.035)^2$$

$$A(3) = \$100(1.035)^3$$

$$A(t) = \$100(1.035)^t$$

$$A(t) = A_0(1 + r)^t$$

A bank pays 3% interest per year, and they pay you each month, what is the monthly interest rate?

$$\frac{0.03}{\cancel{year}} * \frac{\cancel{year}}{12\ months} \rightarrow \frac{0.03}{12\ month} \rightarrow \frac{0.03}{12} per\ month$$

$$\rightarrow 0.0025\ per\ month$$

A bank pays 5% interest per year, and they pay you each month, what is the monthly interest rate?

$$0.05\ \ per\ year \ \rightarrow \frac{0.05}{12} per\ month \rightarrow 0.0042 \quad per\ month$$

The exponential growth equation for money in a bank for account where the bank pays you more frequently than at the end of the year is:

$$A(t) = A_0(1 + r/k)^{k*t}$$

- Amount of $$ in the account as a function of time → $A(t)$
- Initial value → A_0
- Annual interest rate → r
- Years after the deposit → t
- # of times the bank pays you each year → k

"Compounding period" → the number of times the bank pays you each year.

Values of "k"	
Words to look for	K
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Daily	365

"A bank pays 3% per year compounded monthly."

$$A(t) = A_0(1 + 0.03/12)^{12*t}$$

You deposit $100 money into an account that pays 3.5% interest per year. The interest is “compounded” monthly. How much money will be in the account at the end of the 5th year?

$$A(t) = A_0(1 + r/k)^{k*t}$$

$$A(t) = A_0(1+r)^t$$

$$A(5) = 100\left(1 + \frac{0.035}{12}\right)^{12*5}$$

$$A(5) = 100(1+0.035)^{(5)}$$

$$A(5) = \$119.09$$

$$A(5) = \$118.77$$

Interest paid at the end of each month

Interest paid at the end of each year

You deposit $200 money into an account that pays 5.5% interest per year. How much money will be in the account at the end of the 20th year?

$$A(t) = A_0(1+r)^t$$

$$A(20) = \$200(1+0.055)^{(20)}$$

$$A(20) = \$583.55$$

You buy a car for $18,500. It depreciates at 15% per year. What is the value of the car (what you could sell it for) after 7 years?

$$V(t) = V_0(1-r)^t$$

$$V(t) = 18{,}500(1-0.15)^{(t)}$$

$$V(t) = 18{,}500(0.85)^{(t)}$$

What is the growth factor?

Is it “growth” or “decay”?

$$V(7) = 18{,}500(0.85)^{(7)}$$

$$V(7) = \$5930.68$$

The population of a town grows at 5%. In 1990 the population was 750. What will be the population if 2020?

$$P(t) = P_0(1+r)^t$$

$$P(t) = 750(1.05)^{(t)}$$

$$P(30) = 750(1.05)^{(30)}$$

What is the growth factor?

Is it “growth” or “decay”?

$$P(30) = 3241$$