Math 1010
Lesson 3-1

Function Composition
And Introduction to Inverse Functions

$$f(x) = x^2 - 3x + 2$$ → f(-2) = ?

Means: wherever you see an ‘x’ in the function, replace it with a ‘-2’.

1. Replace the ‘x’ with a set of parentheses.

$$f(x) = (\quad)^2 - 3(\quad) + 2$$

2. Put the input value ‘-2’ into the parentheses.

$$f(x) = (-2)^2 - 3(-2) + 2$$

3. Find the output value.

f(-2) = 12

Find the outputs

$f(x) = 2x + 1$

$f(2) = 5$

$f(3) = 7$

$f(x - 1) = 2x - 1$

$f(3x) = 6x + 1$

(Input) x	(rule) 2x + 1	(output) f(x)
2	2(2) + 1	5
3	2(3) + 1	7
x – 1	2(x – 1) + 1	2x – 1
3x	2(3x) + 1	6x + 1

If your input is an <u>expression</u> instead of a number: replace 'x' with parentheses and "plug in" the expression.

Find the outputs $f(x) = x^2 + 1$

$f(2) = ?$ $= 5$

$f(x^3) = ?$ $= x^6 + 1$

$f(x+2) = ?$ $= (x+2)^2 + 1$ $= x^2 + 4x + 5$

$f(-2x+3) = ?$ $= (-2x+3)^2 + 1$

$= 4x^2 - 12x + 10$

Compositions of Functions

f(x) = 3x and $g(x) = x^2$

Let's use f(x) as the input to g(x) $g(f(x)) = ?$

$$g(..) = (..)^2$$

1. Replace the 'x' with a set of parentheses.

$$g(3x) = (3x)^2$$

2. Put the input value "3x' into the parentheses.

$$g(f(x)) = 9x^2$$

3. Find the output value.

<u>Compositions</u> of Functions

f(x) = 2x + 3 and $g(x) = x^2$

f(g(x)) = ?

$$f(..) = 2(..) + 3$$

$$f(x^2) = 2(x^2) + 3$$

$$f(g(x)) = 2x^2 + 3$$

1. The input value to f(x) is g(x).
2. Replace the ‘x’ in f(x) with a set of parentheses.
3. Put the input value (g(x)) into the parentheses.
4. Find the output value.

Composition of Functions

f(x) = 2x + 1 g(x) = 3x + 2 h(x) = x + 5

$f(g(x)) = ?$ $= 2(\qquad) + 1 = 2(3x + 2) + 1$

$h(g(x)) = ?$ $= (\qquad) + 5 = (3x + 2) + 5$

$h(f(x)) = ?$ $= (\qquad) + 5 = (2x + 1) + 5$

$g(h(x)) = ?$ $= 3(\qquad) + 2 = 3(x + 5) + 2$

$f(f(x)) = ?$ $= 2(\qquad) + 1$ $= 2(2x + 1) + 1$

Compositions of functions. $f(x) = 3x$ $g(x) = x^2$

$f(g(4))$

4

$g(\ ) = (\ )^2$

$g(4) = (4)^2$

16

$f(\ ) = 3(\ )$

$f(16) = 3(16)$

48

$f(g(x)) = 3(g(x)) = 3x^2$

$f(g(4)) = 3(g(4)) = 3(4)^2$ **= 48**

$$g(x) = x^3 - 1$$

$$f(x) = 5x + 2$$

$$g(f(2)) = ?$$

$$f(g(-1)) = ?$$

$$f(f(3)) = ?$$

Vocabulary

Inverse Relation: A relation that interchanges the input and output values of the original relation.

Relation: (-2, 5), (5, 6), (-2, 6), (7, 6)

Inverse Relation: (5, -2), (6, 5), (6, -2), (6, 7)

Find the inverse of: f(x) = 4x + 2 Exchange ‘x’ and ‘y’

$$x = 4y + 2$$

This IS the inverse function
(written as: “x as a function of y”)

Rewrite it so that it is written as: “y as a function of x”)

subtract ‘2’ (left and right)

$$x - 2 = 4y$$

Divide (all of the) left and right by 4

$$\frac{x}{4} - \frac{2}{4} = \frac{4y}{4}$$

Reduce the fractions

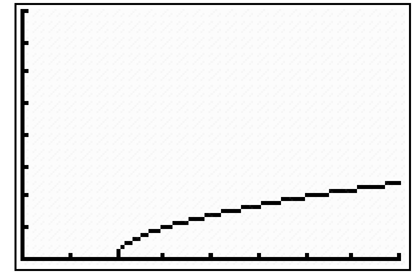
$$\frac{x}{4} - \frac{1}{2} = y$$

Rearrange into “slope intercept form”

$$y = \frac{x}{4} - \frac{1}{2}$$

This is the inverse of: y = 4x + 2

$$f(x) = \sqrt{x-2} \qquad f^{-1}(x) = ?$$



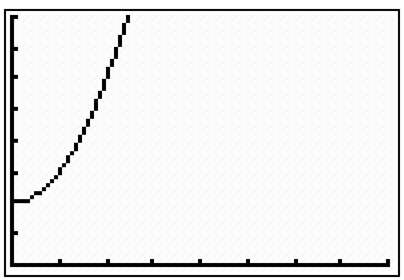
Exchange ‘x’ and ‘y’ in the original relation.

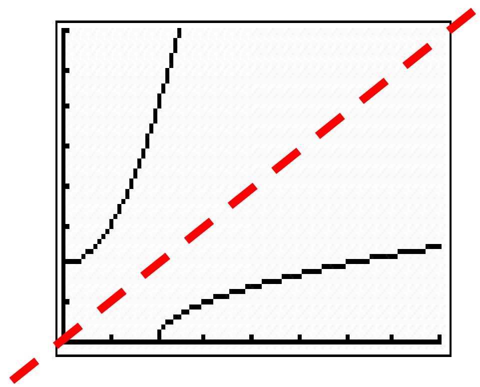
$$x = \sqrt{y-2}$$

This IS the inverse function (written as: “x as a function of y”)

Rewrite it so that it is written as: “y as a function of x”)

$$(x)^2 = \left(\sqrt{y-2}\right)^2 \qquad x^2 = y - 2 \qquad y = x^2 + 2$$





Why do we only graph the right side of the parabola?

Since the x-y pairs of SQRT are all positive, then the x-y pairs of the inverse of the SQRT (square function) will be positive.

Tell me everything you know about *inverse relations.*

When both are graphed on the same axes they are:

reflections of each other across the line $y = x$

If both are given as x-y pairs:

the x-y values are reversed.

If one relation "fails" the horizontal line test:

Its inverse is not a function.

When the two relations are "composed" the output is:

'x'

Function Notation: "the inverse of f(x)"

$f(x)$ $\qquad$ $f^{-1}(x)$

$f^{-1}(x)$ means "the inverse of f(x)"

Do not confuse this notation with the negative exponent property:

$$2^{-1} = \frac{1}{2^1}$$

Negative exponent on a number or an expression means the reciprocal of the number.

The inverse of a function means "exchange 'x' and 'y' (then solve for 'y')."

f(x) = x – 2 $f^{-1}(x) = ?$ Exchange ‘x’ and ‘y’

There’s no “y” !!! Remember: y = f(x) → y = x – 2

$$x = y - 2$$

This IS the inverse function
(written as: “x as a function of y”)

Rewrite it so that it is written as: “y as a function of x”)

Add ‘2’ (left and right)

$$x + 2 = y$$

Rearrange into “slope intercept form”

$$y = x + 2$$

This is the inverse of: y = x - 2

h(x): (-3, 5), (5, 6), (-2, 7), (8, 5)

h(5) = ? = 6

h(-2) = ? = 7

$h^{-1}(7) = ?$ = -2

You must determine the x-y values of the inverse relation before you can determine the output value.

$h^{-1}(x):$ (5, -3), (6, 5), (7, -2), (5, 8)

Is the inverse of h(x) a function? no

If the inverse of h(x) is not a function, we say that h(x) does NOT have an inverse (that is a function).

h(x): (-3, 5), (5, 6), (-2, 7), (9, 8)

$h^{-1}(x):$ (5, -3), (6, 5), (7, -2), (8, 9)

If the inverse of h(x) is not a function, we say that h(x) does NOT have an inverse (that is a function).

$h^{-1}(5) = ?$ = -3

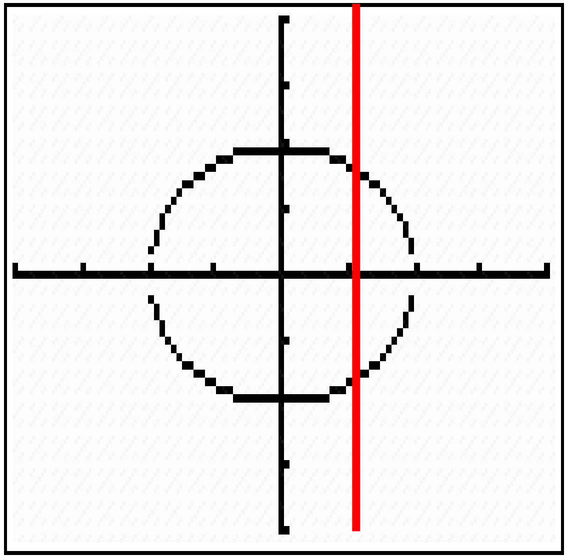
$h(h^{-1}(5)) = ? = h(-3)$ = 5

The input to "h" is the output of $h^{-1}(5)$

$h(h^{-1}(7)) = ? = h(-2)$ = 7

The input to "h" is the output of $h^{-1}(7)$

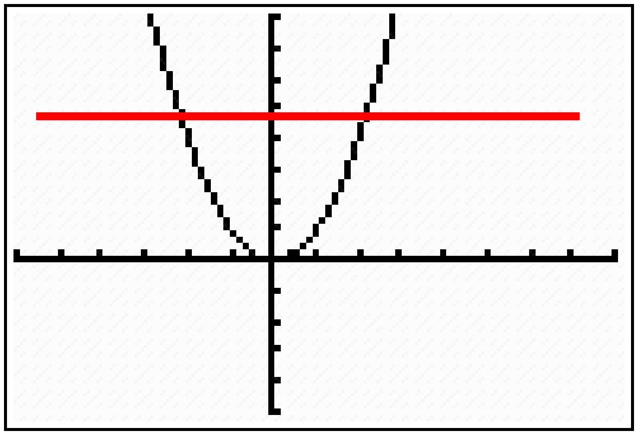
If you have the graph of a relation; how can you tell if the relation is a function?



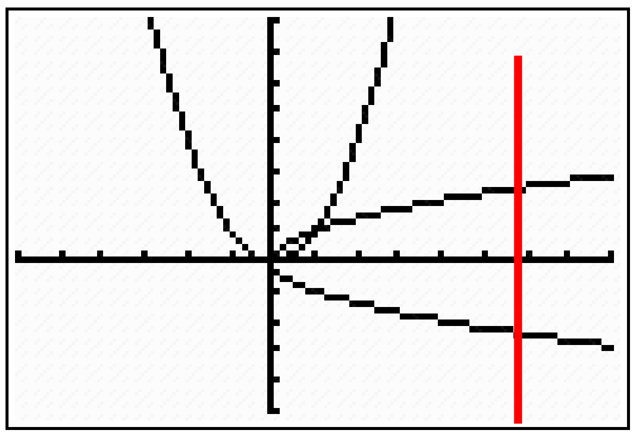
Vertical Line Test if the line intersects the graph more than once, it is NOT a function.

If you have a graph; <u>how can you tell</u> if the inverse of the graphed function is also a function?

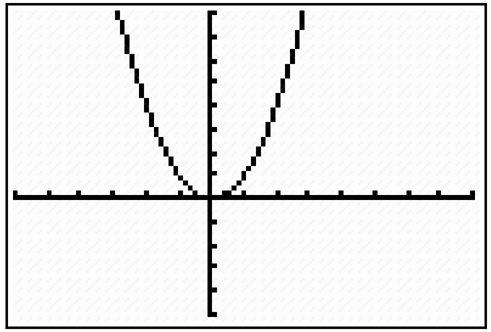
$$f(x) = x^2$$



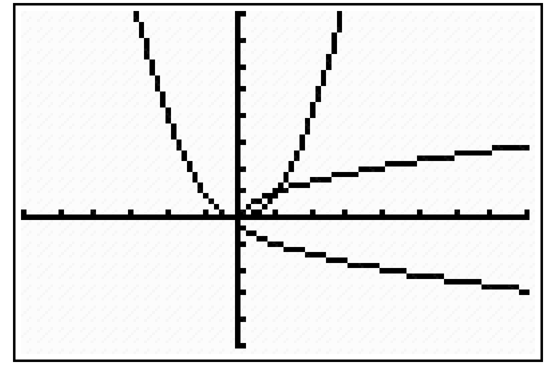
<u>Horizontal Line Test</u>: if the line intersects the graph more than once, then the
<u>Inverse</u> of the function is <u>NOT</u> a function.



$$f(x) = x^2$$



$$f^{-1}(x) = ?$$



$$x = y^2 \rightarrow y = ??$$

$$y = \pm\sqrt{x}$$

$$x = (??)^2$$

$$x = (\sqrt{x})^2 \qquad x = (-\sqrt{x})^2$$

Is the inverse of f(x) a function?

Function A: heating by 10 degrees

What is the inverse of this function?

"Cooling something down by 10 degrees"

Function B: cooling by 10 degrees

The temperature of a bowl of soup is 100 degrees.

The temperature of a bowl of soup is 100 degrees. Apply function A then function B (in sequence) to the bowl of soup. What is the final temperature of the soup?

Temperature = 100 + 10 – 10 = 100

Composition of *inverse functions*

Function A and Function B are inverses of each other.

Function A: *"does something"* to the input.

Function B: *"undoes whatever function A* did to the input.

2

Function A "does something" to input value 2

Function B "undoes (whatever A did) to the input value 2

What is the output of function B? **2**

If you compose a function with its inverse, the output will be the same as the input.

$$f(x) = \sqrt{x}$$

$$g(x) = x^2$$

"A Function "undoes" whatever its inverse "did" to the input value".

$$f(g(x)) = \sqrt{x^2}$$

$$f(g(x)) = x$$

A salesman receives a weekly salary of $500 plus a 2% commission on his gross sales.

Determine a function, f, for a sales representative's weekly gross pay, P (before taxes), as a function of S, his weekly sales in dollars, P = f(S).

$$P = f(S) = 500 + 0.02s$$

Is this correct?

NO. What’s wrong?

$$P = f(S) = 500 + 0.02S$$

Is this correct?

YES. The silly MathXLforSchool.com program is case sensitive!