Math-1010
Lesson 3-2

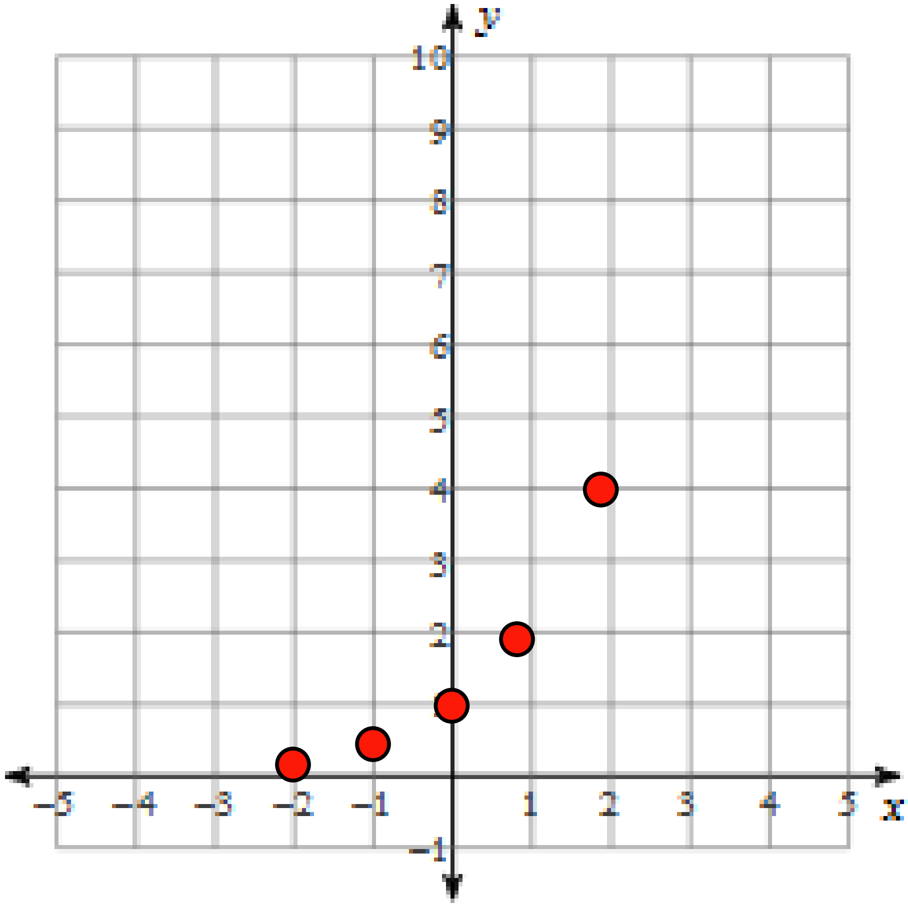
(Textbook Sections 3.1 and 3.2)
The Exponential Function

What does the graph look like?

$$f(x) = 2^x$$

Fill in the rest of the table.

x	2^x	y
-2	2^{-2}	0.25
-1	2^{-1}	0.5
0	2^0	1
1	2^1	2
2	2^2	4



Why does $2^{-2} = 0.25$?

$$\left(\frac{2}{1}\right)^{-2} = \left(\frac{1}{2}\right)^{2} = \frac{1}{4} = 0.25$$

negative exponent property

Why does $2^0 = 1$?

zero exponent property

How fast does it grow? $f(x) = 2^x$

When the integer input value increases by 1 unit, the y-value becomes twice as big.

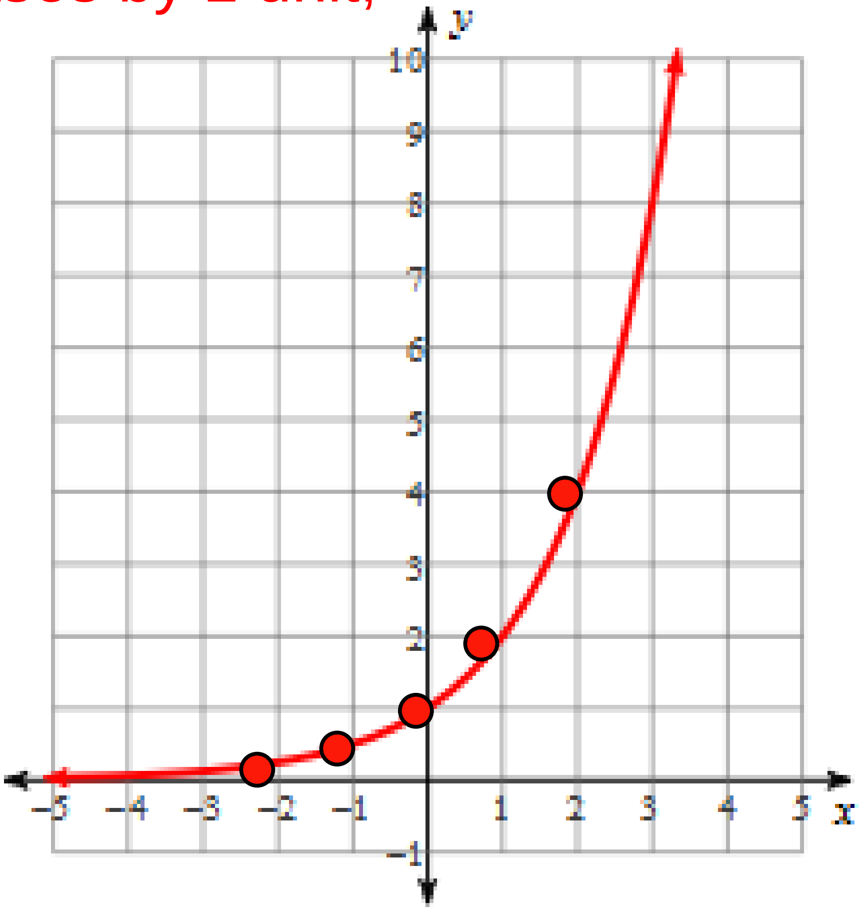
x	2^x	y
-2	2^{-2}	0.25
-1	2^{-1}	0.5
0	2^0	1
1	2^1	2
2	2^2	4

2
2
2
2

What do we call the number "2" in the original equation?

1) "Base" of the exponential.

2) "Growth Factor" of the exponential function.



What is the y-intercept?

$$f(0) = 2^0 = 1$$

Exponential Function $f(x) = 2^x$

Fill in the rest of the table:

x	2^x	y
-5	2^{-5}	≈ 0.03
-10	2^{-10}	≈ 0.001
-15	2^{-15}	≈ 0.00003
-20	2^{-20}	≈ 0.000001
-25	2^{-25}	≈ 0.00000003

$$\frac{1}{2^5} = \frac{1}{32}$$

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$$\frac{1}{2^5} = \frac{1}{32}$$

→What conclusion can you come to by looking at the trend in the output values?

Exponential Function $f(x) = 2^x$

Will the 'y' value ever reach zero (on the left end of the graph)?

Why not?

As the input value becomes a larger negative number,

2^{-5} 2^{-10} 2^{-15} 2^{-20}

→ denominator gets bigger

$1/2^5 = 1/32$ $1/2^{10} = \frac{1}{1024}$ $1/2^{15}$ $1/2^{20}$

→decimal equivalent of the fraction gets smaller.

≈ 0.03 ≈ 0.001 ≈ 0.00003 ≈ 0.000001

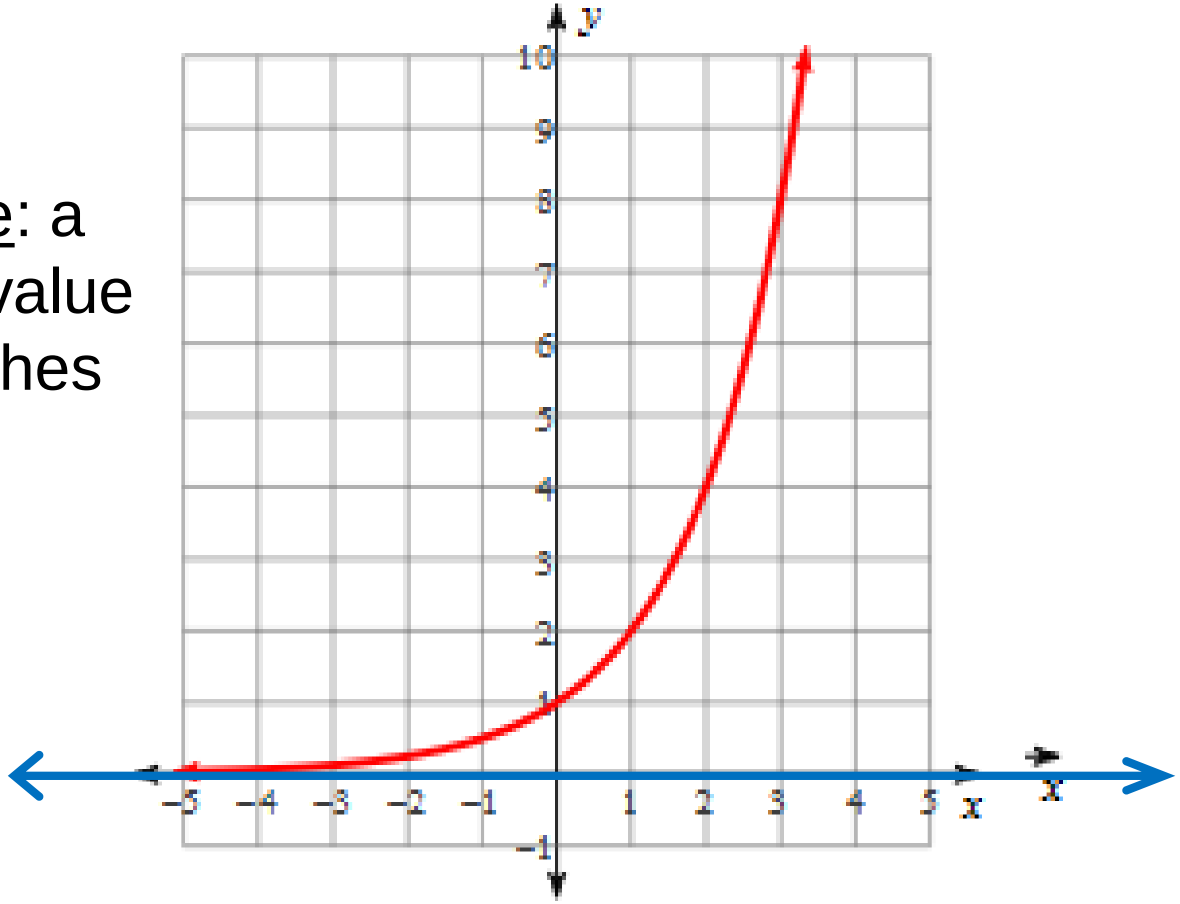
→ decimal equivalent of the of fraction approaches zero but NEVER reaches zero.

The denominator of a fraction can never make a fraction = 0.

$$f(x) = 2^x$$

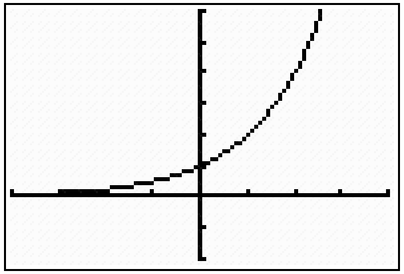
Vocabulary

<ins>Horizontal Asymptote</ins>: a horizontal line the y-value of the graph approaches but never reaches.



<u>General Form</u> of an
"any base" exponential function:

$$f(x) = b^x$$



base of the exponential

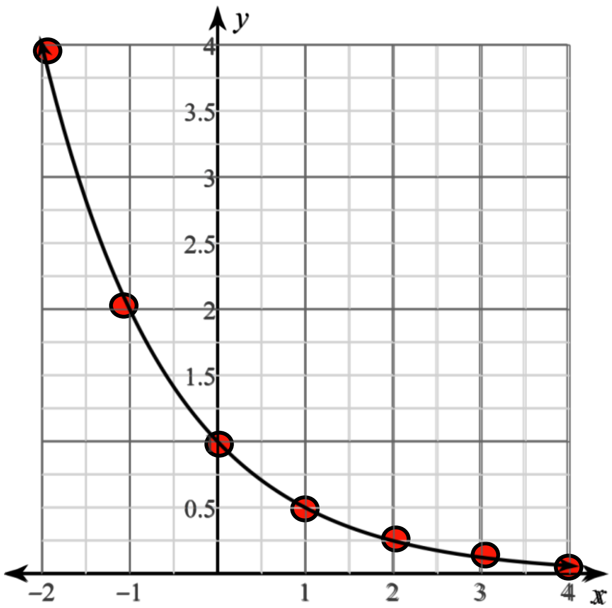
$$Domain = ? \quad x = (-\infty, \infty)$$

$$range = ? \quad y = (0, \infty)$$

y-intercept = ? f(0) = 1

Growth factor = ? b, the base of the exponential

What does the graph look like?



As the integer input value increases by a value of one, by what factor is the output value changing by?

What is the y-intercept?

$$f(x) = b^x$$

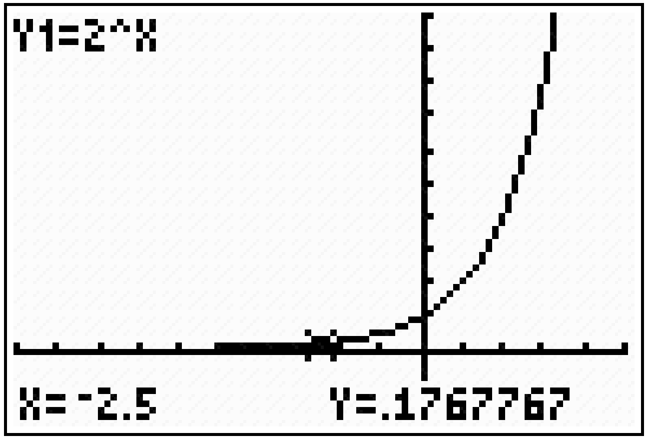
$$f(x) = (0.5)^x$$

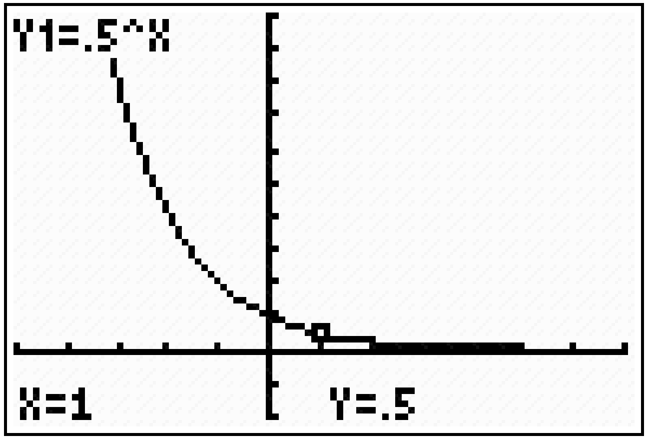
Fill in the Table of values

x	$(0.5)^x$	f(x)
-2	$(0.5)^{-2}$	4
-1	$(0.5)^{-1}$	2
0	$(0.5)^0$	1
1	$(0.5)^1$	0.5
2	$(0.5)^2$	0.25
3	$(0.5)^3$	0.125
4	$(0.5)^4$	0.0625

½ ½ ½ ½ ½ ½

Exponential Growth and Decay





$$f(x) = b^x$$

For what <u>interval of values </u>of growth factor ‘b’ will result in <u>exponential growth</u> ?

b > 1

For what <u>interval of values </u>of growth factor ‘b’ will result in <u>exponential decay</u> ?

0 < b < 1

Exponential Data: what is the equation?

x	f(x)
-2	9
-1	3
0	1
1	0.333
2	0.111
3	0.037
4	0.0124

x-values increment by one each time.

y-values increment by the same factor each time.

⅓ ⅓

This number is the "growth factor"

"growth factor": the "base" of the exponential.

Calculate the base:

$$base = \frac{y_{next}}{y_{previous}}$$

$$base = \frac{3}{9} = \frac{1}{3} = \frac{0.3333}{1}$$

$$f(x) = \left(1/3\right)^x$$

Exponential Data: what is the equation?

x	f(x)
-2	0.0625
-1	0.25
0	1
1	4
2	16
3	64
4	256

4

4

What is the base of the exponential function?

$$base = \frac{y_{next}}{y_{previous}}$$

$$base = \frac{16}{4} = \frac{4}{1} = \frac{1}{0.25}$$

The "growth factor" is the base of the exponential.

$$f(x) = 4^x$$

Exponential Data: what is the equation?

x	f(x)
-2	0.44444
-1	0.66667
0	1
1	1.5
2	2.25
3	3.375
4	5.0625

$$base = \frac{y_{next}}{y_{previous}}$$

1.5

$$base = \frac{2.25}{1.5} = \frac{1.5}{1} = \frac{1}{0.6667}$$

$$f(x) = 1.5^x$$

In what portion of the table is it easier to find the growth factor?

When comparing the growth (or decay) between x = 0 and either x = -1 or x = 1

The "amount" as a function of time (since the 1st data point)

t	A(t)
0	$A_0 = 5$
1	$A_1 = 5.5$
2	
3	
4	

1.1

$$A_1 = A_0 + r * A_0)$$

$$A_1 = A_0(1 + r)$$

Amount (at t = 0)

Rate of change

$$5.5 = 5(1 + r)$$

Rate of change = ?

$$\frac{5.5}{5} = 1 + r$$

Growth factor = ?

Rate of change = 0.1

$$1.1 - 1 = r$$

Growth factor = (1 + r)

Growth factor = (1 + 0.1)

Growth factor = (1.1)

The "amount" as a function of time (since the 1st data point)

t	A(t)
0	$A_0 = 5$
1	$A_1 = 5.5$
2	$A_2 = 6.05$
3	
4	

1.1

1.1

$$A_2 = A_1 + r * A_1)$$

$$A_2 = A_1(1 + r)$$

Amount (at t = 0)

Rate of change

$$A_2 = 5.5(1.1)$$

$$A_2 = 5(1.1)^2$$

$$A_2 = A_0(1.1)^2$$

$$A_2 = 6.05$$

The "amount" as a function of time (since the 1st data point)

t	A(t)
0	$A_0 = 5$
1	$A_1 = 5.5$
2	$A_2 = 6.05$
3	$A_3 = 6.655$
4	

1.1

1.1

1.1

$$A_3 = A_2 + r * A_2)$$

$$A_3 = A_2(1 + r)$$

Amount (at t = 0)

Rate of change

$$A_3 = 6.05(1.1)$$

$$A_3 = A_0(1.1)^3$$

$$A_3 = 6.655$$

The "amount" as a function of time (since the 1st data point)

t	A(t)
0	$A_0 = 5$
1	$A_1 = 5.5$
2	$A_2 = 6.05$
3	$A_3 = 6.655$
4	$A_4 = 7.3205$

1.1

1.1

1.1

1.1

$$A_4 = A_3 + r * A_3)$$

$$A_4 = A_3(1 + r)$$

Amount (at t = 0)

Rate of change

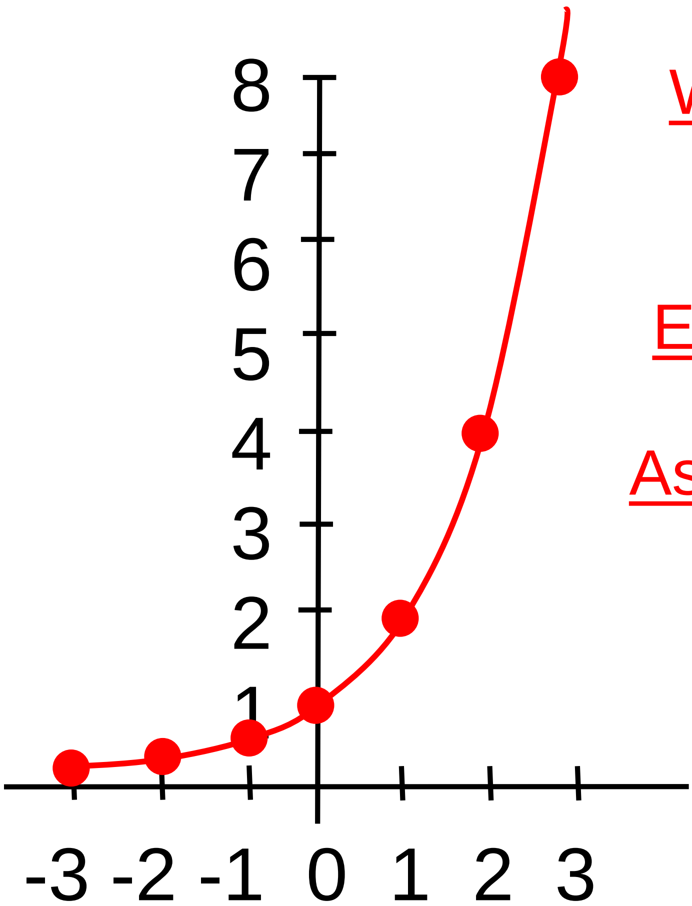
$$A_4 = 6.655(1.1)$$

$$A_4 = A_0(1.1)^4$$

$$A_3 = 7.3205$$

Exponential function

$$f(x) = (2)^x$$



Domain? $D: \; x = (-\infty, \infty)$

Range? $R: \; y = (0, \infty)$

Where increasing?

$$f(x) \uparrow \text{ for } x = (-\infty, \infty)$$

Extrema? $none$

Asymptotes? $y = 0$

Average rate of change:

from x = 0 to x = 1? $\dfrac{\Delta y}{\Delta x} = \dfrac{1}{1} = 1$

from x = 0 to x = 2? $\dfrac{\Delta y}{\Delta x} = \dfrac{3}{2}$

from x = 0 to x = 3? $\dfrac{\Delta y}{\Delta x} = \dfrac{7}{3}$

Half-Life of Medicine: After taking a dose of medicine, your body metabolizes the medicine and it decays away. The time it takes for the medicine in your body to decay to ½ of its original value.

$$A(t) = A_0 (1/2)^t$$

A_o means the "original amount"

A(t) means the amount as a function of time (how much is in your body at any given time after t = 0).

<u>Half-Life of Medicine:</u> After taking a dose of medicine, your body metabolizes the medicine and it decays away. The <u>*half-life*</u> is the time it takes for the medicine in your body to decay to ½ of its original value.

If a medicine has a half-life of 4 days, how much medicine will be in your body after: 4 days? 8 days? 16 days?

Build a table of values for this relation.

Number of days since 1st dose	Number of ½ lives since 1st dose	Fraction of medicine remaining in your body
0	0	1 (100%)
4	1	$\frac{1}{2}$
8	2	$\frac{1}{4}$
16	4	$\frac{1}{8}$

½

½

½

You are injected with 5 ml of radioactive Iodine for a thyroid scan. The half-life of the iodine in your blood is 40 hours.

a) Build a table of values for this relation.

b) What is the equation that models the amount of iodine in your blood as a function of time? $A(t) = A_0(0.5)^t$

c) How much iodine will be in your blood in 10 days?

d) Draw a graph of this scenario

$$10 * days = 6 * \frac{1}{2} lives$$

Number of hours since 1st dose	Number of ½ lives since 1st dose	Fraction of medicine remaining in your body
0	0	1 (100%)
40	1	$^1/_2$
80	2	$^1/_4$
160	4	$^1/_8$

½ ½ ½

$$A(6) = A_0(0.5)^6$$

$$A(6) = 0.015625A_0$$

$$A(6) = 1.5625\% \; A_0$$

$A = amount\ (\%\ of\ original)$

$t = time\ (number\ of\ \frac{1}{2}\ lives)$

$$10 * days * \frac{24 * hrs}{1 * day} * \frac{1 * \frac{1}{2} life}{40 * hrs}$$

Comparing Rates of Growth: How would you figure this out?

Which function grows the fastest?

Table of values?

$$f(x) = 2^x \qquad g(x) = x^2 \qquad h(x) = x^3$$

x	f(x)
1	2
2	4
3	8
4	16

x	g(x)
1	1
2	4
3	8
4	16

x	h(x)
1	1
2	8
3	27
4	64

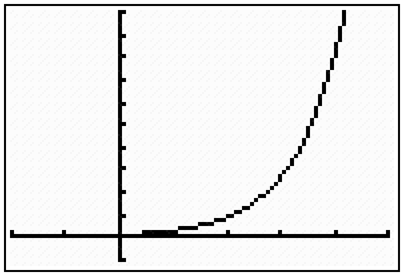
<u>Comparing Rates of Growth</u>:

Which function grows the fastest?

How would you figure this out?

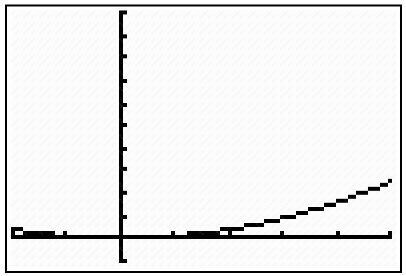
Graph?

$f(x) = 3^x$



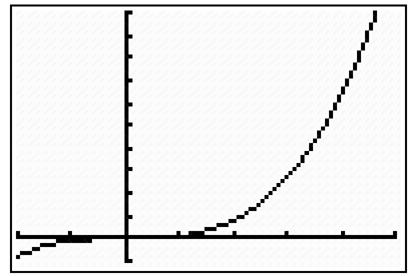
$y = [-10, 100]$

$g(x) = x^2$

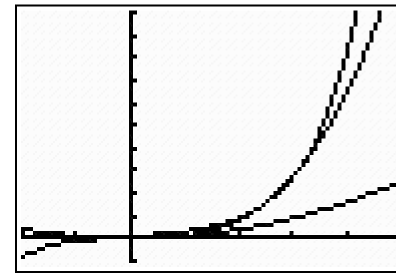


$y = [-10, 100]$

$h(x) = x^3$

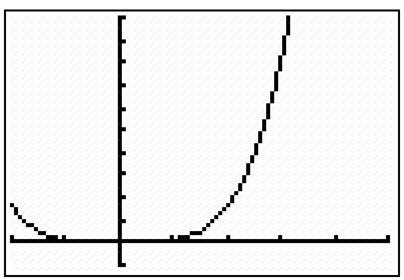


$y = [-10, 100]$



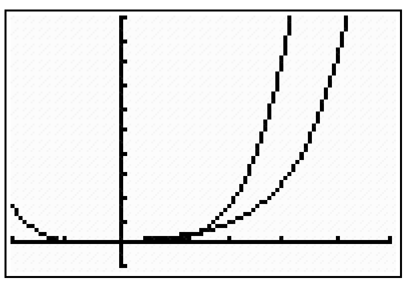
$y = [-10, 100]$

$k(x) = x^4$

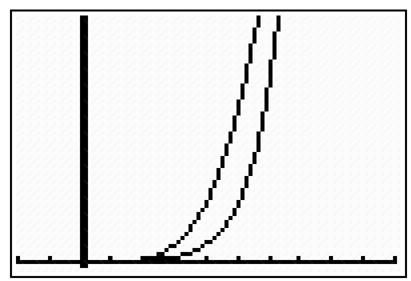


$y = [-10, 100]$

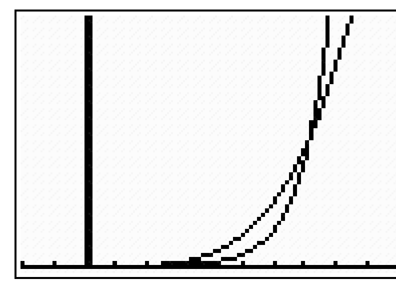
$f(x) = 3^x$



$y = [-10, 100]$



$y = [-10, 1000]$



$y = [-10,\ 10{,}000]$